

Interference

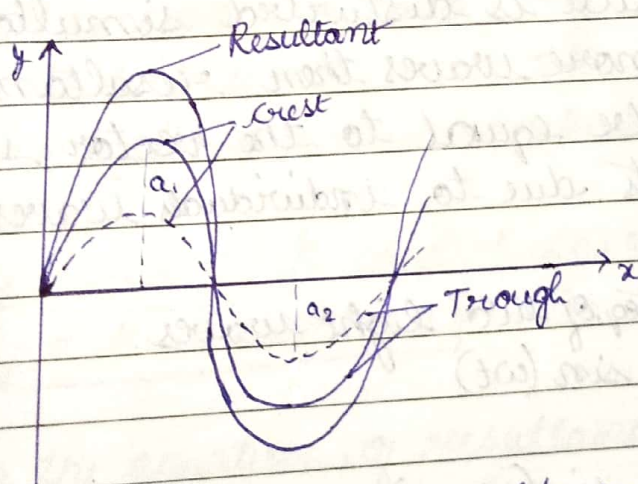
Interference

The phenomenon of superposition of two or more light waves having same frequency travelling in the same direction having constant initial phase difference in the region of superposition resulting in modification of intensity is called interference.

Types of interference

1. Constructive interference

When two light waves superpose in such a way that crest of one wave falls on the crest of another wave and intensity of resultant wave is more than the sum of the intensities of superposing waves.

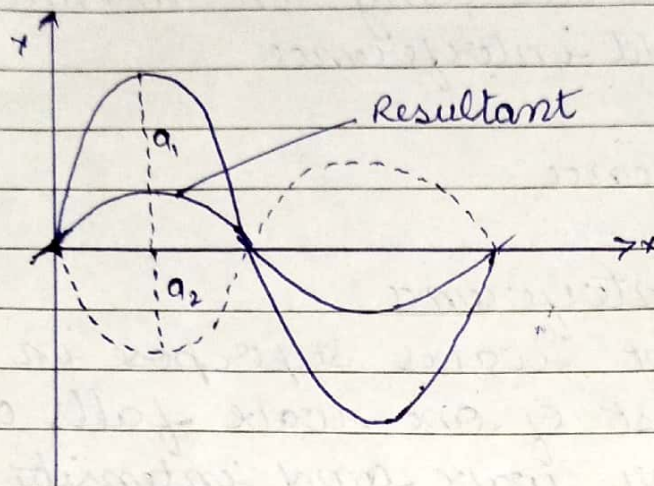


Waves are in same phase

$$I_{\text{resultant}} > I_1 + I_2$$

2. Destructive interference

When two light waves superpose in such a way that crest of one wave falls on the trough of another wave and vice versa and intensity of resultant wave is less than the sum of the intensities of superposing waves.



Waves are in opposite phase

$$I_R < I_1 + I_2$$

Principle of superposition

When a particle is disturbed simultaneously by two or more waves then resultant displacement will be equal to the vector sum of the displacements due to individual waves.

Displacement eq. of two light waves.

$$y_1 = a_1 \sin(\omega t)$$

$$y_2 = a_2 \sin(\omega t + \phi)$$

Resultant displacement

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \dots + \vec{y}_n$$

! Expression of resultant displacement and resultant intensity due to superposition of two light waves.

Let us consider two light waves of amplitudes a_1 and a_2 , angular frequency ω travelling in the same direction and has phase difference ϕ at time t .

Displacement equations of two light wave at time t are

$$y_1 = a_1 \sin(\omega t) \quad - (1)$$

$$y_2 = a_2 \sin(\omega t + \phi) \quad - (2