

In transmitting continuous signal more power is consumed but if we transmit it in the form of samples then it consumes less power.



Types of signal

→ Continuous and discrete signal

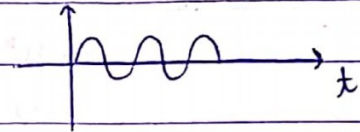
$f \rightarrow (t)$

continuous in nature

Sinusoidal

$f(t)$ or

$x(t)$

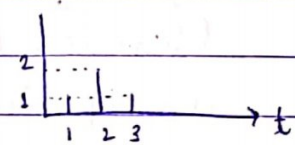


value on particular instant of time

$x(nT)$ or $x(n)$

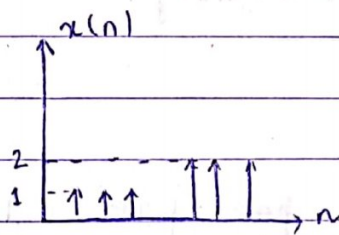
↳ integer

$x(n)$



Representation of discrete signal

Graphical representation



Mathematical representation

$$x(n) = \begin{cases} 1 & 1 \leq n \leq 3 \\ 2 & 4 \leq n \leq 6 \end{cases}$$

Tabular representation

n	1	2	3	4	5	6
$x(n)$	1	1	1	2	2	2

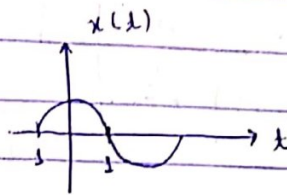
→ Even and odd signal

(Symmetrical and Asymmetrical signal)

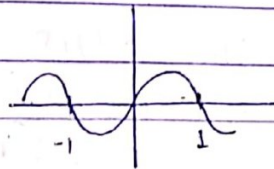
Signals which are symmetric on the time axis are even signal. Example - cosine

Signals which are asymmetric on the time axis are odd signal. Example - sine

$$x(-t) = x(t)$$



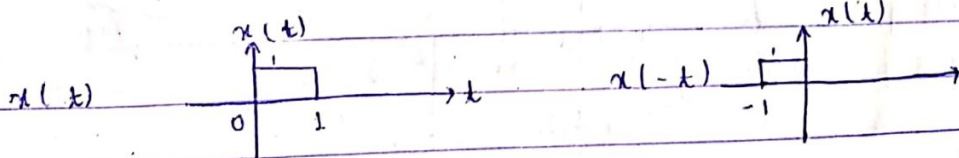
$$x(-t) = -x(t)$$



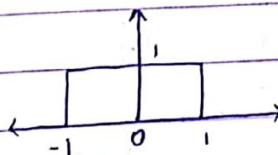
Ques: Prove it

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$$

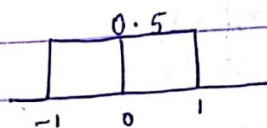
$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$



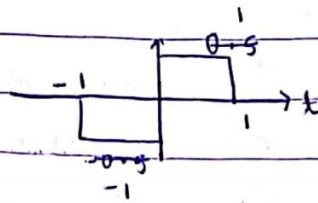
$$x(t) + x(-t)$$



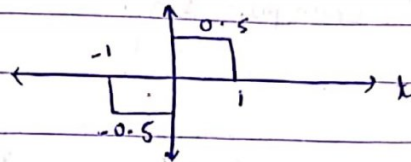
$$x_e(t)$$



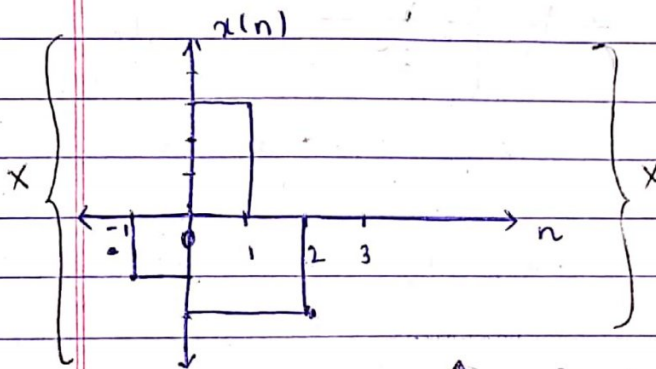
$$x(t) - x(-t)$$



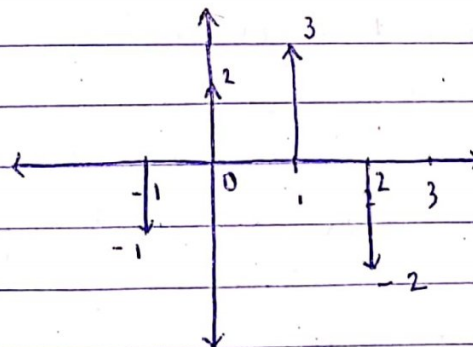
$$x_0(t)$$



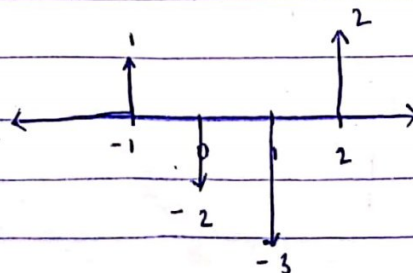
Ques: $x[n] =$ $-1, 2, 3, -2$
 $n = -1 \quad \uparrow \quad n = 1 \quad n = 2$
 $n = 0$

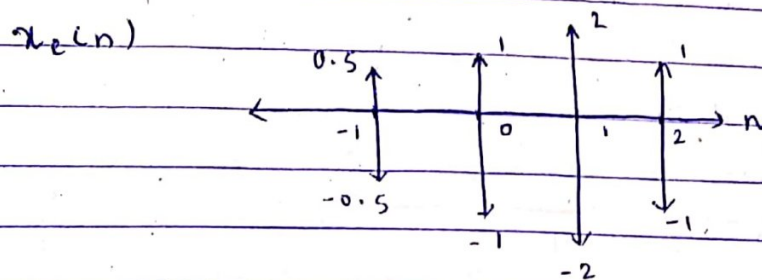
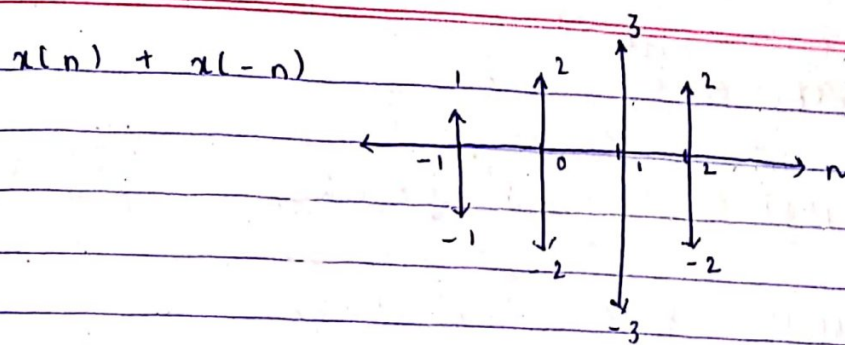


$$x[n]$$



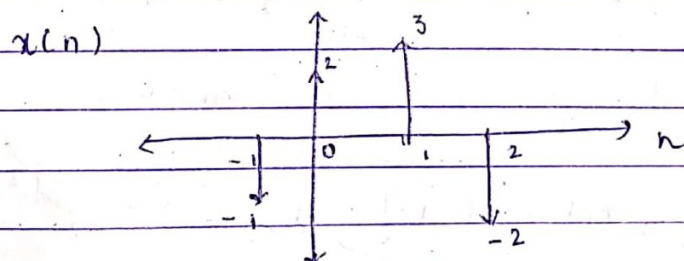
$$x(-n)$$





Ques: $x(n) = \left\{ \begin{matrix} -1 & 2 & 3 & -2 \\ n=-1 & \uparrow & n=1 & n=2 \end{matrix} \right\}$

$n=0$



$x(-n) = \left\{ \begin{matrix} -2 & 3 & 2 & -1 \\ n=-2 & n=-1 & \uparrow & n=1 \end{matrix} \right\}$

$n=0$

$x_e(n) = \frac{1}{2} \{ 0-2, -1+3, 2+2, 3-1, 0-2+0 \}$

$= \frac{1}{2} \{ -2, 2, 4, 2, -2 \} = \{ -1, 1, 2, 1, -1 \}$

$x_o(n) = \frac{1}{2} \{ 2, -1-0, 2-2, 3+1, -2-0 \}$

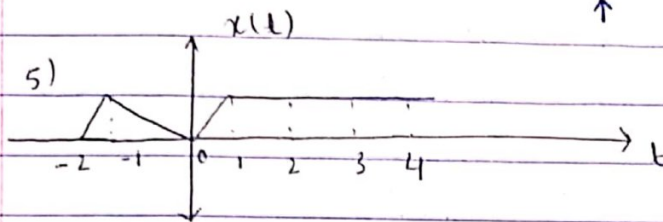
$= \frac{1}{2} \{ 2, -1, 0, 4, -2 \} = \{ 1, -0.5, 0, 2, -1 \}$

Ques: 1) $x(t) = e^{j2t}$

2) $x(t) = (1 + t^2 + t^3) \cos^2 10t$

3) $x(n) = \{-3, 1, 2, -4, 2\}$

4) $x(n) = \{5, 4, 3, 2, 1\}$



1) $x(-t) = e^{-j2t}$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)] = \frac{1}{2} [e^{j2t} + e^{-j2t}]$$

$$= \cos 2t$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)] = \frac{1}{2} [e^{j2t} - e^{-j2t}]$$

$$= \sin 2t$$

2) $x(-t) = (1 + t^2 - t^3) \cos^2(-10t)$

$$= (1 + t^2 - t^3) \cos^2 10t$$

$$x_e(t) = \frac{1}{2} [(1 + t^2 + t^3) \cos^2 10t + (1 + t^2 - t^3) \cos^2 10t]$$

$$= \frac{(1 + t^2 + t^3) \cos^2 10t + (1 + t^2 - t^3) \cos^2 10t}{2}$$

$$x_o(t) = \frac{1}{2} [(1 + t^2 + t^3) \cos^2 10t - (1 + t^2 - t^3) \cos^2 10t]$$

$$= \frac{(1 + t^2 + t^3) \cos^2 10t - (1 + t^2 - t^3) \cos^2 10t}{2}$$

$$3) \quad x(n) = \{ -3, 1, 2, -4, 2 \}$$

$\uparrow$ $n=1$ $n=2$ $n=3$ $n=4$
 $n=0$

$$x(-n) = \{ 2, -4, 2, 1, -3 \}$$

$n=-4$ $n=-3$ $n=-2$ $n=-1$ $\uparrow$
 $n=0$

$$x(n) + x(-n) = \{ 2, -4, 2, 1, -6, 1, 2, -4, 2 \}$$

$$x_e(n) = \frac{1}{2} [2, -4, 2, 1, -6, 1, 2, -4, 2]$$

$$x(n) - x(-n) = \{ -2, 4, -2, -1, 0, 1, 2, -4, 2 \}$$

$$x_o(n) = \frac{1}{2} [-2, 4, -2, -1, 0, 1, 2, -4, 2]$$

$$4) \quad x(n) = \{ 5, 4, 3, 2, 1 \}$$

$n=-4$ $n=-3$ $n=-2$ $n=-1$ $\uparrow$ $n=0$

$$x(-n) = \{ 1, 2, 3, 4, 5 \}$$

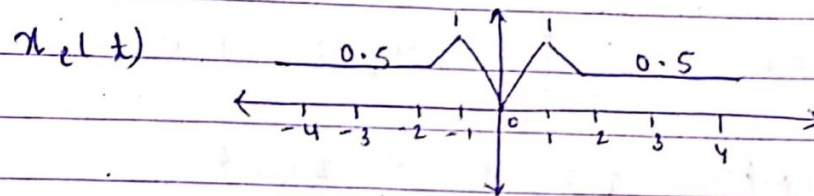
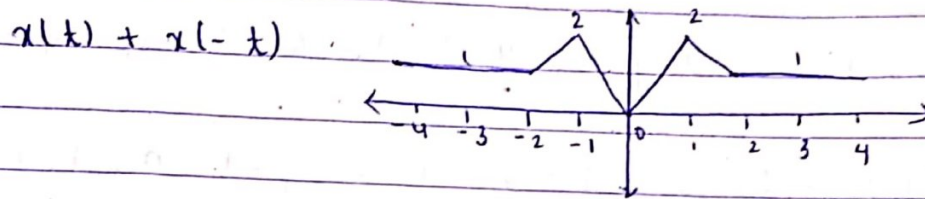
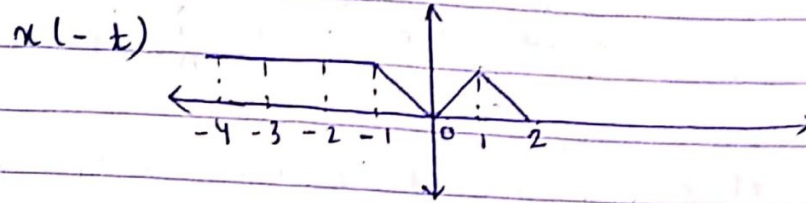
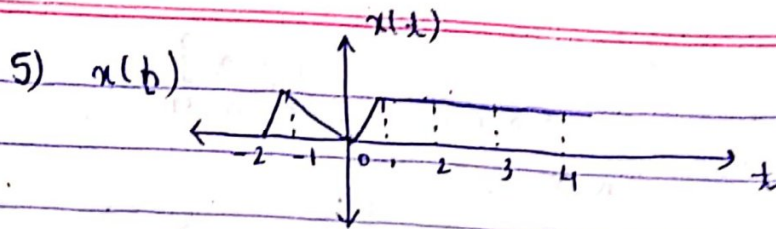
$\uparrow$ $n=1$ $n=2$ $n=3$ $n=4$
 $n=0$

$$x(n) + x(-n) = \{ 5, 4, 3, 2, 2, 2, 3, 4, 5 \}$$

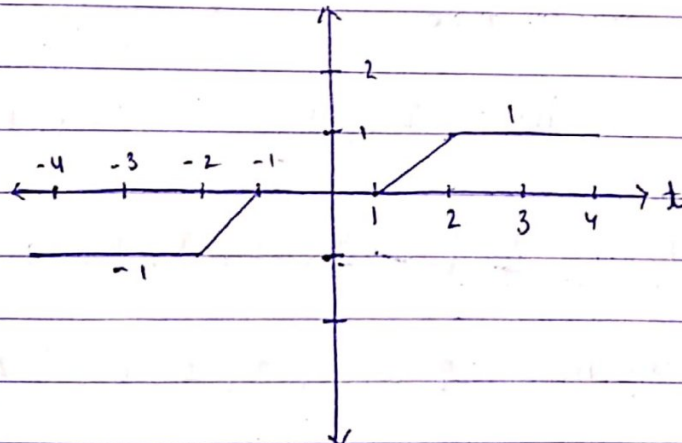
$$x_e(n) = \frac{1}{2} [5, 4, 3, 2, 2, 2, 3, 4, 5]$$

$$x(n) - x(-n) = \{ 5, 4, 3, 2, 0, -2, -3, -4, -5 \}$$

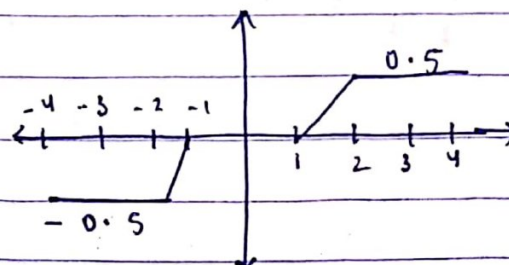
$$x_o(n) = \frac{1}{2} [5, 4, 3, 2, 0, -2, -3, -4, -5]$$



$x_o(t) = x(t) - x(-t)$



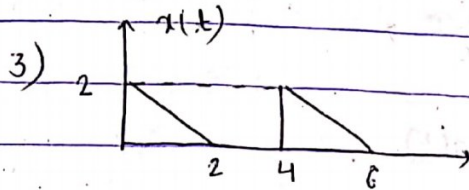
$x_o(t)$



H.W

$$1) x(t) = \sin 2t + \sin 2t \cos 2t + \cos 2t$$

$$2) x(n) = \{-2, 5, 1, -3\}$$



$$4) \sin(2t + \frac{\pi}{2})$$

$$1) x(t) = \sin 2t + \sin 2t \cos 2t + \cos 2t$$

$$x(-t) = -\sin 2t - \sin 2t \cos 2t + \cos 2t$$

$$x(t) + x(-t) = 2\cos 2t$$

$$x_e(t) = \cos 2t$$

$$x(t) - x(-t) = 2(\sin 2t + \sin 2t \cos 2t)$$

$$x_o(t) = \sin 2t + \sin 2t \cos 2t$$

$$2) x(n) = \{-2, 5, 1, -3\}$$

$\uparrow$ $n=1$ $n=2$ $n=3$
 $n=0$

$$x(-n) = \{-3, 1, 5, -2\}$$

$\uparrow$ $n=-2$ $n=-1$ $\uparrow n=0$
 $n=-3$

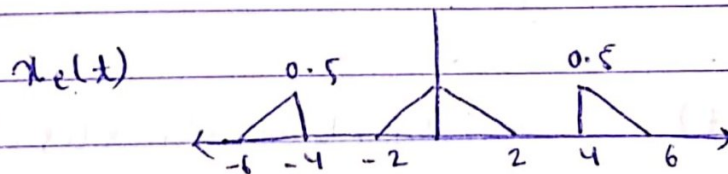
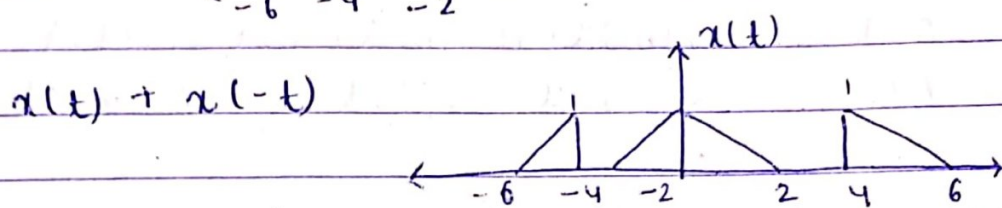
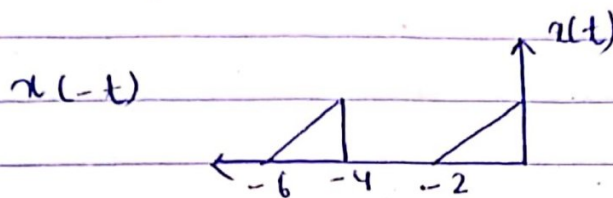
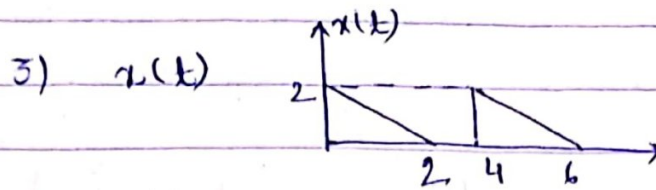
$$x(n) + x(-n) = \{-3, 5, 1, -2\}$$

$$x(n) + x(-n) = \{-3, 1, 5, -4, 5, 1, -3\}$$

$$x_e(n) = \frac{1}{2} [-3, 1, 5, -4, 5, 1, -3]$$

$$x(n) - x(-n) = \{3, -1, -5, 0, 5, -1, -3\}$$

$$x_0(n) = \frac{1}{2} [3, -1, -5, 0, 5, -1, -3]$$

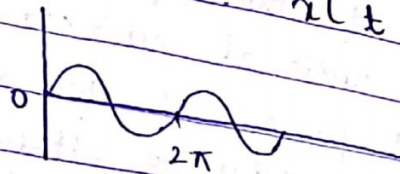


$$x(t) - x(-t)$$

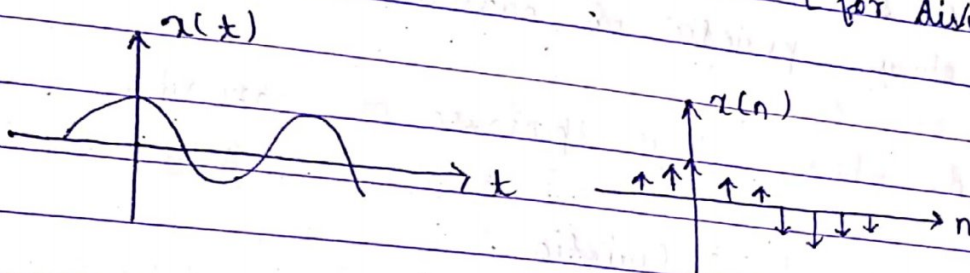
Periodic and Non Periodic Signal

A signal which repeats its pattern after a regular interval of time is known as periodic signal and that interval of time is known as fundamental period.

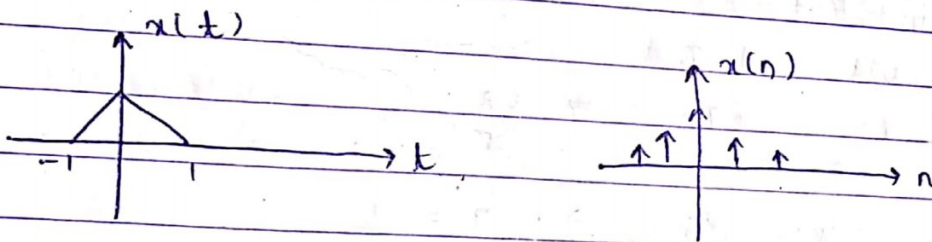
Property: $x(t) = x(t \pm mT)$ [for continuous]



$x(n) = x(n \pm N)$ [for discrete]



Periodic



Aperiodic

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$N = \frac{2\pi}{\omega}$$

} Fundamental period

- * Sinusoidal function is periodic in nature
- * complex exponential signal is always periodic in nature.

Property of the periodic and Aperiodic signal

1. The sum of two continuous periodic signal may or may not be periodic.
2. The sum of two periodic signals is periodic only if the ratio of their fundamental period i.e. T_1/T_2 is a rational number.
3. The fundamental period of the summation of the signals is the LCM of their individual fundamental period i.e. T_1 and T_2 .
4. The sum of two discrete ^{time} periodic sequence is always periodic in nature.

Ques: Find whether it is periodic or aperiodic

1) $\sin 12\pi t$: Periodic

$$x(t) = x(t + T)$$

$$\sin 12\pi t = \sin(12\pi(t+T))$$

$$\omega t = 12\pi t$$

$$\omega = 2\pi f \Rightarrow \frac{2\pi}{T}$$

$$12\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{6}$$

2) $e^{j4\pi t}$

$$x(t) = x(t + T)$$

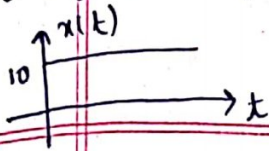
$$e^{j4\pi t} = e^{j4\pi t} \cdot e^{j4\pi T}$$

it is possible if $e^{j4\pi T} = 1$

$$\cos 4\pi tT + j \sin 4\pi tT = 1$$

$$\text{compare, } \cos 4\pi T = \cos 2\pi = 1$$

DC signal is periodic



$$e^{a+ib} = e^a \cdot e^{ib} \\ e^{jx} = \cos x + j \sin x$$

$$\omega = 4\pi$$

$$\frac{2\pi}{T} = 4\pi$$

$$T = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$3) \cos 2t + \sin \sqrt{3}t = x(t)$$

$$x_1(t)$$

↓

$$T_1$$

$$x_2(t)$$

↓

$$T_2$$

$$\omega_1 = 2$$

$$\frac{2\pi}{T_1} = 2 \Rightarrow T_1 = \pi$$

$$\omega_2 = \sqrt{3}$$

$$\frac{2\pi}{T_2} = \sqrt{3} \Rightarrow T_2 = \frac{2\pi}{\sqrt{3}}$$

$$\frac{T_1}{T_2} = \frac{\pi}{\frac{2\pi}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$$

The ratio is not rational so $x(t)$ is not periodic.

Note:

→ continuous signal

non-repeating and non-terminating are irrational number.

$$4) 3 \sin 200\pi t + 4 \cos 100t$$

$$200\pi = \omega$$

$$\frac{2\pi}{T_1} = 200\pi \Rightarrow T_1 = \frac{1}{100}$$

$$100 = \omega$$

$$\frac{2\pi}{T_2} = 100 \Rightarrow T_2 = \frac{\pi}{50}$$

$$\frac{T_1}{T_2} = \frac{1}{2\pi}$$

Ratio is irrational so signal is aperiodic

$$x \left\{ \begin{array}{l} T = \text{LCM}(T_1, T_2) \\ T = \frac{\pi}{50} \end{array} \right\} x$$

H.W

$$1) \sin(10t + 1) - 2 \cos(5t - 2)$$

$$2) x(t) = e^{j\pi t}$$

$$3) 6 e^{j(4t + \pi/3)} + 8 e^{j(3\pi t + \pi/4)}$$

$$4) 2 + \cos 2\pi t$$

$$5) \cos 4\pi$$

$$1) x_1(t) = \sin(10t + 1)$$

$$x_2(t) = 2 \cos(5t - 2)$$

$$\omega_1 = 10$$

$$\omega_2 = 5$$

$$\frac{2\pi}{T_1} = 10$$

$$\frac{2\pi}{T_2} = 5$$

$$T_1 = \frac{\pi}{5}$$

$$T_2 = \frac{2\pi}{5}$$

$$\frac{T_1}{T_2} = \frac{1}{2}$$

Ratio is rational, so signal is periodic

functional period $T_0 = 2T_1 = \frac{2\pi}{5}$

2) $x(t) = j e^{j6t}$

$x(t)$ is complex exponential function so it is periodic

$$\omega = 6$$

$$\frac{2\pi}{T} = 6 \Rightarrow T = \frac{\pi}{3}$$

3) $x_1(t) = 6 e^{j(4t + \pi/3)}$, $x_2(t) = 8 e^{j(3\pi t + \pi/4)}$

$$\omega_1 = 4$$

$$\frac{2\pi}{T_1} = 4$$

$$T_1 = \frac{\pi}{2}$$

$$\omega_2 = 3\pi$$

$$\frac{2\pi}{T_2} = 3\pi$$

$$T_2 = \frac{2}{3}$$

$$\frac{T_1}{T_2} = \frac{3\pi}{4}$$

Ratio is irrational so signal is aperiodic.

4)

Energy and Power signal

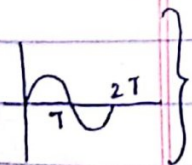
Energy signal have finite energy and zero average power. Aperiodic signals come into the category of energy signal.

Power signals have finite value of average power and infinite energy. Periodic signals come into the category of power signal.

Both signals are mutually exclusive that is a signal can't be both energy signal as well as power signal.

For continuous time

$$\text{avg. power } P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$



$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

sine and cosine both comes under sinusoidal signal

Ques: Prove that : $x_e(t) = \frac{1}{2} [x(t) + x(-t)]$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$

$$x(t) = x_e(t) + x_o(t) \quad \text{--- (1)}$$

$$x(-t) = x_e(t) - x_o(t) \quad \text{--- (2)}$$

$$\text{eq}^n(1) + \text{eq}^n(2)$$

$$x(t) + x(-t) = 2x_e(t)$$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$$

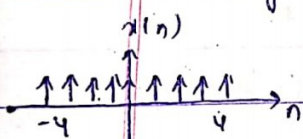
$$\text{eq}^n(1) - \text{eq}^n(2)$$

$$x(t) - x(-t) = 2x_o(t)$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$

For discrete Time

$$\text{avg. Power } P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$



$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

Ques: find out the energy and power of a sinusoidal signal in continuous time

$$x(t) = A \sin \omega t$$

* all sinusoidal signals are power signal
 $(\sin^2 x = (1 - \cos 2x) \frac{1}{2})$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T A^2 \sin^2 \omega t dt$$

$$P = \lim_{T \rightarrow \infty} \frac{A^2}{2T} \int_{-T}^T \frac{1 - \cos 2\omega t}{2} dt$$

$$P = \lim_{T \rightarrow \infty} \frac{A^2}{2T} \left[\left[\frac{t}{2} \right]_{-T}^T - \int_{-T}^T \frac{\cos 2\omega t}{2} dt \right]$$

(integration of cosine function over a cycle is zero)

$$\therefore \int_{-T}^T \frac{\cos 2\omega t}{2} dt = 0$$

$$P = \lim_{T \rightarrow \infty} \frac{A^2}{2T} \times \frac{2T}{2} = \frac{A^2}{2}$$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T A^2 \sin^2 \omega t dt = \int_{-\infty}^{\infty} A^2 \sin^2 \omega t dt$$

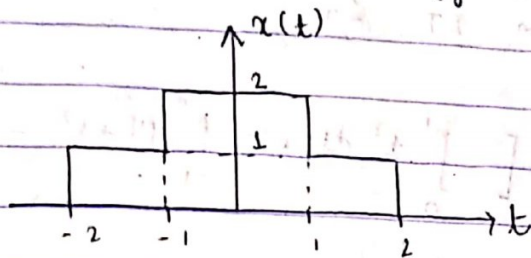
$$E = A^2 \int_{-\infty}^{\infty} \frac{1 - \cos 2\omega t}{2} dt$$

$$E = A^2 \left[\int_{-\infty}^{\infty} \frac{1}{2} dt - \int_{-\infty}^{\infty} \frac{\cos 2\omega t}{2} dt \right]$$

$$E = A^2 \left[\frac{t}{2} \right]_{-\infty}^{\infty} = \infty$$

fundamental period matlab jis time bad signal repeat ho
 uha hai.

Ques: Find whether it is energy signal or power signal



$$x(t) = \begin{cases} 1 & -2 \leq t \leq -1 \\ 2 & -1 \leq t \leq 1 \\ 1 & 1 \leq t \leq 2 \end{cases}$$

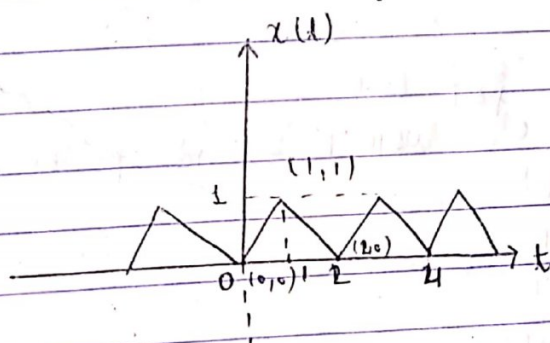
Since the signal is aperiodic so it is energy signal.

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$$

$$E = \int_{-2}^{-1} 1 dt + \int_{-1}^1 4 dt + \int_1^2 1 dt$$

$$E = 1 + 8 + 1 = 10 \text{ joules}$$

Ques: Find whether it is energy signal or power signal



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

fundamental period, $T = 2$

$$x(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 2 - t & 1 \leq t \leq 2 \end{cases}$$

Since the signal is periodic so it is power signal.

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\int_0^1 t^2 dt + \int_1^2 (4+t^2-4t) dt \right]$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\frac{1}{3} + [4t]_1^2 + \left[\frac{t^3}{3} \right]_1^2 - [2t^2]_1^2 \right]$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2 \times 2} \left[\frac{1}{3} + 4 + \frac{7}{3} - 6 \right]$$

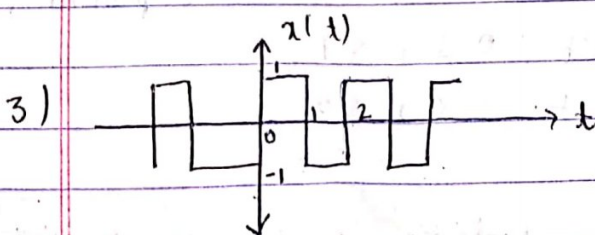
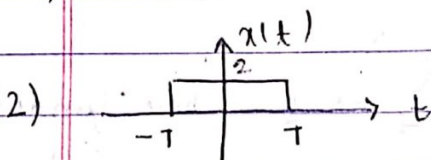
$$P = \frac{1}{4} \left[\frac{8}{3} - 2 \right] = \frac{1}{4} \times \frac{8-6}{3} = \frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$$

$$P = \frac{1}{4} \left[\frac{8-6}{3} \right]$$

$$= \frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$$

H.W

- 1) $x(t) = e^{j2t}$ w.s $10T$: Find power and RMS



1) $x(t) = e^{j2t} \cos 10t = (\cos 2t + j \sin 2t) \cos 10t$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T (\cos 2t + j \sin 2t) \cos 10t dt$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \cos 2t \cos 10t + j \sin 2t \cos 10t dt$$

$$x(t) = \cos 2t \cos 10t + j \sin 2t \cos 10t$$

$$x(t) = \frac{\cos 12t + \cos 8t}{2} + j \frac{(\sin 12t - \sin 8t)}{2}$$

$$P = \frac{(\frac{1}{2})^2 + (\frac{1}{2})^2}{2} + \frac{(\frac{1}{2})^2}{2} + \frac{(\frac{1}{2})^2}{2} \quad \boxed{P = \frac{A^2}{2}}$$

$$P = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

$$RMS = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

2) since the signal is aperiodic so it is energy signal

$$x(t) = \begin{cases} 2 & -T < t < T \end{cases}$$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \int_{-T}^T 2^2 dt$$

$$E = \lim_{T \rightarrow \infty} 4[2T] = \lim_{T \rightarrow \infty} 8T = \infty$$

3) The signal is periodic so it is power signal

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\int_0^1 1^2 dt + \int_1^2 (-1)^2 dt \right]$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2 \times 2} \left[\left[\frac{t}{2} \right]_0^1 + \left[\frac{t}{2} \right]_1^2 \right]$$

$$P = \frac{1}{4} \left[\frac{1}{2} + 1 - \frac{1}{2} \right] = \frac{1}{4} \quad P = \frac{1}{4} [1 + 1] = \frac{2}{4} = \frac{1}{2}$$

Ques: $x(n) = \begin{cases} (1/2)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$

It is aperiodic signal so signal is energy signal

$$E = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x(n)|^2$$

$$E = \lim_{N \rightarrow \infty} \sum_{n=0}^{\infty} \left(\frac{1}{2} \right)^{2n} = \lim_{N \rightarrow \infty} \sum_{n=0}^{\infty} \left(\frac{1}{4} \right)^n$$

$$E = \left(\frac{1}{1 - 1/4} \right) = \frac{4}{3} \quad \left(1 + \frac{1}{4} + \left(\frac{1}{4} \right)^2 + \dots \right)$$

Ques: Check whether it is energy signal or power signal

$$x(n) = \begin{cases} n & n > 0 \\ 0 & n < 0 \end{cases}$$

$$E = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x(n)|^2$$

$$E = \lim_{N \rightarrow \infty} \sum_{n=0}^{\infty} (n)^2 = \lim_{N \rightarrow \infty} \sum_{n=0}^{\infty} n^2$$

$$E = \lim_{N \rightarrow \infty} \frac{n(n+1)(2n+1)}{6} = \infty$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} n^2 = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \frac{N(2N+1)(N+1)}{6}$$
$$P = \infty$$

Both power and energy of the signal are infinite
so this signal is neither energy signal nor power
signal.

Ques: $x(n) = \begin{cases} n^2 & 0 \leq n \leq 3 \\ 10-n & 4 \leq n \leq 7 \\ n & 7 \leq n \leq 9 \\ 0 & \text{otherwise} \end{cases}$

It is aperiodic signal so it is energy signal

$$E = \lim_{N \rightarrow \infty} \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \lim_{N \rightarrow \infty} \sum_{n=0}^3 n^4 + \sum_{n=4}^7 (10-n)^4 + \sum_{n=7}^9 n^2$$

$$E = \lim_{N \rightarrow \infty} 1 + 16 + 81 + 36 + 9 + 25 + 16 + 49 + 64 + 81$$

$$E = 370 \text{ joules}$$

Ques: $x(n) = e^{j(\frac{\pi}{3}n + \frac{\pi}{2})}$

$$x(n) = \cos\left(\frac{\pi}{3}n + \frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{3}n + \frac{\pi}{2}\right)$$

$$P = \frac{(1)^2}{2} + \frac{(1)^2}{2} = 1$$

$$E = \lim_{N \rightarrow \infty} \sum_{n=-\infty}^{\infty} (1)^2 = 2N+1 = \infty$$

Deterministic and Random signal

The signals which can be expressed by mathematical equation or in graphical form is known as deterministic signal.

Sinusoidal signal, exponential signal.

The signals which cannot be expressed by mathematical equation are known as random signals.

Random signals are analysed by the probability theory.

Example - Noise.

Causal and Non-causal signal

The signals whose value depends on the positive time are known as causal signals.

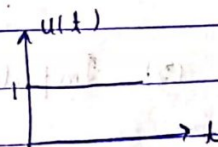
While the signals whose value depend on the negative time are known as non-causal signal or anti-causal signal.

Elementary Signal

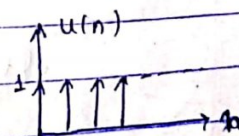
→ use to reconstruct complex signal

(1) unit step signal

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

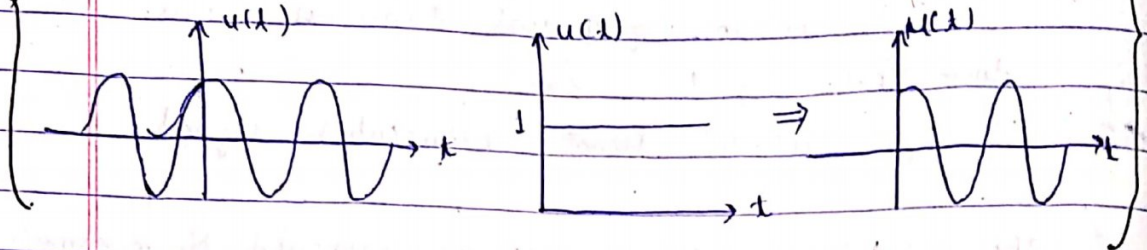


$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



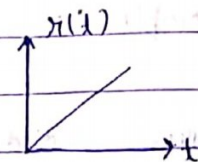
comp signal is used to increase the strength of the signal

if we multiply cosine signal with unit step signal then result -

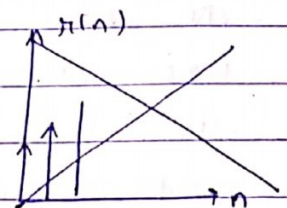
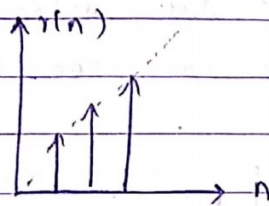


(2) ramp signal

$$r(t) = \begin{cases} t & t > 0 \\ 0 & t < 0 \end{cases}$$



$$r(n) = \begin{cases} n & n > 0 \\ 0 & n < 0 \end{cases}$$



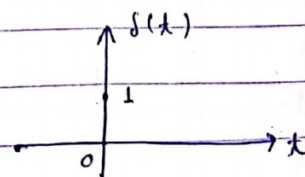
Relation between unit and ramp signal

$$r(t) = \int_{-\infty}^t u(t) dt \Rightarrow r(t) = t u(t)$$

(3) Impulse signal

→ area under curve is unity

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$



area under curve

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

properties

$$1) \int_{-\infty}^{\infty} x(t) \delta(t) dt = x(0) \int_{-\infty}^{\infty} \delta(t) dt = x(0)$$

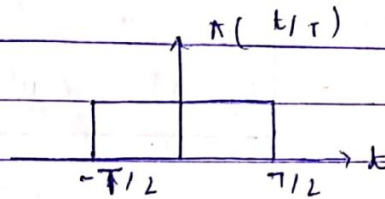
$$2) x(t) \delta(t) = x(0)$$

$$x(t) \delta(t - t_0) = x(t_0) \quad \left\{ \text{put } t = t_0 \right\}$$

(4) rectangular function

$$\pi\left(\frac{t}{T}\right) \begin{matrix} \nearrow \text{independent} \\ \text{variable} \end{matrix} = \begin{cases} 1, & |t| \leq T/2 \\ 0, & \text{otherwise} \end{cases}$$

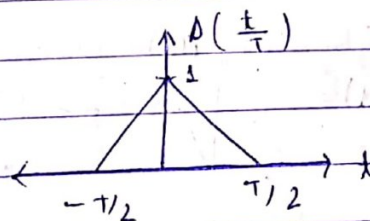
width of the function



(5) triangular function

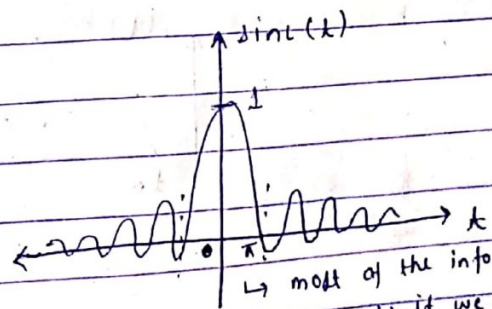
$$\Delta\left(\frac{t}{T}\right) \begin{matrix} \nearrow \text{independent} \\ \text{variable} \end{matrix} = \begin{cases} 1 - 2\left(\frac{|t|}{T}\right), & |t| \leq T/2 \\ 0, & \text{otherwise} \end{cases}$$

width of the function



(6) sinc function

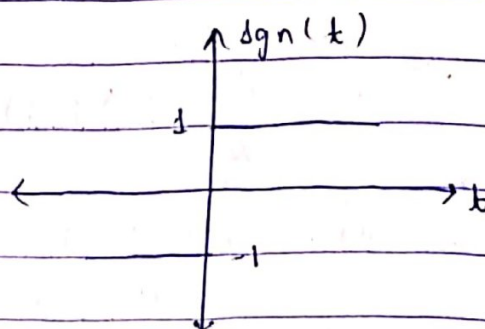
$$\text{sinc } t = \frac{\sin t}{t}$$



on this part only so if we apply filter then we may get full signal

(7) signum function

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \\ 0 & t = 0 \end{cases}$$



Relation between signum function and unit step signal

$$\text{sgn}(t) = u(t) - u(-t) - 1$$

$$\text{sgn}(t) = u(t) + u(t) - 1$$

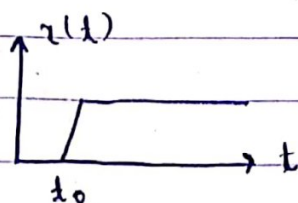
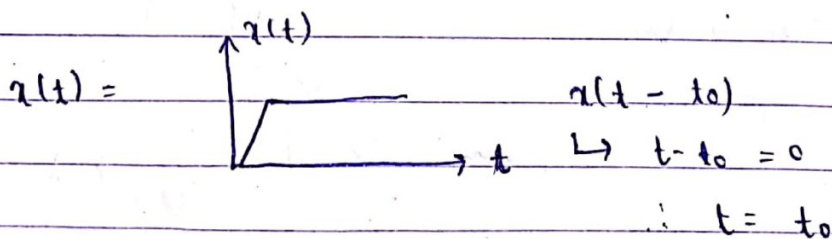
$$\boxed{\text{sgn}(t) = 2u(t) - 1}$$

Transformation of independent variables

(i) Shifting

$$x(t) \Rightarrow x(t - t_0) \Rightarrow \text{Delayed}$$

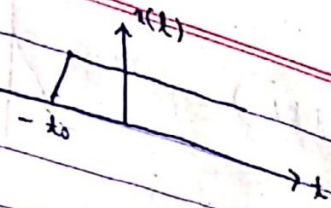
$$x(t + t_0) \Rightarrow \text{Advanced}$$



$$x(t + t_0)$$

$$\rightarrow t + t_0 = 0$$

$$\therefore t = -t_0$$



$$x(n)$$

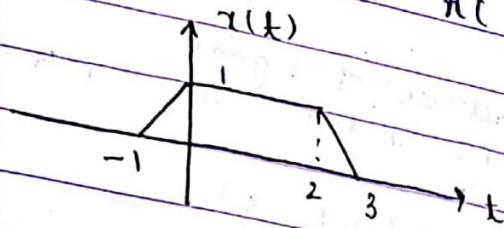
$\Rightarrow$

$$x(n + n_0)$$

$\Rightarrow$ advanced by n_0

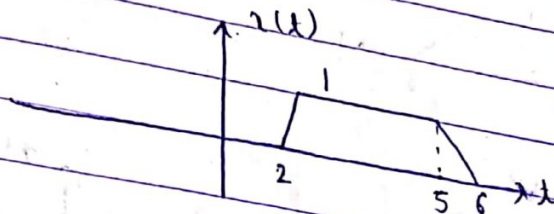
$$x(n - n_0) \Rightarrow \text{delayed by } n_0$$

Ques:

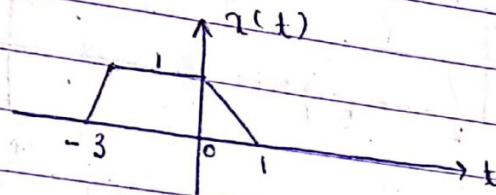


Draw $x(t-3)$ and $x(t+2)$

$$x(t-3)$$



$$x(t+2)$$



(ii) Time Scaling

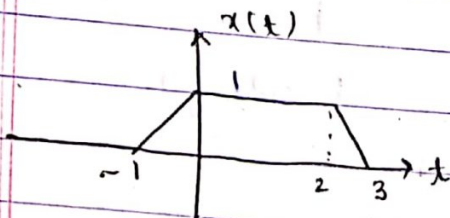
$$x(at)$$

If

$a > 1$, signal is compressed

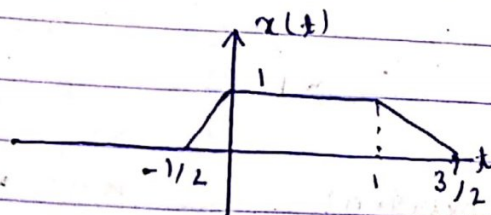
$a < 1$, signal is extended

Ques:



Draw $x(2t)$ and $x(t/3)$

$$x(2t)$$



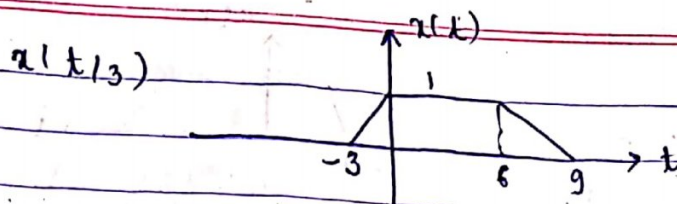
$$\text{put } t = 1/2$$

$$\frac{t}{a} \leftarrow t$$

$$a = 2$$

$\therefore a > 1$
compressed

shifting $\rightarrow$ scaling $\rightarrow$ inversion $\neq$ less mistake
 inversion $\rightarrow$ scaling $\rightarrow$ shifting



put $t = 3$

$$a = 1/3$$

$\therefore a < 1$
 extended

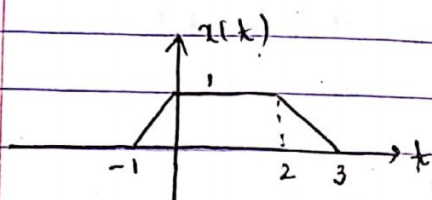
(iii) Time reversal

$x(t)$

$x(-t)$

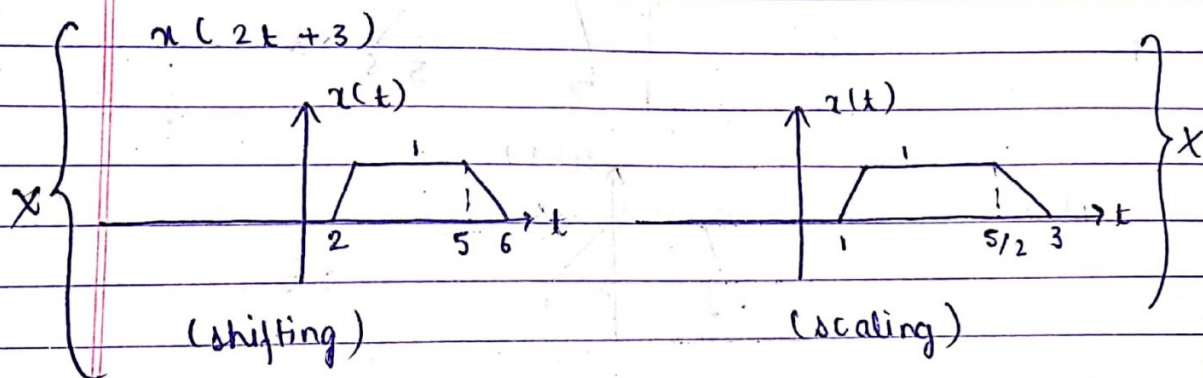
reversal of signal
 i.e. mirror image on time axis

Ques:

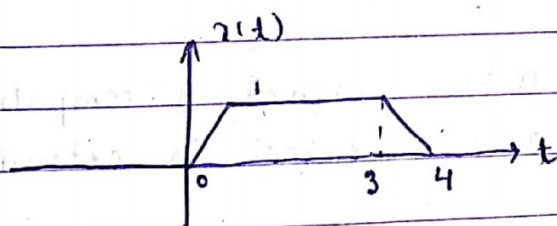


Draw $x(2t+3)$

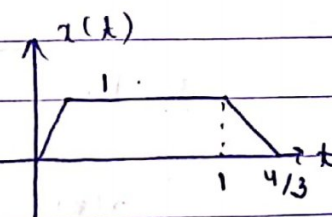
$x(-3t-1)$



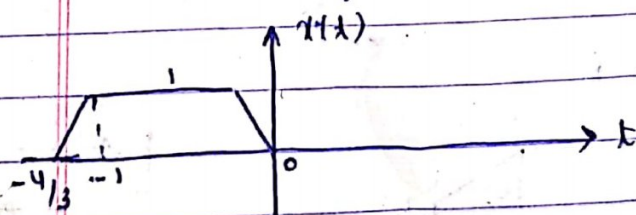
$x(-3t-1)$



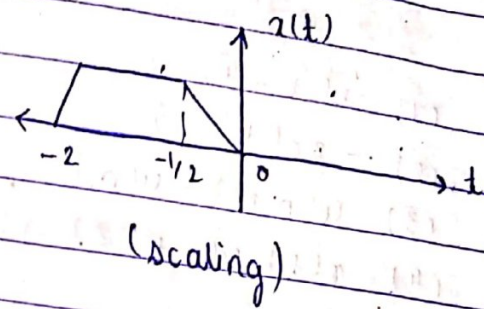
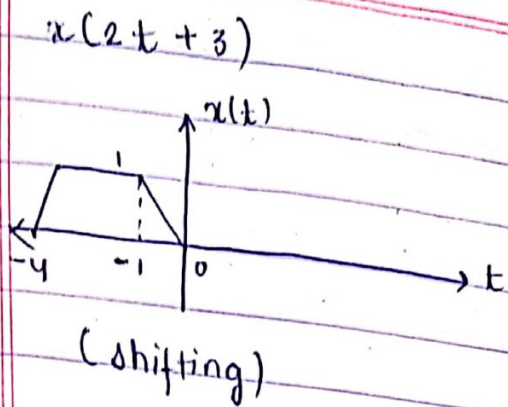
(shifting)



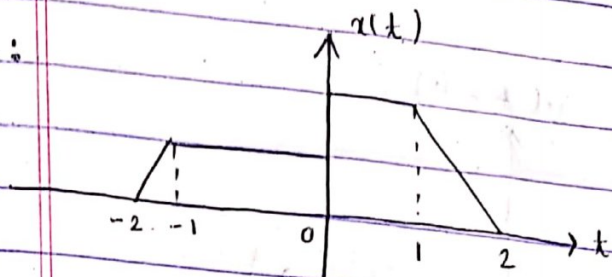
(scaling)



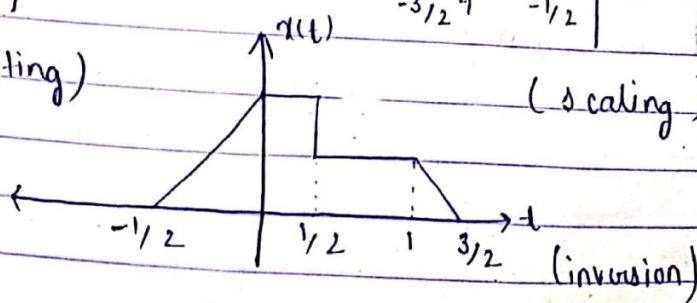
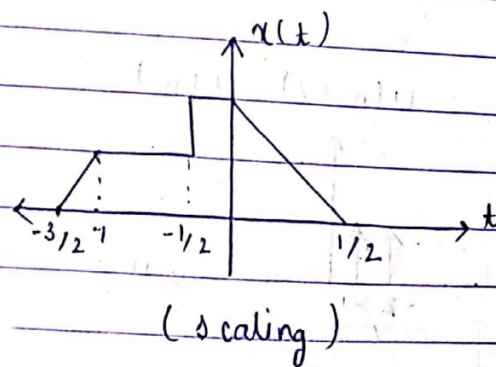
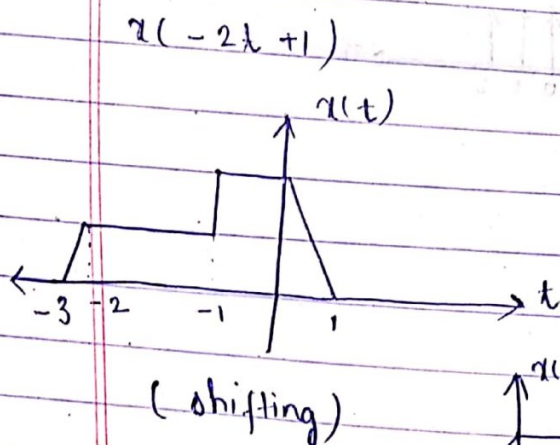
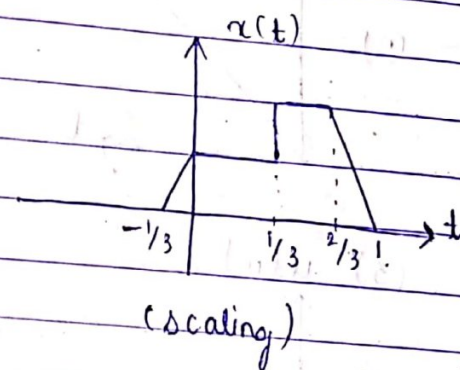
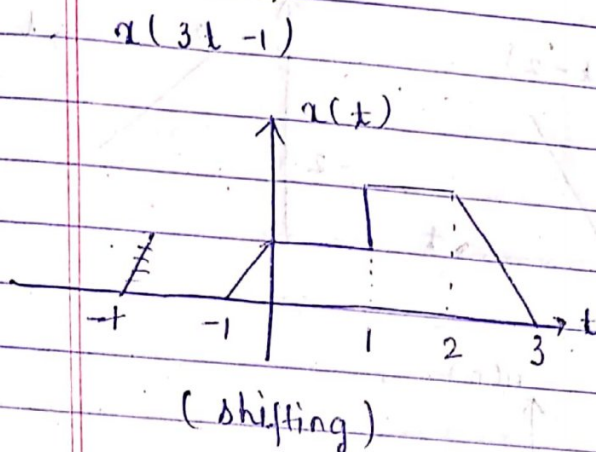
(inversion)



Ques:



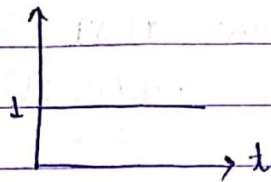
Draws $x(3t-1)$ and $x(1-2t)$



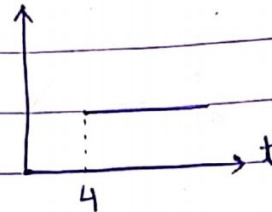
Ques: Sketch the following

- (1) $u(t-4)$
- (2) $-2r(t-2)$
- (3) $u(n+2) - u(n)$
- (4) $r(t)u(-t+3)$

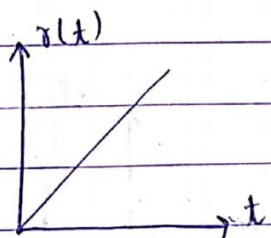
(1) $u(t)$



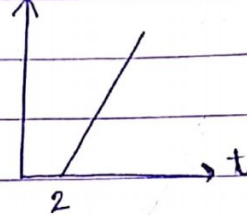
$u(t-4)$



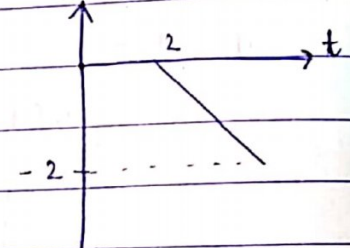
(2) $r(t)$



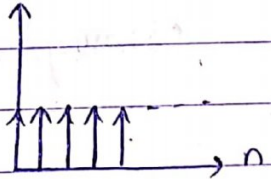
$r(t-2)$



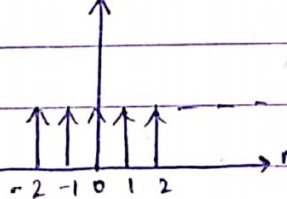
$-2r(t-2)$



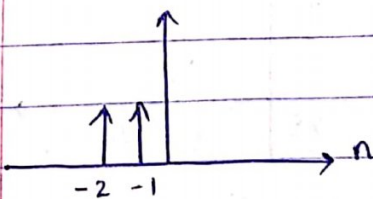
(3) $u(n)$

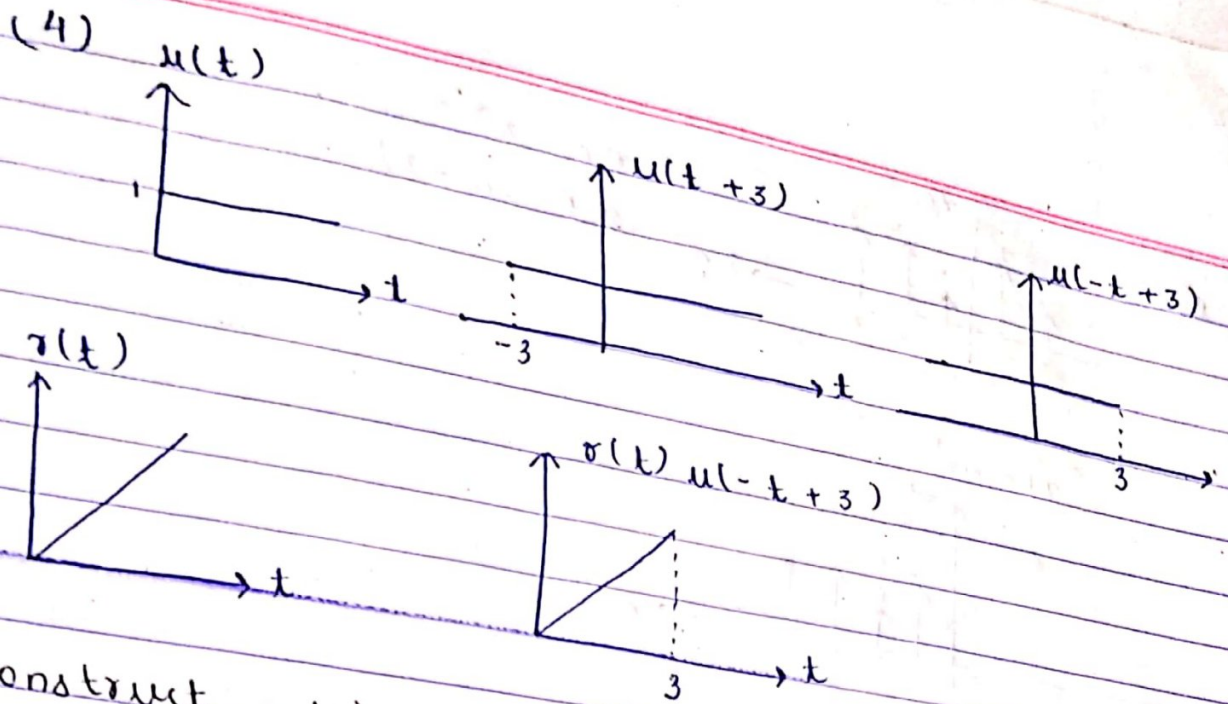


$u(n)$



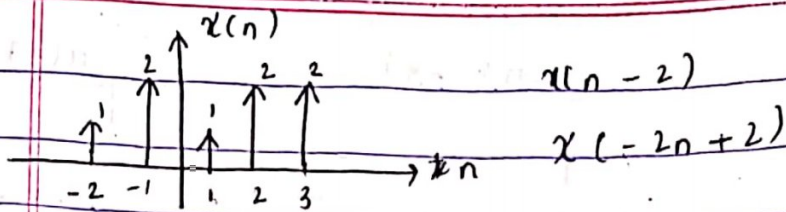
$u(n+2) - u(n)$



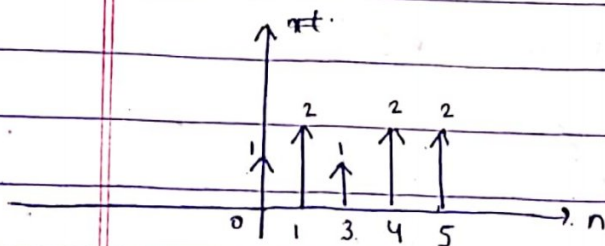


Ques: Construct triangular signal with the help of unit step and ramp signal.

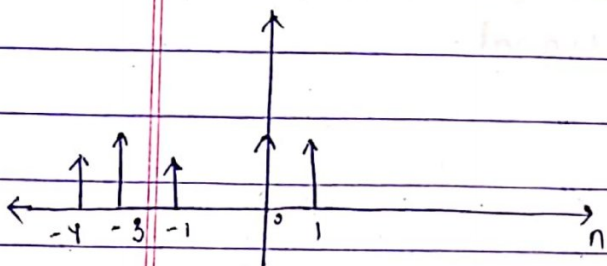
Ques:



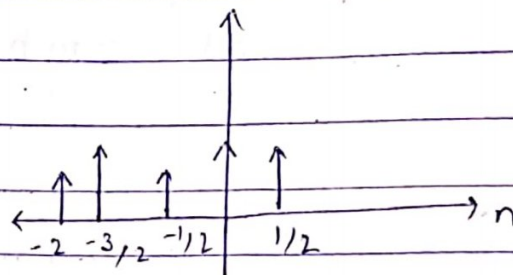
$x(n-2)$



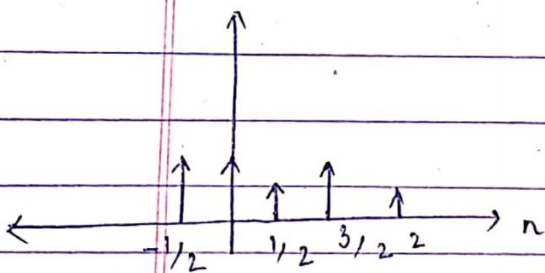
$x(n+2)$



$x(2n+2)$



$x(-2n+2)$



Date
16/09/2019

II - Mid

Date
Page

Module - 2

Fourier Series and Fourier Transform
(Periodic) (Non-Periodic)

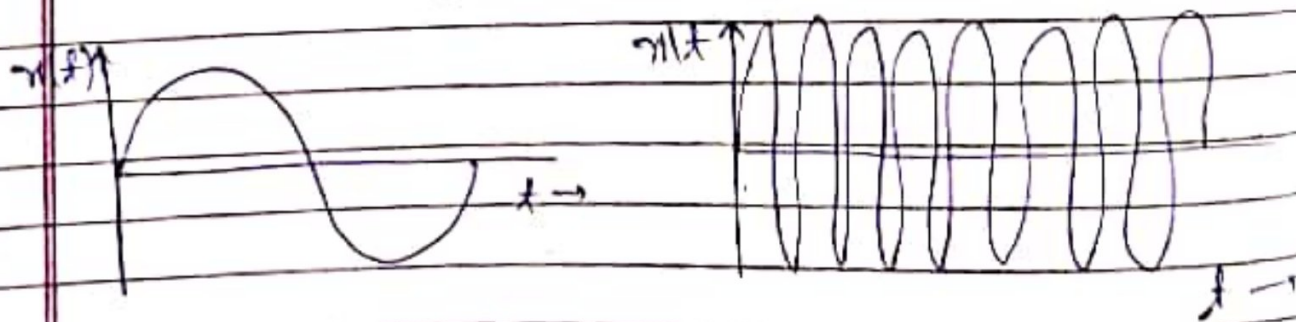
Laplace Transform
Z Transform

• Fourier Series :-

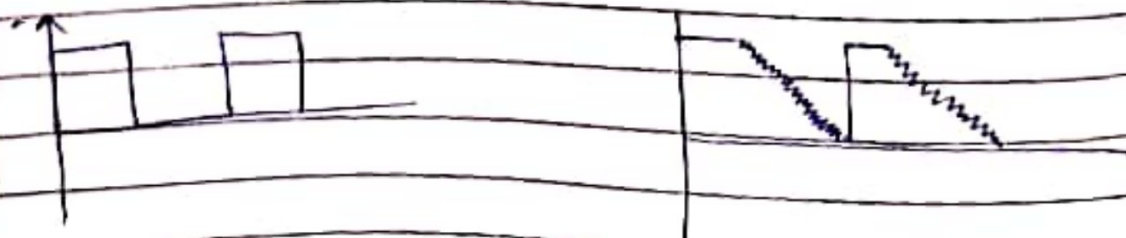
Fourier Series expansion is used for Periodic Signal to expand them in terms of their harmonics which are sinusoidal and orthogonal to one another.

Every periodic signal could not be expanded by the Fourier series because they have to satisfy a condition known as ~~Dirichlet~~ Dirichlet Condition.

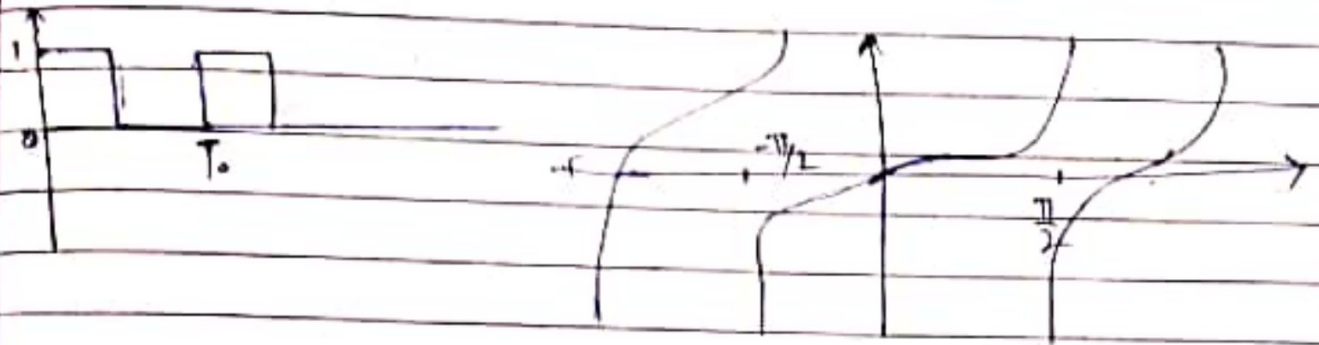
1. The signal should have finite no. of maxima and minima within a time period.



- ② The signal should have finite no. of discontinuity within a time period.



③ The signal should be absolutely integrable with in a time period,



1. Trigonometric Fourier Series
2. Complex exponential Fourier Series
3. Polar harmonic Fourier Series

• Trigonometric Fourier Series :-

$x(t) = \text{dc value} + \text{cosine terms} + \text{sine terms}$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

a_0 , a_n & b_n are the Fourier Constant.

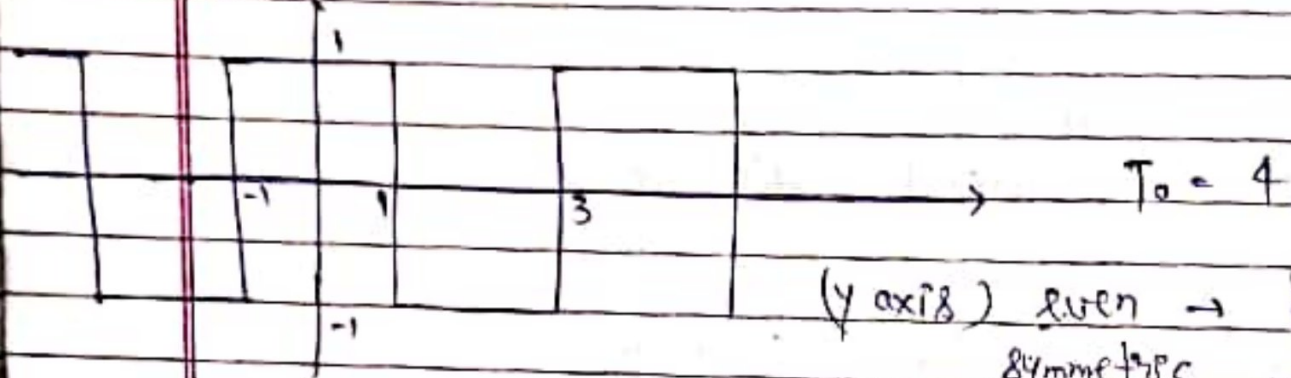
$$a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t dt$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t dt$$

- ① If signal is symmetric about time axis (x -axis)
then $a_0 = 0$
- ② If the signal is symmetric about y -axis (even)
then $b_n = 0$
- ③ If the signal is not symmetric about y -axis (odd)
then $a_n = 0$

$x(t)$



(y axis) even $\rightarrow b_n = 0$
 symmetric $\rightarrow a_0 = 0$
 about x -axis

$$a_n = \frac{2}{4} \int_{-1}^3 x(t) dt$$

$$= \frac{1}{2} \left[\int_{-1}^1 1 \cdot dt + \int_1^3 (-1) dt \right]$$

$$= \frac{1}{2} \left[\left[t \right]_{-1}^1 - \left[t \right]_1^3 \right] = \frac{1}{2} \left[1 - (-1) - (3 - 1) \right]$$

$$= \frac{1}{2} [1 + 1 - (3 - 1)] =$$

$$a_n = \frac{2}{4} \int_{-1}^3 x(t) \cos n\omega_0 t \, dt$$

$$T_0 = 4$$

$$\omega_0 = \frac{2\pi}{4}$$

$$= \frac{1}{2} \left[\int_{-1}^1 x(t) \cos \frac{n\pi}{2} t \, dt - \int_1^3 x(t) \cos \frac{n\pi}{2} t \, dt \right] = \frac{\pi}{2}$$

$$= \frac{1}{2} \left[\left[\frac{\sin \frac{n\pi}{2} t}{\frac{n\pi}{2}} \times \frac{2}{n\pi} \right]_{-1}^1 - \left[\frac{\sin \frac{n\pi}{2} t}{\frac{n\pi}{2}} \times \frac{2}{n\pi} \right]_1^3 \right]$$

$$\frac{1}{2} \times \frac{2}{n\pi} \left[\frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}} + \frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}} - \frac{\sin \frac{3n\pi}{2}}{\frac{n\pi}{2}} + \frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}} \right]$$

$$\frac{1}{n\pi} \left[\frac{3 \sin \frac{n\pi}{2}}{\frac{n\pi}{2}} - \frac{\sin \frac{3n\pi}{2}}{\frac{n\pi}{2}} \right] = \frac{1}{n\pi} \left[\frac{3 \sin \frac{n\pi}{2}}{\frac{n\pi}{2}} - \frac{\sin (n\pi + \frac{n\pi}{2})}{\frac{n\pi}{2}} \right]$$

• Case-I : If n is even

$$\frac{1}{n\pi} \left[\frac{3 \sin \frac{n\pi}{2}}{\frac{n\pi}{2}} + \frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}} \right] = \frac{2 \sin \frac{n\pi}{2}}{n\pi}$$

$$a_n = \frac{1}{n\pi} \left(\frac{2 \sin \frac{n\pi}{2}}{\frac{n\pi}{2}} \right) = 0 \quad \left\{ \text{for all value of } n \text{ (even)} \right\}$$

• Case-II : If n is odd

$$\frac{1}{n\pi} \left[\frac{3 \sin \frac{n\pi}{2}}{\frac{n\pi}{2}} + \frac{\sin \frac{n\pi}{2}}{\frac{n\pi}{2}} \right] = \frac{1}{n\pi} \left(\frac{4 \sin \frac{n\pi}{2}}{\frac{n\pi}{2}} \right)$$

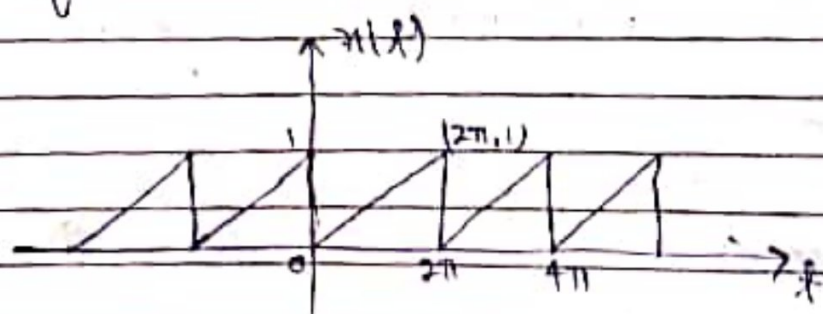
$$x(t) = \sum_{n=1}^{\infty} \frac{1}{n\pi} \left(\frac{4 \sin \frac{n\pi}{2}}{\frac{n\pi}{2}} \right) \cos n\omega_0 t$$

if $n = 1, 5, 9, \dots$
 $\frac{\sin n\pi}{2} = 1$

if $n = 3, 7, 11, \dots$
 $\frac{\sin n\pi}{2} = -1$

$$x(t) = \frac{4}{\pi} \cos \omega_0 t - \frac{4}{3\pi} \cos 3\omega_0 t + \frac{4}{5\pi} \cos 5\omega_0 t - \frac{4}{7\pi} \cos 7\omega_0 t + \dots$$

Ques. Find out the trigonometric fourier series of the waveform



Ans. ~~Fourier series~~ ~~is as follows~~

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} \frac{2\pi t}{2\pi} dt$$

$$= \frac{1}{4\pi^2} \left[t^2 \right]_0^{2\pi}$$

$T_0 = 2\pi$
 $(0,0) \quad (2\pi, 1)$

$$y - 0 = \frac{1 - 0}{2\pi - 0} (x - 0)$$

$$y = \frac{x}{2\pi}$$

$$x(t) = \frac{t}{2\pi}$$

$$(2\pi, 0)$$

$$(2\pi, 1)$$

$$y - 0 = \frac{1-0}{2\pi-2\pi} x$$

$$1 = 2\pi$$

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$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2\pi} = 1$$

$$\frac{1}{4\pi^2} \left(\frac{1\pi^2}{2} \right) = \frac{1}{2}$$

$$a_n = \frac{2}{2\pi} \int_0^{2\pi} f(t) \cos nt \, dt$$

$$= \frac{1}{2\pi^2} \left[\left[\frac{t \cdot \sin nt}{n} \right]_0^{2\pi} - \int_0^{2\pi} \frac{1 \cdot \sin nt}{n} dt \right]$$

$$= \frac{1}{2\pi^2 n} \left[-\cos nt \right]_0^{2\pi}$$

$$= -\frac{1}{2\pi^2 n^2} (\cos 2\pi n - \cos 0) = 0$$

$$a_n = 0$$

$$b_n = \frac{2}{2\pi} \int_0^{2\pi} f(t) \sin nt \, dt$$

$$= \frac{1}{2\pi^2} \int_0^{2\pi} t \cdot \sin nt \, dt$$

$$= \frac{1}{2\pi^2} \left[\left[-\frac{t \cos nt}{n} \right]_0^{2\pi} - \int_0^{2\pi} \frac{1 \cdot (-\cos nt)}{n} dt \right]$$

$$= \frac{1}{2\pi^2} \left[\frac{-2\pi}{n} + \frac{1}{n} \left[\frac{\sin nt}{n} \right]_0^{2\pi} \right]$$

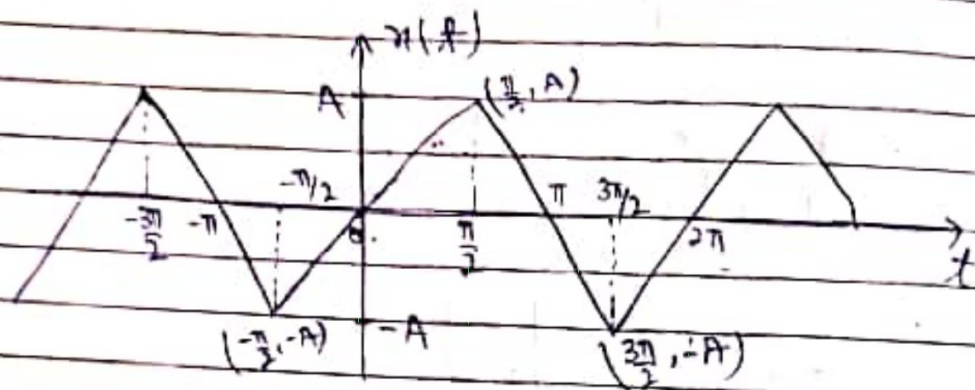
$$= \frac{-2\pi}{2\pi^2 n} = -\frac{1}{n\pi}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$



$$x(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \left(\frac{-1}{n\pi} \right) \sin n\omega_0 t$$

Ques.



1. odd function

2. Symmetric about
 ab. & x axis

$$a_n = 0$$

$$a_0 = 0$$

$$T_0 = 2\pi$$

$$\left(-\frac{\pi}{2}, -A \right) \quad \left(\frac{\pi}{2}, A \right)$$

$$y - 0 = \frac{A - 0}{\pi/2 - 0} (x - 0)$$

$$y + A = \frac{A + A}{\left(\frac{\pi}{2} + \frac{\pi}{2} \right)} \left(x + \frac{\pi}{2} \right)$$

$$y = \frac{A}{\pi/2} x$$

$$x(t) = \frac{2A}{\pi} t$$

$$y + A = \frac{2A}{\pi} \left(x + \frac{\pi}{2} \right)$$

$$x(t) + A = \frac{2A}{\pi} \left(t + \frac{2\pi}{2} \right)$$

$$x(t) = \frac{2A}{\pi} t + A - A$$

$$= \frac{2A}{\pi} t$$

$$b_n = 2 \times \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} \frac{2A}{\pi} f \sin nt \, dt$$

$$= \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} \frac{2A}{\pi} f \sin nt \, dt$$

$$= \frac{4A}{\pi^2} \left[\left[\frac{f \cos nt}{-n} \right]_{-\pi/2}^{\pi/2} - \int_{-\pi/2}^{\pi/2} \frac{\cos nt}{n} \, dt \right]$$

$$= -\frac{4A}{\pi^2 n} \left[\left(\frac{\pi}{2} \cos \frac{n\pi}{2} + \frac{\pi}{2} \cos \frac{n\pi}{2} \right) - \left[\frac{\sin nt}{n} \right]_{-\pi/2}^{\pi/2} \right]$$

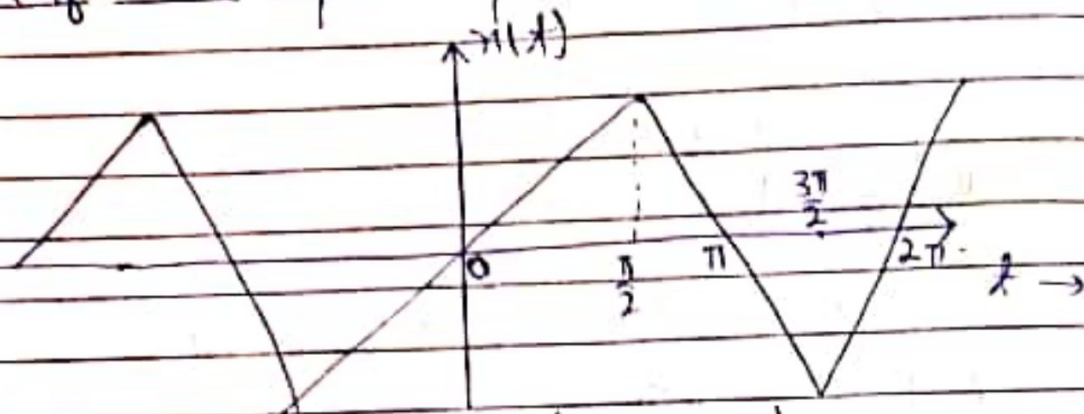
$$= -\frac{4A}{n\pi^2} \left[\frac{\pi \cos n\pi}{2} - \frac{1}{n} \left(\frac{\sin n\pi}{2} + \frac{\sin n\pi}{2} \right) \right]$$

$$= -\frac{4A}{n\pi^2} \left[\frac{\pi \cos n\pi}{2} - \frac{2}{n} \frac{\sin n\pi}{2} \right]$$

if n is even,



Half Wave Symmetry :-



Symmetric about x -axis (time axis)

So, $a_0 = 0$

Odd signal, i.e., $a_n = 0$

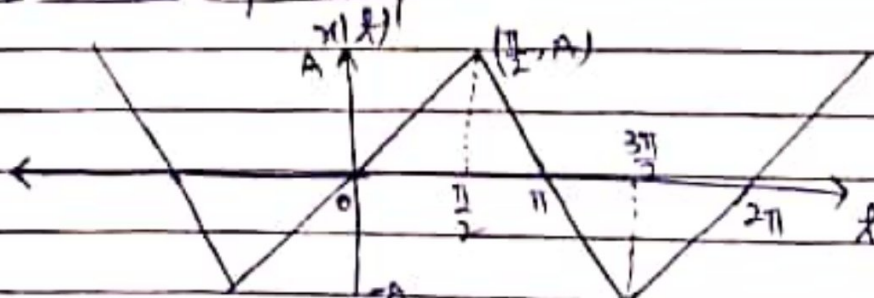
$$b_n = \frac{2}{T_0} \int_0^{T_0} x(t) \sin n\omega t dt$$

$$b_n = \frac{4}{T_0} \int_0^{\pi} x(t) \sin n\omega t dt$$

$$\text{if } x(t) = -x\left(t \pm \frac{T}{2}\right)$$

Quarter

• Quarter Wave Symmetry :-



Signal is symmetric about x -axis $\Rightarrow a_0 = 0$
odd signal $\Rightarrow a_n = 0$

$$b_n = \frac{2}{T_0} \int_0^{T_0} x(t) \sin n\omega t dt$$

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$$b_n = \frac{8}{T_0} \int_0^{T_0/2} x(t) \sin n\omega t \, dt$$

$$T_0 = 2\pi$$

$$(0,0) \quad \left(\frac{\pi}{2}, A\right)$$

$$= \frac{8}{2\pi} \int_0^{\pi/2} \frac{2A}{\pi} t \sin nt \, dt$$

$$y=0 = \frac{A}{\frac{\pi}{2}} (x=0)$$

$$= \frac{8A}{\pi^2} \int_0^{\pi/2} t \sin nt \, dt$$

$$y = \frac{2A}{\pi} t$$

$$= \frac{8A}{\pi^2} \left[\left[\frac{-t \cos nt}{n} \right]_0^{\pi/2} - \int_0^{\pi/2} \frac{-\cos nt}{n} \, dt \right]$$

$$= \frac{8A}{\pi^2 n} \left[\frac{-\pi \cos n\pi}{2} + \left[\frac{\sin nt}{n} \right]_0^{\pi/2} \right]$$

$$= \frac{8A}{n\pi^2} \left[\frac{-\pi \cos n\pi}{2} + \frac{1}{n} \frac{\sin n\pi}{2} \right]$$

• Complex Exponential Series :-

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt$$

$$C_n = \frac{1}{T_0} \int_{T_0} x(t) [\cos n\omega_0 t - j \sin n\omega_0 t] dt$$

$$C_n = \frac{1}{2} \left[\frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t dt - j \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t dt \right]$$

$$C_n = \frac{1}{2} [a_n - j b_n]$$

$$C_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt$$

$$C_{-n} = \frac{1}{T_0} \int_{T_0} x(t) e^{jn\omega_0 t} dt$$

$$C_n + C_{-n} = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t dt$$

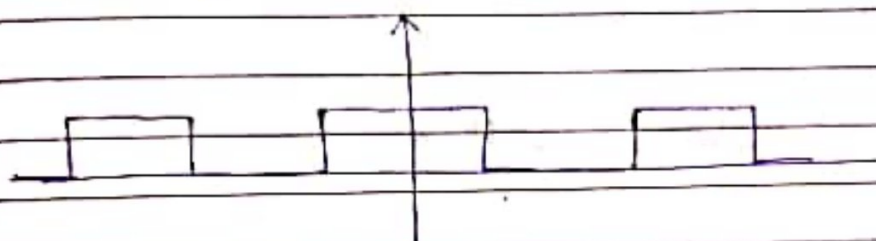
$$C_n + C_{-n} = a_n$$

$$C_n - C_{-n} = \int_{T_0}^T \frac{2}{T_0} \int x(t) \sin n\omega t dt = \int b_n$$

$$\boxed{\frac{C_n - C_{-n}}{f} = b_n}$$

Tutorial

Ques. The fourier expansion of the following signal will contain the following non-zero term.



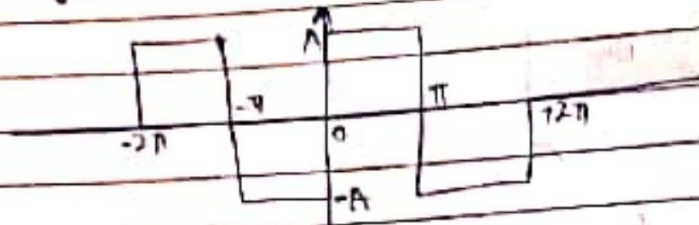
Hidden symmetry

- | | |
|---|-----------------------------|
| (a) a_n and b_n | $n = 1, 3, 5, \dots \infty$ |
| (b) a_0 and a_n | $n = 1, 2, 3, \dots \infty$ |
| (c) a_0, a_n and b_n | $n = 1, 2, 3, \dots \infty$ |
| ☒ (d) a_0 and a_n | $n = 1, 3, 5, \dots \infty$ |

② $x(t)$ is a real valued function with period T . Its trigonometric fourier series expansion contain no terms of frequency $\omega = 2\pi \left(\frac{2K}{T} \right)$ where $K = 1, 2, 3, \dots$ also no sine terms are present.

- (a) $x(t) = -x(t-T)$
 (b) $x(t) = -x(-t)$
☒ (c) $x(t) = -x\left(t - \frac{T}{2}\right)$
 (d) $x(t) = x\left(t - \frac{T}{2}\right)$

③ Find out the exponential fourier series expansion of the following signal and also draw the frequency spectrum.



$$a_0 = \frac{1}{T_0} \int_0^{2\pi} x(t) dt$$

$$= \frac{1}{2\pi} \left[\int_0^{\pi} A dt - \int_{\pi}^{2\pi} A dt \right]$$

$$= \frac{1}{2\pi} \left[A(\pi - 0) - A(2\pi - \pi) \right]$$

$$= \frac{A\pi - A\pi}{2\pi} = 0$$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$\omega_0 = 1$$

$$C_n = \frac{1}{T_0} \int_0^{2\pi} x(t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{2\pi} \left[\int_0^{\pi} A e^{-jn\omega_0 t} dt - \int_{\pi}^{2\pi} (-A) e^{-jn\omega_0 t} dt \right]$$

$$= \frac{A}{2\pi} \left[\int_0^{\pi} e^{-jn\omega_0 t} dt + \int_{\pi}^{2\pi} e^{-jn\omega_0 t} dt \right]$$

$$= \frac{A}{2\pi} \left[\left. \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right|_0^{\pi} + \left. \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right|_{\pi}^{2\pi} \right]$$

$$C_n = A \left[\frac{(e^{-jn\pi} - 1)}{-jn} - \frac{(e^{-jn2\pi} - e^{-jn\pi})}{-jn} \right]$$

~~$$= \frac{A}{2\pi} \left[\frac{1 - e^{-jn\pi}}{jn} + \frac{e^{-jn\pi} - e^{-jn2\pi}}{jn} \right]$$~~

$$= \frac{-A}{j2n\pi} \left[(e^{-jn\pi} - 1) - (e^{-jn2\pi} - e^{-jn\pi}) \right]$$

$$= \frac{-A}{j2n\pi} \left[(-1)^n - 1 - 1 + (-1)^n \right]$$

$$= \frac{-A}{j2n\pi} \left[-2 + 2(-1)^n \right]$$

$$= \frac{-A}{jn\pi} \left[-1 + (-1)^n \right]$$

if n is odd

$$C_n = \frac{2A}{jn\pi}$$

if n is even

$$C_n = 0$$

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{2A}{jn\pi} e^{j\omega_0 n t} \quad (n \text{ is odd})$$

$$x(t) = \frac{2A}{j\pi} \sum_{n=-\infty}^{\infty} \frac{1}{n} e^{j\omega_0 n t}$$

Note :-

If the exponential Fourier series coefficient are conjugate symmetric then the function $x(t)$ is real.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

$$C_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega t} dt \quad \text{--- (1)}$$

Put $n = -n$

$$C_{-n} = \frac{1}{T_0} \int_{T_0} x(t) e^{jn\omega t} dt$$

Take * of C_n

$$C_n^* = \left(\frac{1}{T_0} \int_{T_0} x(t) e^{jn\omega t} dt \right)^*$$

$$C_n^* = \frac{1}{T_0} \int_{T_0} x^*(t) e^{-jn\omega t} dt \quad \text{--- (2)}$$

if $x(t)$ is real $\Rightarrow x(t) = x^*(t)$
 $C_n = C_n^*$

$$C_n = a + jb$$

$$C_{-n} = a - jb$$

$$C_n^* = a - jb$$

$$|C_n| = |C_{-n}|$$

if C_n is conjugate symmetric then the magnitude of C_n follows the even property.

$$\angle C_n = \tan^{-1} \left(\frac{-b_n}{a_n} \right) = \theta_n$$

$$\angle C_{-n} = \tan^{-1} \left(\frac{-b_{-n}}{a_{-n}} \right)$$

$$= \tan^{-1} \left(\frac{b_n}{a_n} \right) = -\tan^{-1} \left(\frac{-b_{-n}}{a_{-n}} \right) = -\theta_n = -\angle C_n$$

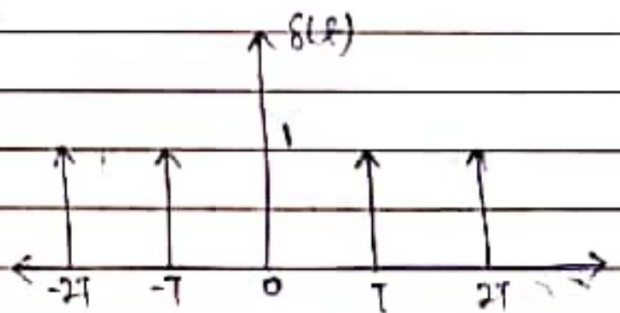
The phase spectrum is an odd function.

Ques. Find the Complex exponential Fourier Series of unit impulse train. $S(t) = \sum_{n=-\infty}^{\infty} S(t-nT)$. From this result draw the line spectrum.

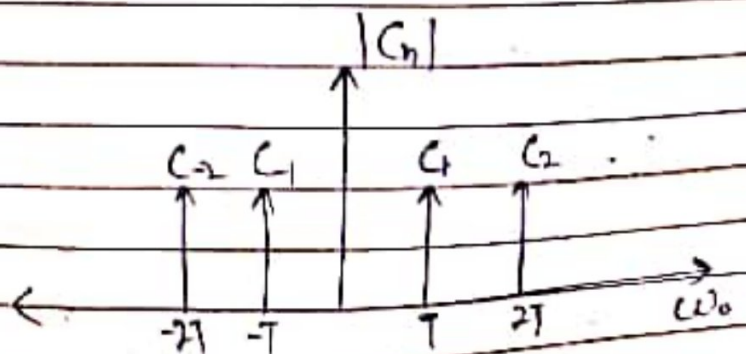
$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

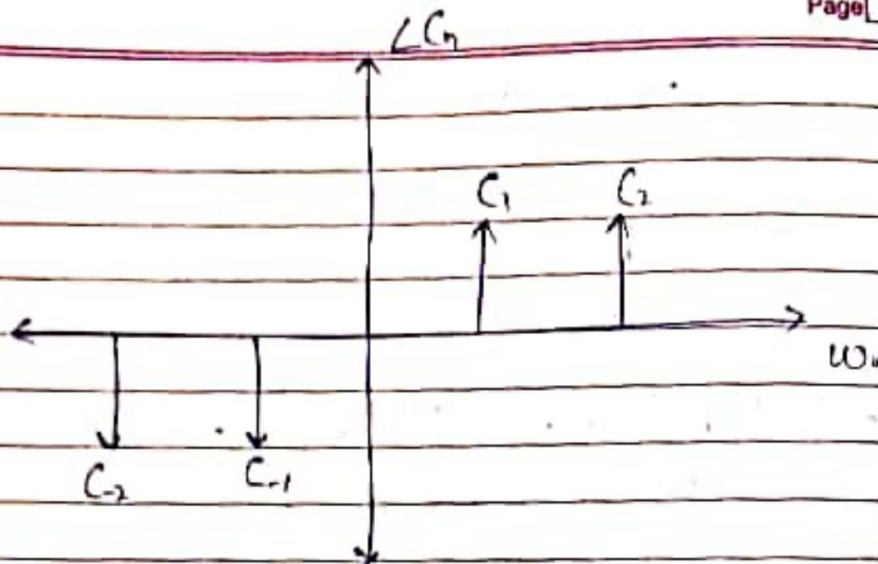
$$C_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt$$

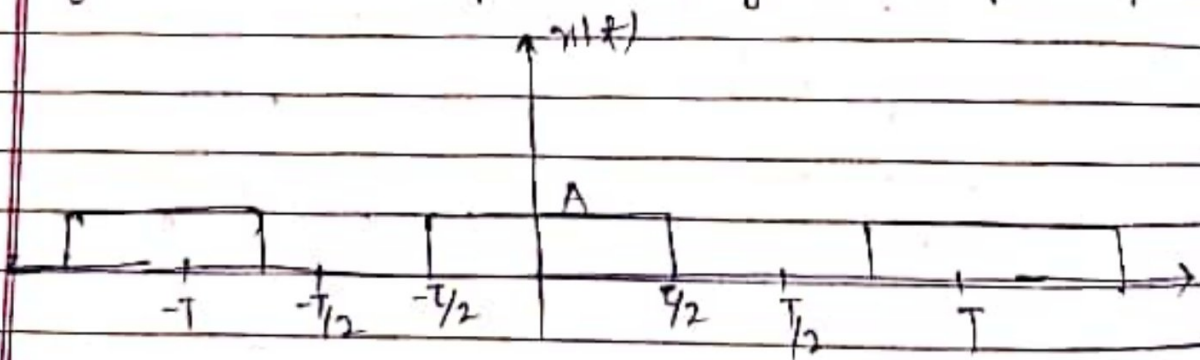


$$C_n = \frac{1}{T}$$





Que. For this periodic gate function, find out the exponential fourier series and plot the magnitude and phase spectrum.



$$A \text{ rect}(t/T)$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt$$

$$C_n = \frac{1}{T} \int_{-T/2}^{-T/4} 0 \cdot e^{-jn\omega_0 t} dt + \int_{-T/4}^{T/4} A \cdot e^{-jn\omega_0 t} dt + \int_{T/4}^{T/2} 0 \cdot dt$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

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$$= \frac{A}{T} \int_{-T/2}^{T/2} e^{-jn\omega_0 t} dt$$

$$= \frac{A}{T} \left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right]_{-T/2}^{T/2}$$

$$= \frac{A}{T} \frac{e^{-jn\omega_0 T/2} - e^{+jn\omega_0 T/2}}{-jn\omega_0}$$

$$= \frac{A}{T(jn\omega_0)} 2 \left[j \sin n\omega_0 T/2 \right]$$

$$= \frac{A}{Tjn\omega_0} \times \frac{2jn\omega_0 T}{2} = \frac{2A}{T(n\omega_0)} \sin\left(\frac{n\omega_0 T}{2}\right)$$

$$= \frac{2A \times \sin\left(\frac{n\omega_0 T}{2}\right)}{T(n\omega_0)}$$

$$C_n = \frac{2A \times T/2}{T n\omega_0 T/2} \sin n\omega_0 T/2$$

$$C_n = \frac{AT}{2T} \text{sinc} \left(\frac{n\omega_0 T}{2} \right)$$

$$\frac{\sin x}{x} = \text{sinc}(x)$$

• Properties of Fourier Series :-

1. Linearity -

$$x_1(t) \Rightarrow C_n$$

$$x_2(t) \Rightarrow C_n'$$

$$\alpha x_1(t) + \beta x_2(t) \Rightarrow \alpha C_n + \beta C_n'$$

$$C_n = \frac{1}{T} \int x_1(t) e^{-jn\omega t} dt$$

$$C_n' = \frac{1}{T} \int x_2(t) e^{-jn\omega t} dt$$

$$\alpha C_n + \beta C_n' = \frac{1}{T} \int (\alpha x_1(t) + \beta x_2(t)) e^{-jn\omega t} dt$$

2. Time Shifting :-

$$x(t) \Rightarrow C_n$$

$$x(t-t_0) \Rightarrow C_n e^{-jn\omega t_0}$$

$$x(t-t_0) \Rightarrow C_n'$$

then $C_n' = C_n e^{-jn\omega t_0}$

$$C_n = \frac{1}{T} \int x(t) e^{-jn\omega t} dt$$

Put $t = t - t_0$

$$C_n = \frac{1}{T} \int x(t-t_0) e^{-jn\omega(t-t_0)} dt$$



$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\begin{aligned} x(t-t_0) &= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0(t-t_0)} \\ &= \sum_{n=-\infty}^{\infty} (C_n e^{-jn\omega_0 t_0}) e^{jn\omega_0 t} \end{aligned}$$

$$x(t-t_0) = \sum_{n=-\infty}^{\infty} C'_n e^{jn\omega_0 t}$$

Where $C'_n = C_n e^{-jn\omega_0 t_0}$

Ques. If a signal $x(t)$ have fourier coefficient C_n , then find out the fourier coefficient of $y(t)$ where $y(t) = x(t-1) + x(t+1)$

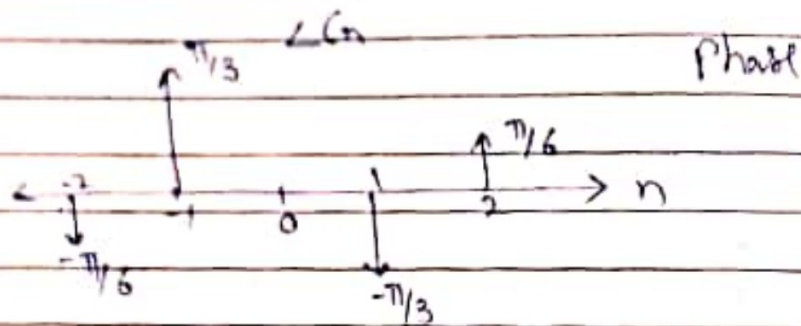
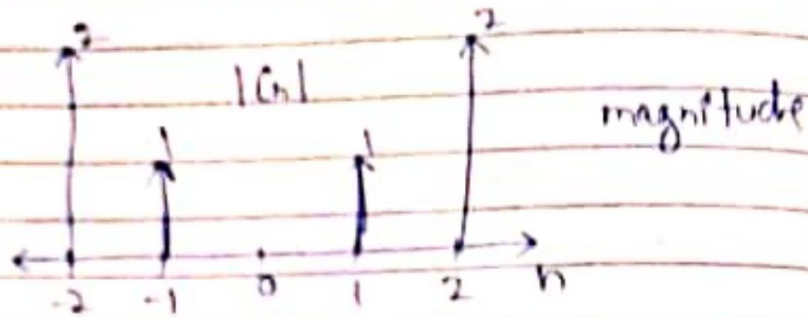
Sol. $y(t) = C_n e^{-jn\omega_0} + C_n e^{jn\omega_0}$

3. Time Reversal :-

If $x(t) \Rightarrow C_n$
then $x(-t) \Rightarrow C_n^*$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$x(-t) = \sum_{n=-\infty}^{\infty} C_n e^{-jn\omega_0 t}$$



$$x(t) = \sum_{n=-\infty}^{\infty} G_n e^{j\omega n t}$$

$$x(t) = \sum_{n=-2}^2 G_n e^{j\omega n t}$$

$$x(t) = G_{-2} e^{-2j\omega t} + G_{-1} e^{-j\omega t} + G_0 e^{j\omega t} + G_2 e^{2j\omega t}$$

$$G_n = |G_n| e^{j\theta}$$

$$G_{-2} = 2 e^{j\pi/6} = 2 \left\{ \cos \frac{\pi}{6} - j \sin \frac{\pi}{6} \right\}$$

$$G_{-2} = 2 \left\{ \frac{\sqrt{3}}{2} - j \right\} = \sqrt{3} - j$$

$$G_{-1} = 1 \cdot e^{j\pi/3} = \cos \frac{\pi}{3} + j \sin \frac{\pi}{3} = \frac{1}{2} + j \frac{\sqrt{3}}{2}$$

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$$G_1 = 1 \cdot e^{-j\pi/3} = \cos \frac{\pi}{3} - j \sin \frac{\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{2} j$$

$$G_2 = 2 \cdot e^{j\pi/6} = 2 \left\{ \cos \frac{\pi}{6} + j \sin \frac{\pi}{6} \right\}$$

$$= 2 \left\{ \frac{\sqrt{3} + j}{2} \right\} = \sqrt{3} + j$$

$$x(t) = (\sqrt{3} - j) e^{-2j\omega t} + \frac{(1 + j\sqrt{3})}{2} e^{-j\omega t} + \frac{(1 - \sqrt{3}j)}{2} e^{j\omega t} +$$

$$(\sqrt{3} + j) e^{2j\omega t}$$

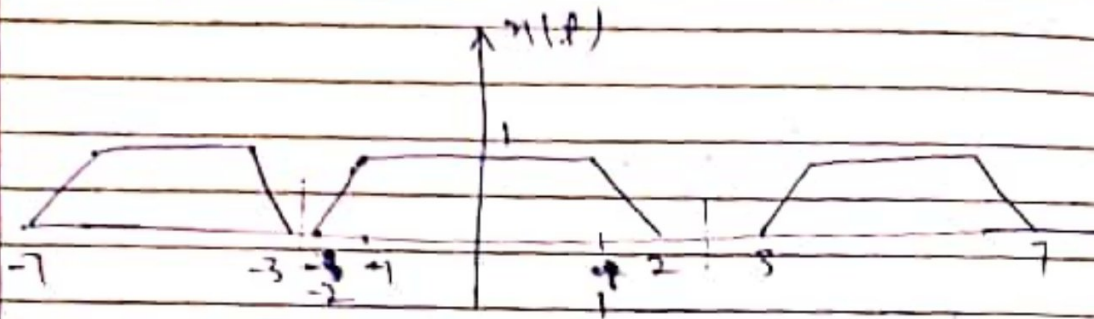
$$C_0 = a_0 = 0$$

$$C_n = \frac{a_n - j b_n}{2}$$

$$C_{-n} = \frac{a_n + j b_n}{2}$$

$$x(t) = a_2 \cos 2\omega t + a_1 \cos \omega t + b_2 \sin 2\omega t + b_1 \sin \omega t$$

Ques. Find out the exponential fourier series of the following wave form.



$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{5} \left[\int_{-2.5}^{-3} 0 \cdot dt + \int_{-3}^{-1} x(t) dt e^{-jn\omega_0 t} + \int_{-1}^1 x(t) \cdot e^{-jn\omega_0 t} dt + \int_1^2 x(t) e^{-jn\omega_0 t} dt + \int_2^{2.5} 0 \cdot dt \right]$$

$$= \frac{1}{5} \left[\int_{-3}^{-1} (t+2) e^{-jn\omega_0 t} dt + \int_{-1}^1 1 \cdot e^{-jn\omega_0 t} dt + \int_1^2 (t-2) e^{-jn\omega_0 t} dt \right]$$

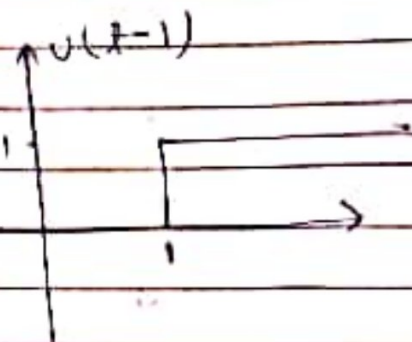
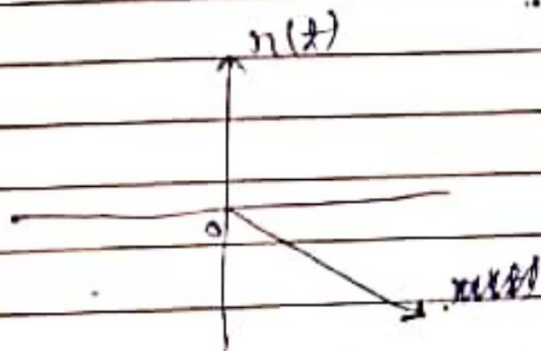
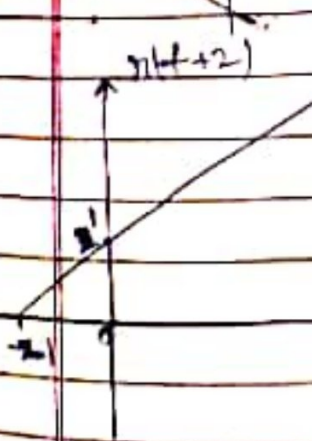
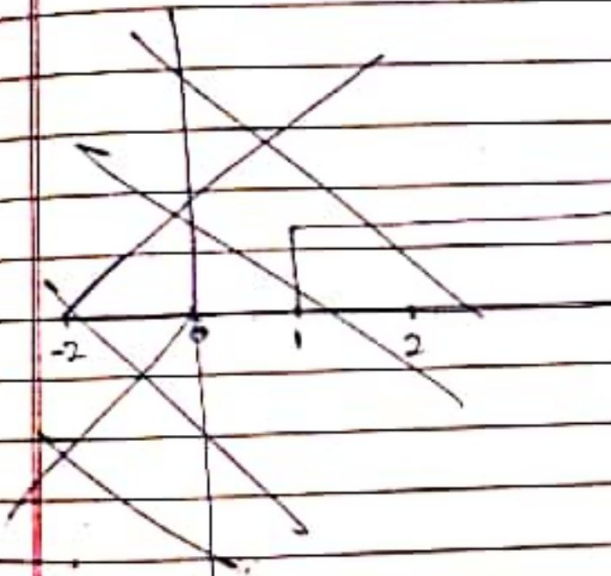
$$= \frac{1}{5} \left[\left[\frac{(t+2) \cdot e^{-jn\omega_0 t}}{(-jn\omega_0)} \right]_{-3}^{-1} - \int_{-3}^{-1} e^{-jn\omega_0 t} dt + \left[\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)} \right]_{-1}^1 \right]$$

$$- \left[\frac{(t-2) \cdot e^{-jn\omega_0 t}}{(-jn\omega_0)} \right]_1^2 + \int_1^2 e^{-jn\omega_0 t} dt \right]$$

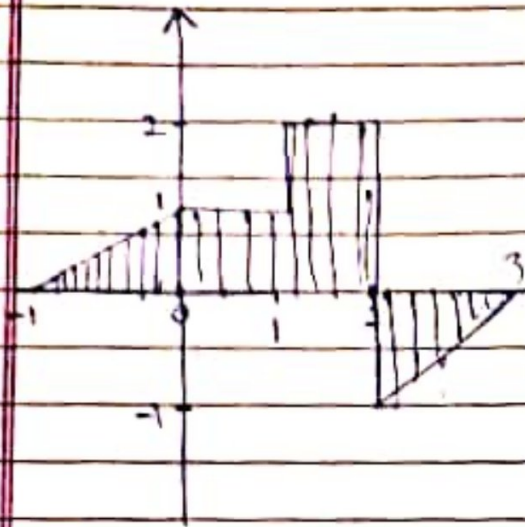
$$x(t) = t u(t)$$

Ans. Plot $y(t) = x(t+2) - x(t) + u(t-1) - 3u(t-2) + x(t-2) - x(t-3)$

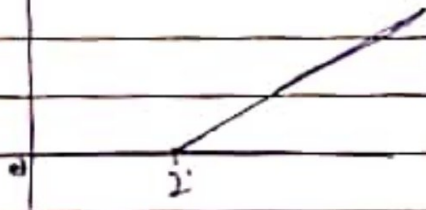
~~$y(t) = x(t+2) - x(t) + u(t-1) - 3u(t-2) + x(t-2) - x(t-3)$~~



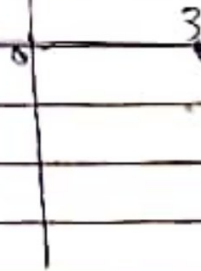
$$x(t+1) = x(t) + u(t-1) - 3u(t-2)$$



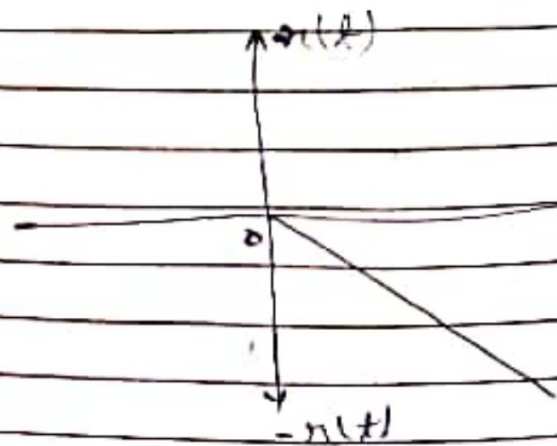
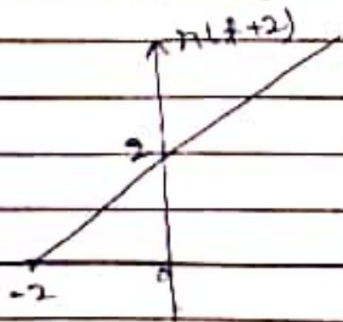
$$x(t-2)$$

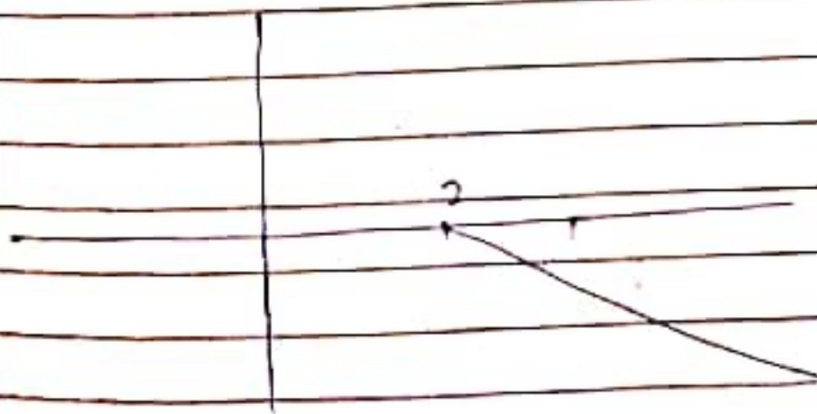
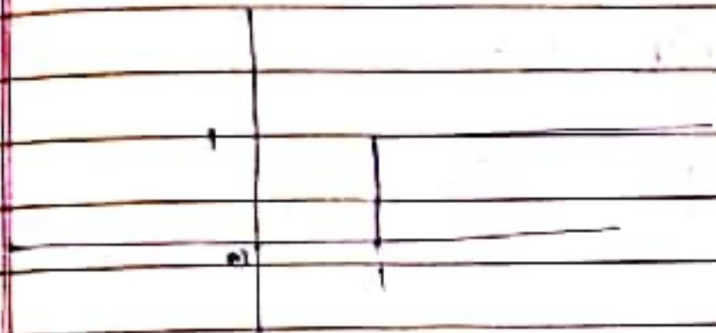
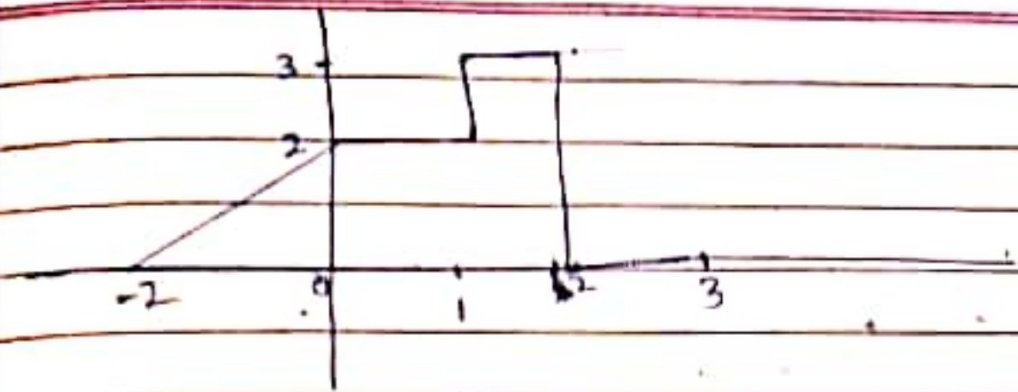


$$-x(t-3)$$

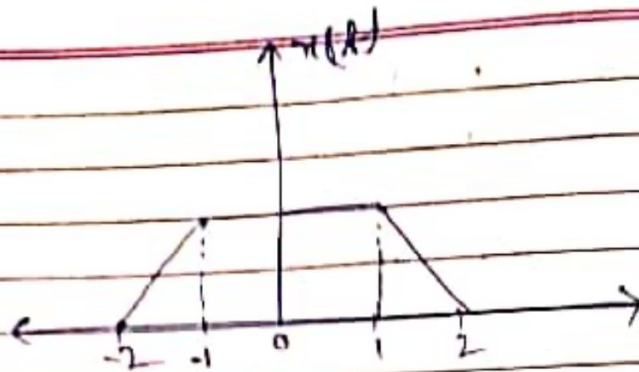


$$② \quad x(t+2) + -x(t) + u(t-1) - 3u(t-2) + x(t-2)$$





①



$$x(t) = x(t+2) + x(t+1) + x(t-1) + x(t-2)$$

Time Reversal :-

$$x(t) \Rightarrow C_n$$

$$C_n = \frac{1}{T} \int_{T_0}^T x(t) e^{-jn\omega_0 t} dt$$

$$\text{if } x(-t) \Rightarrow C_n'$$

$$C_n' = \frac{1}{T} \int_0^T x(-t) e^{-jn\omega_0 t} dt$$

$$\text{Put } -t = \tau$$

$$dt = -d\tau$$

$$t = -\tau$$

$$C_n' = \frac{1}{T} \int_0^T x(\tau) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{T} \int_0^T x(\tau) e^{-jn\omega_0 \tau} d\tau$$

$$C_n = \frac{1}{T} \int_0^T x(\tau) e^{-jn\omega_0 \tau} d\tau$$

$$x(t) \xrightarrow[\text{differentiation}]{\text{integration}} u(t)$$

$$x(t) \xrightarrow[\text{integration}]{\text{differentiation}} u(t)$$

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4. Differentiation :-

$$x(t) \Rightarrow C_n$$

$$\frac{dx(t)}{dt} = \sum C_n e^{jn\omega_0 t} \cdot jn\omega_0$$

$$= \sum (jn\omega_0 C_n) e^{jn\omega_0 t}$$

$$= \sum C_n' e^{jn\omega_0 t}$$

where,

$$C_n' = jn\omega_0 C_n$$

5. Integration :-

$$x(t) \Rightarrow C_n$$

$$\frac{dx(t)}{dt} \Rightarrow jn\omega_0 C_n$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\int_{-\infty}^t x(t) dt = \int_{-\infty}^t \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} dt$$

$$= \sum_{n=-\infty}^{\infty} C_n \left[\frac{e^{jn\omega_0 t}}{jn\omega_0} \right]_{-\infty}^t$$

$$= \sum_{n=-\infty}^{\infty} C_n \left[\frac{e^{jn\omega_0 t} - e^{-\infty}}{jn\omega_0} \right]$$

$$= \sum_{n=-\infty}^{\infty} \frac{C_n}{jn\omega_0} e^{jn\omega_0 t}$$

$$\boxed{\int_{-\infty}^t x(t) dt = \sum_{n=-\infty}^{\infty} \left(\frac{C_n}{jn\omega_0} \right) e^{jn\omega_0 t}}$$

6. Conjugation :-

$$x(t) \Rightarrow C_n$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{-jn\omega_0 t}$$

$$x^*(t) = \sum_{n=-\infty}^{\infty} (C_n)^* e^{jn\omega_0 t}$$

$$x^*(t) \Rightarrow (C_n)^*$$

$$C_n = \frac{1}{T} \int_T x(t) e^{-jn\omega_0 t} dt$$

$$C_n^* = \frac{1}{T} \int_T x^*(t) e^{jn\omega_0 t} dt$$

$$(C_n)^* = \frac{1}{T} \int_T x^*(t) e^{-jn\omega_0 t} dt$$

$$\boxed{x^*(t) \Rightarrow (C_n)^*}$$

7. Convolution :-

$$\left. \begin{array}{l} x_1(t) \Rightarrow C_n \\ x_2(t) \Rightarrow C_n \end{array} \right\} * T.$$

$$\begin{aligned} x(t) &= x_1(t) * x_2(t) \\ &= T_0 [C_n \times C_n] \end{aligned}$$

$$C_{12} = \frac{1}{T_0} \int_{T_0} x_1(t) \cdot e^{-j\omega_0 t} dt$$

$$C_{21} = \frac{1}{T_0} \int_{T_0} x_2(t) \cdot e^{-j\omega_0 t} dt$$

Let C_n is Fourier Coefficient of $x(t)$.

$$C_n = \frac{1}{T_0} \int_{T_0} x(t) \cdot e^{-j\omega_0 t} dt$$

$$C_n = \frac{1}{T_0} \int_{T_0} (x_1(t) * x_2(t)) \cdot e^{-j\omega_0 t} dt$$

$$x_1(t) * x_2(t) = \int_0^t x_1(\tau) * x_2(t-\tau) d\tau$$

$$= \frac{1}{T_0} \int_0^{T_0} \left[\int_0^t x_1(\tau) * x_2(t-\tau) d\tau \right] e^{-j\omega_0 t} \cdot \frac{e^{j\omega_0 t}}{e^{j\omega_0 t}} dt$$

$$= \frac{1}{T_0} \int_0^{T_0} \int_0^t [x_1(\tau) \cdot e^{-j\omega_0 \tau} d\tau] [x_2(t-\tau) \cdot e^{-j\omega_0 (t-\tau)} dt]$$

Put $t-\tau = m$

$$\frac{1}{T_0} \int_0^{T_0} \int_0^t x_1(m) \cdot e^{-j\omega_0 m} dm \cdot x_2(t) \cdot e^{-j\omega_0 t} dt$$

$$\frac{1}{T_0} \int_0^{T_0} \int_0^t x_2(m) \cdot e^{-j\omega_0 m} dm \cdot \frac{T_0}{T_0} x_1(t) \cdot e^{-j\omega_0 t} dt$$

$$T_0 [C_{21} \cdot C_{12}]$$

Ques. If a signal $x(t)$ is having an exponential fourier coefficient as C_n then find out the value of exponential fourier coefficient of the signal.

1. $y(t) = \text{odd}[x(t)]$

2. $y(t) = \text{Real}[x(t)]$

Sol. $\text{odd}[x(t)] = \frac{x(t) - x(-t)}{2}$

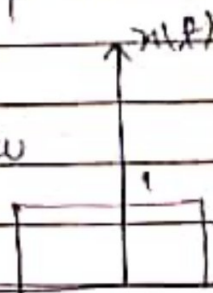
$= \frac{C_n - C_{-n}}{2}$

$\text{Real}[x(t)] = \frac{x(t) + x^*(t)}{2} = \frac{C_n + C_n^*}{2}$

• Fourier Transform:-

Fourier Transform is a mathematical tool which is used to analyse a signal in frequency domain.

at $T_0 \rightarrow \infty$ $n\omega_0 \rightarrow \omega$



$\lim_{T_0 \rightarrow \infty} T_0 C_n = \lim_{T_0 \rightarrow \infty} \int_{-T_0/2}^{T_0/2} x(t) e^{-jn\omega_0 t} dt$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty$$

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$$T_0 C_n = X(\omega)$$

$$C_n = \frac{X(\omega)}{T_0}$$

T_0

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{X(\omega)}{T_0} e^{jn\omega_0 t}$$

$$T = 2\pi$$

ω_0

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{X(\omega)}{2\pi} e^{jn\omega_0 t} \omega_0$$

$$\text{as } T_0 \rightarrow \infty$$

$$n\omega_0 \rightarrow \omega$$

$$\omega_0 \rightarrow d\omega$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

1. Fourier Transform of $\delta(t)$ (Impulse Signal)

$$\delta(t) = \begin{cases} 1 & , t = 0 \\ 0 & , t \neq 0 \end{cases}$$

$$FT(\delta(t)) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt$$

$$= \delta(0) e^{-j\omega \times 0}$$

$$= \delta(0) \cdot 1$$

$$FT(\delta(t)) = 1$$

$$\{ \delta(0) = 1 \}$$

2. $FT[\delta(t-t_0)]$

$$FT[\delta(t-t_0)] = \int_{-\infty}^{\infty} \delta(t-t_0) \cdot e^{-j\omega t} dt$$

$$\delta(t-t_0) = \begin{cases} 1 & t = t_0 \\ 0 & t \neq t_0 \end{cases}$$

$$FT[\delta(t-t_0)] = \delta(t-t_0) \cdot e^{-j\omega t_0} \quad \left\{ \delta(0) = 1 \right\}$$

$$FT[\delta(t-t_0)] = e^{-j\omega t_0}$$

3. $FT[e^{-at} u(t)]$

$$FT[e^{-at} u(t)] = \int_{-\infty}^{\infty} e^{-at} u(t) \cdot e^{-j\omega t} dt$$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\int_{-\infty}^0 e^{-at-j\omega t} u(t) dt + \int_0^{\infty} e^{-(a+j\omega)t} u(t) dt$$

$$0 + \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$\left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

$$= \frac{1}{-(a+j\omega)} [e^{-\infty} - e^0] = \frac{0-1}{-(a+j\omega)}$$

$$FT[e^{-at} u(t)] = \frac{1}{a+j\omega}$$

Que- $FT(e^{-a|t|})$

Sol. $FT[e^{-a|t|}] = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$$= \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t} dt$$

$$= \int_{-\infty}^0 e^{at} e^{-j\omega t} dt + \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_{-\infty}^0 e^{(a-j\omega)t} dt + \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \left[\frac{e^{(a-j\omega)t}}{a-j\omega} \right]_{-\infty}^0 + \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

$$= \frac{e^0 - e^{-\infty}}{(a-j\omega)} - \frac{e^{-\infty} - e^0}{(a+j\omega)}$$

$$= \frac{1-0}{a-j\omega} - \left[\frac{0-1}{a+j\omega} \right]$$

$$= \frac{1}{(a-j\omega)} + \frac{1}{(a+j\omega)}$$

$$= \frac{a+j\omega + a-j\omega}{a^2 - j^2\omega^2}$$

$$\boxed{FT[e^{-a|t|}] = \frac{2a}{a^2 + \omega^2}}$$

• Properties of Fourier Transform :-

1. Linearity :-

$$x_1(t) \leftrightarrow X_1(\omega)$$

$$x_2(t) \leftrightarrow X_2(\omega)$$

$$\Rightarrow x(t) = a x_1(t) + b x_2(t) \leftrightarrow a X_1(\omega) + b X_2(\omega)$$

$$F[a x_1(t) + b x_2(t)] = \int_{-\infty}^{\infty} [a x_1(t) + b x_2(t)] e^{-j\omega t} dt$$

$$= a \int_{-\infty}^{\infty} x_1(t) e^{-j\omega t} dt + b \int_{-\infty}^{\infty} x_2(t) e^{-j\omega t} dt$$

$$= a X_1(\omega) + b X_2(\omega)$$

2. Time Shifting :-

$$x(t) \leftrightarrow X(\omega)$$

$$x(t - t_0) \leftrightarrow e^{-j\omega t_0} X(\omega)$$

$$x(t + t_0) \leftrightarrow e^{j\omega t_0} X(\omega)$$

$$F[x(t - t_0)] = \int_{-\infty}^{\infty} x(t - t_0) e^{-j\omega(t - t_0)} dt$$

Put $t - t_0 = p$

~~$x(p) e^{-j\omega(p + t_0)}$~~

$$\int_{-\infty}^{\infty} x(p) e^{-j\omega(p + t_0)} dp$$

$$e^{-j\omega t_0} \int_{-\infty}^{\infty} x(p) e^{-j\omega p} dp$$

$$= e^{-j\omega t_0} X(\omega)$$

$$F[x(t+t_0)] = \int_{-\infty}^{\infty} x(t+t_0) e^{-j\omega t} dt$$

Put $t+t_0 = p$

$$\int_{-\infty}^{\infty} x(p) e^{-j(p-t_0)\omega} dp$$

$$e^{j\omega t_0} \int_{-\infty}^{\infty} x(p) e^{-j\omega p} dp = e^{j\omega t_0} X(\omega)$$

3. Frequency Shifting :-

$$x(t) \longleftrightarrow X(\omega)$$

$$e^{j\omega_0 t} x(t) \rightarrow X(\omega - \omega_0)$$

$$e^{-j\omega_0 t} x(t) \rightarrow X(\omega + \omega_0)$$

$$F[x(t) e^{j\omega_0 t}] = \int_{-\infty}^{\infty} x(t) e^{j\omega_0 t} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} x(t) e^{-(j\omega - j\omega_0)t} dt$$

$$= \int_{-\infty}^{\infty} x(t) e^{-j(\omega - \omega_0)t} dt$$

~~$$= \int_{-\infty}^{\infty} x(t) e^{-j(\omega - \omega_0)t} dt$$~~

$$= X(\omega - \omega_0)$$

4. Duality Property :-

$$x(t) \leftrightarrow X(\omega)$$

$$X(t) \leftrightarrow 2\pi x(-\omega)$$

$$F^{-1}[X(\omega)] = x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Put $t = -t$

$$x(-t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{-j\omega t} d\omega$$

Interchanging $\omega \leftrightarrow t$

$$X(-\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$2\pi X(-\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$\boxed{FT[X(t)] = 2\pi X(-\omega)}$$

• $FT[\delta(t-t_0)] = e^{-j\omega t_0}$

$$\boxed{FT[e^{j\omega_0 t}] = 2\pi \delta(-\omega - \omega_0)}$$

5. Time Reversal :-

$$\text{If } x(t) \leftrightarrow X(\omega)$$

$$x(-t) \leftrightarrow X(-\omega)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(-t) e^{-j\omega t} dt$$

$$-t = p$$

$$\int_{-\infty}^{\infty} x(p) e^{j\omega p} dp = \int_{-\infty}^{\infty} x(p) e^{-j(-\omega)p} dp = X(-\omega)$$

$$\bullet \quad u(t) \xrightarrow[\text{integration}]{\text{diff.}} \delta(t) \text{ (Impulse)}$$

$$\bullet \quad \text{sgn}(t) = 2u(t) - 1 \Rightarrow \frac{1}{2} \text{sgn}(t) + \frac{1}{2} = u(t)$$

$$\Rightarrow F[-u(t)] = F\left[\frac{1}{2} \text{sgn}(t) - \frac{1}{2}\right]$$

$$= \frac{1}{2} F[\text{sgn}(t)] + \frac{1}{2} F[1]$$

$$= \frac{1}{2} \times \frac{2}{j\omega} + \frac{1}{2} (2\pi \delta(\omega))$$

$$= \frac{1}{j\omega} + \pi \delta(\omega)$$

6. Differentiation :-

$$x(t) \xleftrightarrow{FT} X(\omega)$$

$$\frac{dx(t)}{dt} \xleftrightarrow{FT} j\omega X(\omega)$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$\Rightarrow \frac{dx(t)}{dt} = \frac{j\omega}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$= \int_{-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \right]$$

$$= \int_{-\infty}^{\infty} F^{-1}[X(\omega)]$$

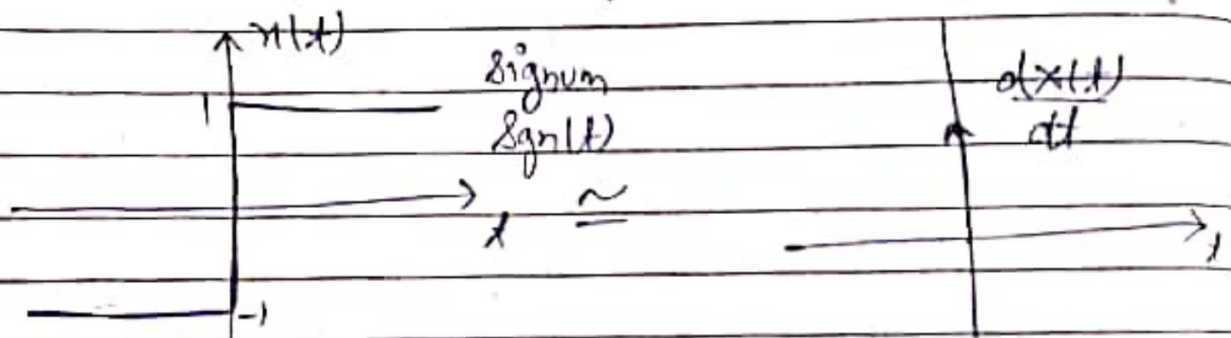
$$\frac{dx(t)}{dt} = \int_{-\infty}^{\infty} \frac{X(\omega)}{F}$$

General

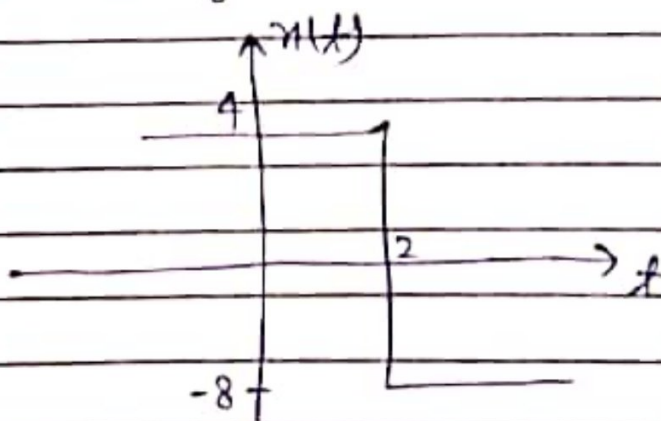
$$\frac{d^n x(t)}{dt^n} \xleftrightarrow{FT} (j\omega)^n X(\omega)$$

$$F\left(\frac{dx(t)}{dt}\right) = j\omega X(\omega)$$

Que.



Find FT of



$$\frac{dx(t)}{dt} = 2\delta(t)$$

$$F\left[\frac{dx(t)}{dt}\right] = 2F[\delta(t)]$$

$$j\omega X(\omega) = 2 \times 1$$

$$X(\omega) = \frac{2}{j\omega}$$

$$F[\text{sgn}(t)] = \frac{2}{j\omega}$$

⑦. Convolution :-

$$F[x_1(t) * x_2(t)] = X_1(\omega) \times X_2(\omega)$$

$$\int_{-\infty}^{\infty} [x_1(t) * x_2(t)] e^{-j\omega t} dt = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\tau) * x_2(t-\tau) e^{-j\omega t} d\tau \right] dt$$

$$\int_{-\infty}^{\infty} x_1(\tau) \left[\int_{-\infty}^{\infty} x_2(t-\tau) e^{-j\omega t} dt \right] d\tau$$

Put $t - \tau = p \Rightarrow \tau = t - p \quad t = p + \tau$

$$\int_{-\infty}^{\infty} x_1(\tau) \int_{-\infty}^{\infty} x_2(p) e^{-j\omega(p+\tau)} dp d\tau$$

$$\int_{-\infty}^{\infty} x_1(\tau) e^{-j\omega\tau} d\tau \int_{-\infty}^{\infty} x_2(p) e^{-j\omega p} dp \quad \text{Hence Proved.}$$

• Differentiation in frequency domain :-

$$x(t) \leftrightarrow X(\omega)$$

$$t x(t) \leftrightarrow j \frac{d}{d\omega} X(\omega)$$

Que. $F[t e^{-at} u(t)]$

$$j \left\{ \frac{d}{d\omega} \left(\frac{1}{a+j\omega} \right) \right\} = j^2 \left(\frac{-1}{(a+j\omega)^2} \right) = \frac{1}{(a+j\omega)^2}$$

Ques (1) Find the Convolution of the signals

$$x_1(t) = t \cdot e^{-t} \cdot u(t)$$

$$x_2(t) = e^{-3t} \cdot u(t)$$

Sol. $X(t) = X_1(t) * X_2(t)$

$$X_1(t) * X_2(t) = X_1(\omega) \cdot X_2(\omega)$$

$$X_1(\omega) = F[X_1(t)]$$

$$= F[t \cdot e^{-t} u(t)]$$

$$= j \frac{d}{d\omega} [X(\omega)]$$

$$t X_1(t) \rightarrow j \frac{d}{d\omega} (X(\omega))$$

$$t \cdot e^{-t} u(t)$$

$$= \frac{1}{(1+j\omega)^2}$$

$$X_2(\omega) = F[X_2(t)]$$

$$= F[e^{-3t} u(t)]$$

$$= \frac{1}{3+j\omega}$$

$$X(\omega) = X_1(\omega) \cdot X_2(\omega)$$

$$X(\omega) = \frac{1}{(1+j\omega)^2 (3+j\omega)}$$

$$F^{-1} X(\omega) = F^{-1} \left[\frac{1}{(1+j\omega)^2 (3+j\omega)} \right]$$

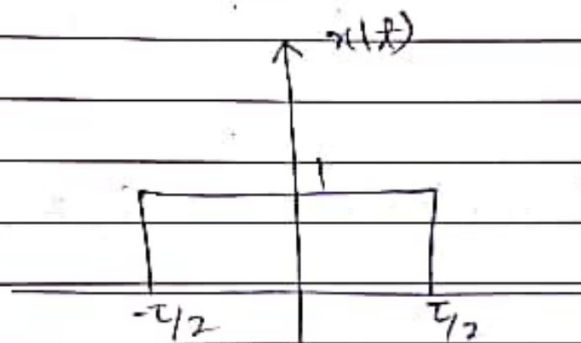
$$\frac{1}{(1+j\omega)^2 (3+j\omega)} = \frac{A}{(1+j\omega)} + \frac{B}{(1+j\omega)^2} + \frac{C}{(3+j\omega)}$$

$$A = -\frac{1}{4}, \quad B = \frac{1}{2}, \quad C = \frac{1}{4}$$

$$F^{-1} \left[\frac{-\frac{1}{4}}{(1+j\omega)} + \frac{\frac{1}{2}}{(1+j\omega)^2} + \frac{\frac{1}{4}}{(3+j\omega)} \right]$$

$$= -\frac{1}{4} e^{-t} u(t) + \frac{1}{2} t e^{-t} u(t) + \frac{1}{4} e^{-3t} u(t)$$

Ques 2) Fourier transform of $\text{rect}\left(\frac{t}{\tau}\right)$



$$x(t) = u\left(t + \frac{\tau}{2}\right) - u\left(t - \frac{\tau}{2}\right)$$

$$F\left[\frac{dx(t)}{dt}\right] = F\left[\delta\left(t + \frac{\tau}{2}\right) - \delta\left(t - \frac{\tau}{2}\right)\right]$$

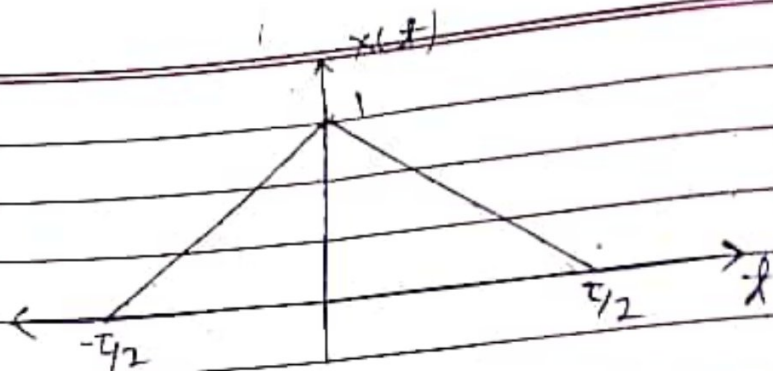
$$j\omega X(\omega) = e^{j\omega\tau/2} - e^{-j\omega\tau/2}$$

$$j\omega X(\omega) = 2j \sin \omega \frac{\tau}{2}$$

$$X(\omega) = \frac{2 \sin \omega \frac{\tau}{2}}{j\omega} \times \frac{1}{2}$$

$$X(\omega) = \tau \text{ sinc } \frac{\omega\tau}{2}$$

(3)



$$x(t) = u\left(t + \frac{\tau}{2}\right) + u\left(t - \frac{\tau}{2}\right) - u(t) - u(t)$$

$$x(t) = u\left(t + \frac{\tau}{2}\right) - 2u(t) + u\left(t - \frac{\tau}{2}\right)$$

$$\frac{dx(t)}{dt} = \delta\left(t + \frac{\tau}{2}\right) - 2\delta(t) + \delta\left(t - \frac{\tau}{2}\right)$$

$$\frac{d^2x(t)}{dt^2} = \delta\left(t + \frac{\tau}{2}\right) - 2\delta(t) + \delta\left(t - \frac{\tau}{2}\right)$$

$$(j\omega)^2 X(\omega) = e^{j\omega\tau/2} - 2 + e^{-j\omega\tau/2}$$

$$(j\omega)^2 X(\omega) = (e^{j\omega\tau/4})^2 - 2e^{j\omega\tau/4} \cdot e^{-j\omega\tau/4} + (e^{-j\omega\tau/4})^2$$

$$(j\omega)^2 X(\omega) = (e^{j\omega\tau/4} - e^{-j\omega\tau/4})^2$$

$$-\omega^2 X(\omega) = 2j \left(\sin \frac{\omega\tau}{4} \right)^2$$

• Properties :-

⑧ Integration -

$$\begin{aligned} x(t) &\rightarrow X(\omega) \\ \int x(t) dt &\rightarrow \frac{1}{j\omega} X(\omega) \end{aligned}$$

⑨ Time Scaling -

$$\begin{aligned} x(t) &\rightarrow X(\omega) \\ x(at) &\rightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right) \end{aligned}$$

⑩ Multiplication -

$$x_1(t) \cdot x_2(t) \xleftrightarrow{FT} \frac{1}{2\pi} [X_1(\omega) * X_2(\omega)]$$

• Parseval's Theorem :-

$$\int_{-\infty}^{\infty} x_1(t) \cdot x_2^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(\omega) X_2^*(\omega) d\omega$$

$$\begin{aligned} x_1(t) &\rightarrow X_1(\omega) \\ x_2(t) &\rightarrow X_2(\omega) \end{aligned}$$

$$\text{If } x_1(t) = x_2(t) = x(t)$$

$$\int_{-\infty}^{\infty} x_1(t) \cdot x_1^*(t) dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(\omega) \cdot X_1^*(\omega) d\omega$$

$$= \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X_1(\omega)|^2 d\omega$$

① $\delta(t) \rightarrow 1$

② $e^{-at} u(t) \rightarrow \frac{1}{a + j\omega}$

③ $e^{j\omega_0 t} \rightarrow 2\pi \delta(\omega - \omega_0)$

④ $1 \rightarrow 2\pi \delta(\omega)$

⑤ $e^{-a|t|} \rightarrow \frac{2a}{a^2 + \omega^2}$

⑥ $\text{sgn}(t) \rightarrow \frac{2}{j\omega}$

⑦ $u(t) \rightarrow \frac{1}{j\omega} + \pi \delta(\omega)$

⑧ $\text{rect}\left(\frac{t}{\tau}\right) \rightarrow \tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$

⑨ $\Delta\left(\frac{t}{\tau}\right) \rightarrow \frac{\tau}{2}$

• Properties :-

① $ax_1(t) + bx_2(t) \leftrightarrow aX_1(\omega) + bX_2(\omega)$

② $x(t - t_0) \leftrightarrow e^{-j\omega t_0} X(\omega)$

③

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⑤

⑥ $t x(t) \leftrightarrow j \frac{d}{d\omega} X(\omega)$

⑦

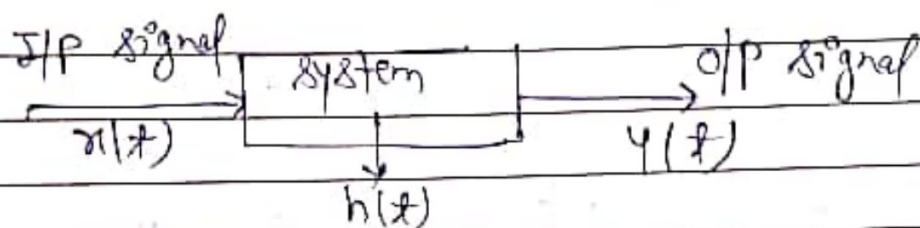
$$\begin{array}{ccc}
 x(t) & \longleftrightarrow & X(\omega) \\
 & \swarrow \quad \searrow & \\
 x(t) & \longleftrightarrow & 2\pi (X(-\omega))
 \end{array}$$

⑧

⑨

⑩

• System -



$$x(t) * h(t) = y(t)$$

• Frequency response :-

$$X(\omega) H(\omega) = Y(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

if $x(t) = \delta(t)$
 $X(\omega) = 1$

$$F^{-1}[Y(\omega)] = F[H(\omega)]$$

$y(t) = h(t) \rightarrow$ impulse response of the system.

Ques. The input and output of the causal LTI system are related by differential equation $\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6y(t) = x(t)$. Find the frequency response and Impulse response.

Sol.

$$FT \left[\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6y(t) \right] = FT [x(t)]$$

$$(j\omega)^2 Y(\omega) + 5j\omega Y(\omega) + 6Y(\omega) = X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$$Y(\omega) = \frac{X(\omega)}{(j\omega)^2 + 5j\omega + 6} = X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{(j\omega)^2 + 5j\omega + 6}$$

$$h(t) = F^{-1} [H(\omega)] = F^{-1} \left[\frac{1}{(j\omega)^2 + 5j\omega + 6} \right]$$

$$h(t) = F^{-1} \left[\frac{1}{(j\omega + 3)(j\omega + 2)} \right]$$

$$h(t) = F^{-1} \left[\frac{A}{(j\omega + 3)} + \frac{B}{(j\omega + 2)} \right]$$

$$h(t) = F^{-1} \left[\frac{-1}{(j\omega + 3)} + \frac{1}{(j\omega + 2)} \right]$$

$$h(t) = e^{-2t} u(t) - e^{-3t} u(t)$$

③ Find out the response of the system if $x(t) = t \cdot e^{-4t} u(t)$

~~$$(j\omega)^2 Y(\omega) + 5j\omega Y(\omega) + 6Y(\omega) = X(\omega) \cdot \frac{1}{(4+j\omega)^2}$$~~

$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

$$Y(\omega) = \frac{1}{(4+j\omega)^2} \times \frac{1}{\{(j\omega)^2 + 5j\omega + 6\}}$$

$$Y(\omega) = \frac{1}{(4+j\omega)^2 (3+j\omega) (2+j\omega)}$$

$$Y(t) = F^{-1} \left[\frac{1}{(4+j\omega)^2 (3+j\omega) (2+j\omega)} \right]$$

$$Y(t) = F^{-1} \left[\frac{A}{(4+j\omega)} + \frac{B}{(4+j\omega)^2} + \frac{C}{(3+j\omega)} + \frac{D}{(2+j\omega)} \right]$$

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Ques.

 $\cos(\omega_0 t) u(t)$

Sol.

$$FT[\cos(\omega_0 t) u(t)] = \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right] u(t)$$

$$= \frac{1}{2} e^{j\omega_0 t} u(t) + \frac{1}{2} e^{-j\omega_0 t} u(t)$$

$$= \frac{1}{2} \left[\frac{1}{j(\omega - \omega_0)} + \pi \delta(\omega - \omega_0) \right] + \frac{1}{2} \left[\frac{1}{j(\omega + \omega_0)} + \pi \delta(\omega + \omega_0) \right]$$

$$(2) \quad (i) \quad \frac{j\omega}{(2+j\omega)^2} = \frac{A}{(2+j\omega)} + \frac{B}{(2+j\omega)^2}$$

$$A=1 \quad \& \quad B=-2$$

$$F^{-1} \left[\frac{1}{(2+j\omega)} - \frac{2}{(2+j\omega)^2} \right]$$

$$= e^{-2t} u(t) - 2t e^{-2t} u(t)$$

$$(ii) \quad e^{-|w|}$$

$$X(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-|w|} e^{j\omega t} dw$$

$$= \frac{1}{2\pi} \left[\int_{-\infty}^0 e^w \cdot e^{j\omega t} dw + \int_0^{\infty} e^{-w} \cdot e^{j\omega t} dw \right]$$

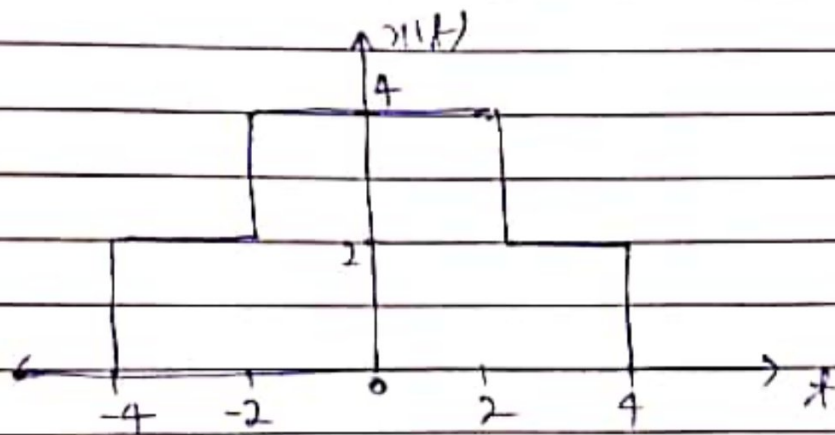
$$= \frac{1}{2\pi} \left[\frac{1}{j\omega} (1 - e^{j\omega t}) + \frac{1}{j\omega} (1 - e^{-j\omega t}) \right]$$

Attenuate (Duality property)

$$\begin{array}{ccc}
 e^{-|t|} & \longrightarrow & 2 \\
 & \searrow & \nearrow \\
 & & 1+t^2 \\
 & \swarrow & \searrow \\
 \frac{2}{1+t^2} & \longleftrightarrow & 2\pi e^{-|w|}
 \end{array}$$

$$e^{-|w|} = \frac{1}{2\pi} \left[\frac{2}{1+t^2} \right]$$

Que.



$$x(t) = \begin{cases} 2 & -4 \leq t \leq -2 \\ 4 & -2 \leq t \leq 2 \\ 2 & 2 \leq t \leq 4 \end{cases}$$

$$FT[x(t)] = \int_{-\infty}^{\infty} e^{-j\omega t} x(t) dt$$

$$= \int_{-4}^{-2} 2 \cdot e^{-j\omega t} dt + \int_{-2}^2 4 \cdot e^{-j\omega t} dt + \int_2^4 2 \cdot e^{-j\omega t} dt$$

$$= 2 \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-4}^{-2} + 4 \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-2}^2 + 2 \left[\frac{e^{-j\omega t}}{-j\omega} \right]_2^4$$

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$$= \frac{2}{-j\omega} \left[e^{2j\omega} - e^{4j\omega} \right] - \frac{4}{j\omega} \left[e^{-2j\omega} - e^{j2\omega} \right] + \frac{2}{j\omega} \left[e^{-4j\omega} - e^{-2j\omega} \right]$$

$$= \frac{\left[2e^{2j\omega} - 2e^{4j\omega} - 4e^{-2j\omega} + 4e^{j2\omega} - 2e^{-4j\omega} + 2e^{-2j\omega} \right]}{(j\omega)}$$

$$= 6e^{2j\omega} - 2e^{-2j\omega} - 2e^{4j\omega} - 2e^{-4j\omega}$$

Assignment

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$$T = 3$$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{3}$$

$$x(t) = \begin{cases} 1 & -1 < t < 1 \\ -1 & 1 < t < 3 \end{cases}$$

$$a_0 = \frac{2}{T} \int_0^T x(t) dt$$

$$= \frac{2}{3} \left[\int_0^1 1 dt + \int_1^3 (-1) dt \right]$$

$$= \frac{2}{3} \left[\int_0^1 1 dt - \int_1^3 1 dt \right]$$

$$= \frac{2}{3} \left[\left[t \right]_0^1 - \left[t \right]_1^3 \right]$$

$$= \frac{2}{3} \left[(1 - 0) - (3 - 1) \right]$$

$$[(0+1) + 1 - 1] \frac{1}{4}$$

$$\frac{1}{4} \text{ Ans}$$

$$a_n \Rightarrow \frac{2}{T} \int_{t_0}^{t_0+T} n(t) \cos n \omega_0 t dt$$

$$\Rightarrow \frac{1}{2} \int_{-1}^2 n(t) \cos\left(\frac{n\pi t}{2}\right) dt$$

$$\frac{1}{2} \left[\int_{-1}^0 \cos\left(\frac{n\pi t}{2}\right) dt + \int_0^1 \cos\left(\frac{n\pi t}{2}\right) dt \right]$$

$$- \int_1^2 \cos\left(\frac{n\pi t}{2}\right) dt$$

$$\frac{1}{2} \frac{2}{n\pi} \left[\left[\sin\left(\frac{n\pi t}{2}\right) \right]_{-1}^0 + \left[\sin\left(\frac{n\pi t}{2}\right) \right]_0^1 \right]$$

$$- \left[\sin\left(\frac{n\pi t}{2}\right) \right]_1^2$$

$$\frac{1}{n\pi} \left[-\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2} - \left[0 - \sin\frac{n\pi}{2} \right] \right]$$

$$\frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

$$b_n = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \sin n\omega t dt$$

$$= \frac{1}{2} \int_{-1}^1 \sin\left(\frac{n\pi}{2}t\right) dt + x(t)$$

$$\Rightarrow \frac{1}{2} \left[\int_{-1}^0 \sin\left(\frac{n\pi}{2}t\right) dt + \int_0^1 \sin\left(\frac{n\pi}{2}t\right) dt \right]$$

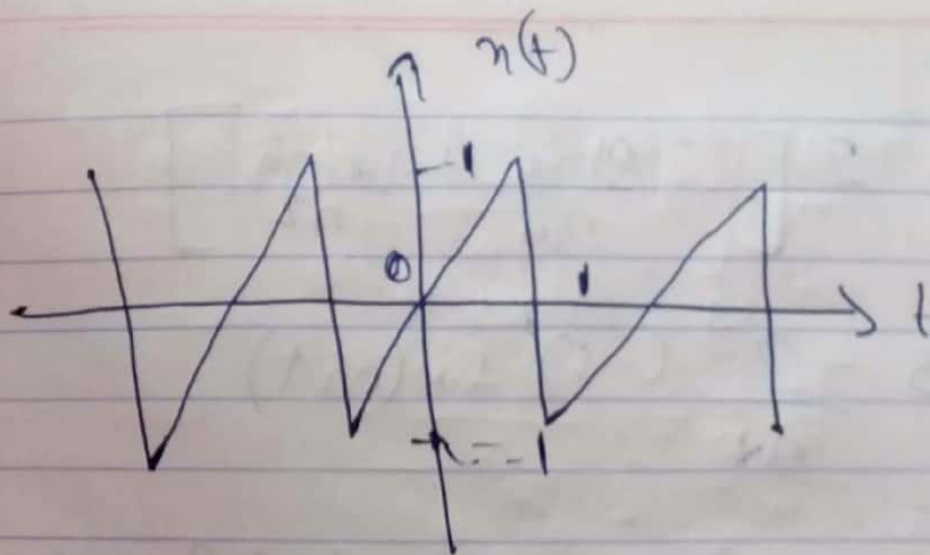
$$\frac{1}{2} \left[\cos\left(\frac{n\pi}{2}t\right) \right]_{-1}^0 + \left[\cos\left(\frac{n\pi}{2}t\right) \right]_0^1$$

$$\frac{2}{n\pi} \frac{1}{2} \left[-\left[\cos\left(\frac{n\pi}{2}t\right) \right]_{-1}^0 - \left[\cos\left(\frac{n\pi}{2}t\right) \right]_0^1 \right]$$

$$= \frac{1}{n\pi} \left[-1 - [-1] + \cos n\pi \right]$$

$$b_n = \frac{\cos n\pi}{n\pi}$$

Q Find the trigonometric series for the periodic signal $x(t)$.



Since the function is odd $a_0 = 0, a_n = 0$

$$b_n = \frac{4}{T} \int_0^{T/2} \sin n\omega_0 t \, dt \, x(t)$$

$$x(t) = \begin{cases} t & 0 \leq t \leq 1 \\ t-2 & 1 \leq t \leq 2 \end{cases}$$

$$\omega_0 = \pi$$

$$T = 2$$

$$\omega_0 = \frac{2\pi}{2} = \pi$$

$$\Rightarrow \frac{4}{2} \int_0^1 t \sin(\pi t) \, dt$$

$$\Rightarrow 2 \int_0^1 t \sin(\pi t) \, dt$$

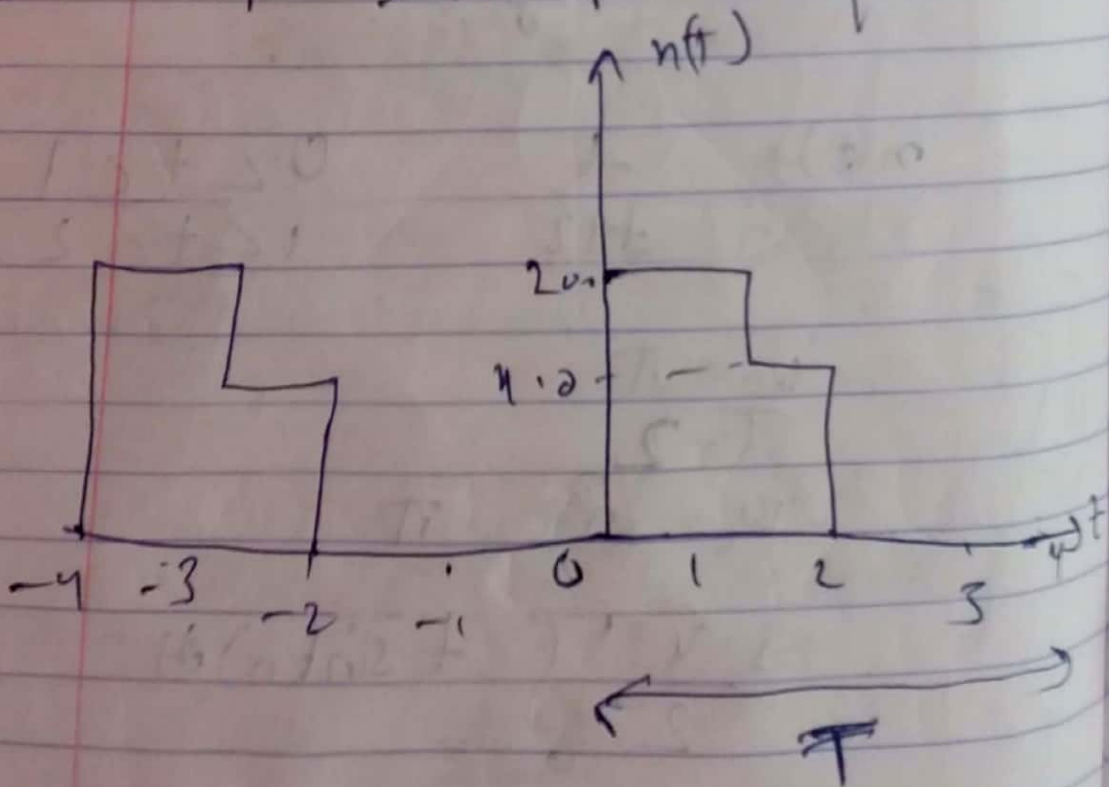
$$2 \left[\left[-t \cos\left(\frac{\pi t}{1}\right) \right]_0^1 + \frac{1}{\pi} \int_0^1 \cos(\pi t) \, dt \right]$$

$$2 \left[\frac{-\cos n\pi}{n\pi} + \frac{\sin n\pi}{n^2} \right]$$

$$b_n \Rightarrow \frac{-2}{\pi n} (-1)^n \sin(n\pi t)$$

$$n(t) \Rightarrow \frac{-2}{\pi n} (-1)^n \sin(n\pi t)$$

3 Compute the exponential Fourier series



$$n(t) = \begin{cases} 2 & 0 \leq t \leq 1 \\ 1 & 1 \leq t \leq 2 \\ 0 & 2 \leq t \leq 4 \end{cases}$$

$$T = W$$

$$W = \pi$$

$$C_n = \frac{1}{T} \int_0^T x(t) dt$$

$$\Rightarrow \frac{1}{T} \int_0^T x(t) dt$$

$$\frac{1}{4} \left[\int_0^1 2 dt + \int_1^2 1 dt + \int_2^3 0 dt \right]$$

$$C_n = \frac{1}{4} \int_0^4 x(t) e^{-j\omega_n t} dt$$

$$\Rightarrow \frac{1}{4} \int_0^4 x(t) e^{-j\omega_n \frac{t}{2}} dt$$

$$\Rightarrow \frac{1}{4} \left[\int_0^1 2 e^{-j\omega_n \frac{t}{2}} dt + \int_1^2 e^{-j\omega_n \frac{t}{2}} dt \right]$$

$$\Rightarrow \frac{1}{4} \left[-2 \left[\frac{e^{-j\omega_n \frac{t}{2}}}{-j\omega_n} \right]_0^1 + - \frac{2}{\omega_n} \left[e^{-j\omega_n \frac{t}{2}} \right]_1^2 \right]$$

$$\Rightarrow \frac{1}{4} \times \frac{-2}{j\omega_n} \left[e^{-j\omega_n \frac{1}{2}} - 1 \right] + \frac{2}{\omega_n} \left[e^{-j\omega_n} - e^{-j\omega_n \frac{1}{2}} \right]$$

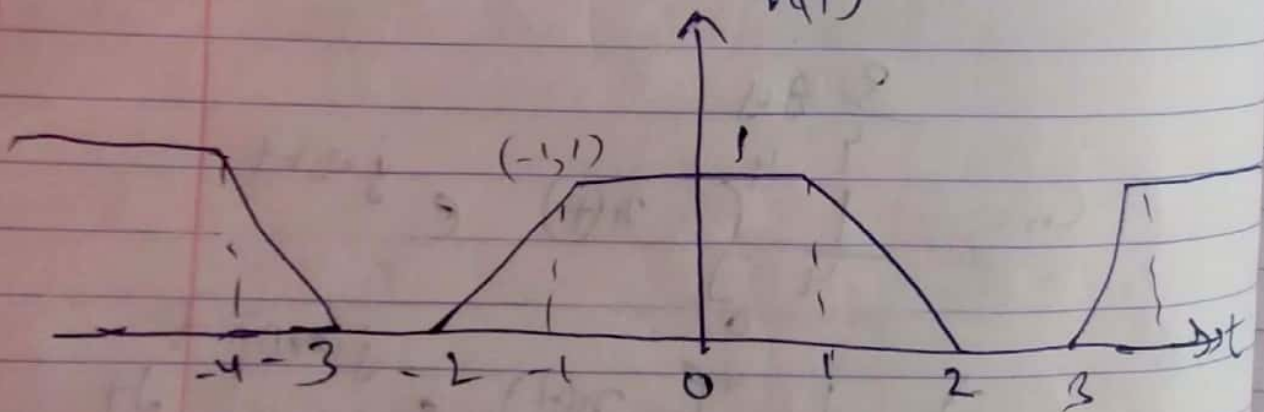
$$x(t) = C_0 + \sum_{n=-\infty}^{\infty} C_n e^{-jn\omega_s t}$$

Putting values of C_0 & C_n

$$x(t) = \frac{3}{4} + \frac{-1}{2} \sum_{n=-\infty}^{\infty} \frac{1}{jn\pi} [e^{-jn\pi} - 1] e^{-jn\pi t/2}$$

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Find exponential fourier series
 $x(t)$



$$x(t) \Rightarrow \begin{array}{ll} t-2 & -2 < t < -1 \\ -t+2 & -1 < t < 1 \\ 0 & 1 < t < 2 \\ & 2 < t < 3 \end{array}$$

$$C_0 = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) dt$$

$$\frac{1}{5} \int_{t_0}^{t_0+T} n(t) dt$$

$$T = 5$$

$$\omega_0 = \frac{2\pi}{5}$$

$$\Rightarrow \frac{1}{5} \int_{-2}^1 n(t) dt$$

$$\Rightarrow \frac{1}{5} \left[\int_{-2}^{-1} n(t) dt + \int_{-1}^1 n(t) dt \right]$$

$$\Rightarrow \frac{1}{5} \left[\int_{-2}^{-1} (t-2) dt + \int_{-1}^1 1 dt \right]$$

$$\frac{1}{5} \left[\left(\frac{t^2}{2} - 2t \right) \Big|_{-2}^{-1} + 2 \right]$$

$$\frac{1}{5} \left[\left(\frac{1}{2} - 2 \right) - \left(2 - 4 \right) + 2 \right]$$

$$\frac{1}{5} \left[\frac{8}{2} - 6 + 2 \right]$$

$$\frac{1}{5} \left[\frac{5}{2} - 4 \right]$$

$$\Rightarrow \frac{1}{5} \left[-\frac{3}{2} \right] \Rightarrow -\frac{3}{5}$$

5 Find the trigonometric series of $x(t) = t^2$ over the interval

$$a_0 = \frac{1}{T} \int_{-1}^1 x(t) dt$$

$$= \frac{1}{2} \int_{-1}^1 t^2 dt$$

$$= \frac{1}{2} \left[\frac{t^3}{3} \right]_{-1}^1 = \frac{1}{3}$$

$$a_n = \frac{1}{T} \int_{-1}^1 x(t) \cos n\pi t dt$$

$$\left[t^2 \frac{\sin n\pi t}{n\pi} \right]_0^1 - \frac{2}{n\pi} \int_0^1 t \cos(n\pi t) dt$$

$$\frac{1}{2} \left[\frac{\sin n\pi}{n\pi} - \frac{2}{n\pi} \int_0^1 t \cos n\pi t dt \right]$$

$$\frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\pi t$$

$$x(t) = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\pi t$$

$$\underline{7} \quad x(t) = 2 \sin(2\pi t - 3) + \sin(6\pi t)$$

$$2 \left[\frac{e^{j(2\pi t - 3)} - e^{-j(2\pi t - 3)}}{2j} \right]$$

$$+ \frac{e^{j6\pi t} - e^{-j6\pi t}}{2j}$$

$$\frac{e^{2j\pi t - 3j} - e^{-2j\pi t + 3j}}{2j} + \frac{e^{j6\pi t} - e^{-j6\pi t}}{2j}$$

$$C_{-3} = j0.5$$

$$C_{-1} = j e^{j3}$$

$$C_1 = -j e^{-j3}$$

$$C_3 = -j0.5$$

Ans

Q Find the exponential Fourier series coefficient

$$x(t) = 2 + \cos\left[\frac{2\pi t}{3}\right] + 4 \sin\left[\frac{5\pi t}{3}\right]$$

$$\Rightarrow 2 + \frac{e^{j2\pi t/3} + e^{-j2\pi t/3}}{2} + 4 \left[\frac{e^{j5\pi t/3} - e^{-j5\pi t/3}}{2j} \right]$$

$$C_0 = 2$$

$$C_2 = \frac{1}{2} \Rightarrow C_{-2} = \frac{1}{2}, \quad C_5 = -2j, \quad C_{-5} = 2j$$

$$\underline{2} \text{ ① } x(t) = 3 \cos 4\omega_0 t$$

$$= 3 \left[\frac{e^{4j\omega_0 t} + e^{-4j\omega_0 t}}{2} \right]$$

$$= \frac{3}{2} e^{4j\omega_0 t} + \frac{3}{2} e^{-4j\omega_0 t}$$

$$C_4 = \frac{3}{2}, C_{-4} = \frac{3}{2}$$

$$C_0 = 0$$

$$\text{b) } x(t) = 2 \sin 3\omega_0 t$$

$$= 2 \left[\frac{e^{3j\omega_0 t} - e^{-3j\omega_0 t}}{2j} \right]$$

$$C_3 = -j$$

$$C_{-3} = j$$

$$C_0 = 0$$

$$\underline{c) } x(t) = \sin \left[3t + \frac{\pi}{3} \right]$$

$$= \frac{e^{j[3t + \frac{\pi}{3}]} - e^{-j[3t + \frac{\pi}{3}]}}{2j}$$

$$= \frac{1}{2j} \left[e^{3jt} \left[\cos \frac{\pi}{3} + j \sin \frac{\pi}{3} \right] - e^{-3jt} \left[\cos \frac{\pi}{3} + j \sin \frac{\pi}{3} \right] \right]$$

$$\frac{1}{2j} \left[e^{3jt} \left[\frac{1}{2} + \frac{j\sqrt{3}}{2} \right] - e^{-3jt} \left[\frac{1}{2} - \frac{j\sqrt{3}}{2} \right] \right]$$

$$C_3 = \frac{1}{4j} + \frac{j\sqrt{3}}{2}, \quad C_{-3} = \left[\frac{1}{4j} - \frac{j\sqrt{3}}{2} \right]_{\text{m}}$$

Ans $x(t) = \sin^2 t$

$$= \frac{1 - \cos 2t}{2}$$

$$= \frac{1}{2} - \frac{1}{2} \left[\frac{1}{2} e^{j2t} + \frac{1}{2} e^{-j2t} \right]$$

$$C_2 = -\frac{1}{4}, \quad C_{-2} = -\frac{1}{4}$$

$$C_0 = \frac{1}{2} \text{ Ans}$$

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$$x(t) = e^{2t}$$

$$T = 2$$

$$\frac{1}{T} \int_0^T e^{2t} dt$$

$$\frac{1}{2} \int_0^2 e^{2t} dt$$

$$e^{-3jt} \left[\cos \frac{\pi}{3} - j \sin \frac{\pi}{3} \right]$$

$$\Rightarrow \frac{1}{2} \left[\frac{e^{2t}}{2} \right]_0^{\infty}$$

$$\Rightarrow \frac{1}{2} \left[\frac{e^4 - 1}{2} \right]$$

$$\Rightarrow \frac{1}{4} [e^4 - 1]$$

$$C_n = \frac{1}{T} \int_0^T e^{-jn\omega t} x(t) dt$$

$$\Rightarrow \frac{1}{4} \left[\frac{e^{(2-2jn)t}}{(1-jn)} \right]_0^2$$

$$\Rightarrow \frac{1}{4} \left[\frac{e^{2(2-2jn)}}{1-jn} - \frac{1}{1-jn} \right]$$

$$\frac{1}{4} \frac{1}{1-jn} (e^{4-4jt} - 1)$$

$$x(t) = \frac{1}{4} [e^4 - 1] + \sum_{n=1}^{\infty} \frac{1}{4} \frac{(e^{4-4jt} - 1)}{(1-jn)}$$

$$\underline{\underline{11}} \quad x(t) = t^2 \quad 0 \leq t \leq 1$$

$$T = 1 \text{ sec}$$

$$\omega_0 = 2\pi$$

$$C_0 = \frac{1}{T} \int_0^T x(t) dt$$

$$= \int_0^1 t^2 dt$$

$$= \left(\frac{t^3}{3} \right)_0^1$$

$$\Rightarrow \frac{1}{3}$$

$$C_n = \int_0^1 t^2 e^{-2\pi i n t} dt$$

$$\left[t^2 \frac{e^{-2\pi i n t}}{-2\pi i n} \right]_0^1 + \frac{1}{2\pi i n} \int_0^1 2t e^{-2\pi i n t} dt$$

$$= \frac{e^{-2\pi i n}}{-2\pi i n} + \frac{1}{\pi i n} \left[-t \frac{e^{-2\pi i n t}}{2\pi i n} \right]_0^1$$

$$+ \frac{1}{2\pi i n} \int_0^1 e^{-2\pi i n t} dt$$

$$= \frac{e^{-2\pi i n}}{-2\pi i n} + \frac{e^{-2\pi i n}}{2\pi i n} - \frac{1}{\pi^2 n^2} [e^{-2\pi i n} - 1]$$

$$x(t) = \frac{1}{3} + \sum_{n=-\infty}^{\infty} \frac{1}{n\pi} (\cos 2n\pi t - \frac{1}{n\pi} \sin 2n\pi t)$$

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$$C_n = |C_n| \angle \theta$$

$$C_0 = 1 \angle -\pi/3 = e^{-j\pi/3}$$

$$C_2 = 2 \angle \pi/6 = 2e^{j\pi/6}$$

$$C_{-1} = 1 \angle \pi/3 = e^{j\pi/3}$$

$$C_{-2} = 2 \angle -\pi/6 = 2e^{-j\pi/6}$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$e^{j\pi t} e^{-j\pi/3} + 2e^{j\pi t} e^{j\pi/6} + e^{-j\pi t} e^{j\pi/3} + 2e^{-j\pi t} e^{-j\pi/6}$$

$$2\cos\left[\pi t - \frac{\pi}{3}\right] + 4\cos\left[2\pi t + \frac{\pi}{6}\right]$$

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$$C_n = \left[-\frac{1}{3}\right]^{n/2} \quad \omega_0 = 1$$

$$x(t) = \sum_{n=-\infty}^{\infty} \left(-\frac{1}{3}\right)^{n/2} e^{jnt}$$

$$21) \sum_{n=-\infty}^{-1} \left[-\frac{1}{3} \right] e^{-jnt} + \sum_{n=0}^{\infty} \left(-\frac{1}{3} \right)^n e^{jnt}$$

$$\Rightarrow \frac{\left(-\frac{1}{3} \right) e^{-jt}}{1 + \frac{1}{3} e^{-jt}} + \frac{1}{1 - \frac{1}{3} e^{jt}}$$

$$\Rightarrow \frac{4}{5 + 3 \cos t} dt$$

1/6

35/2

2/8
1/5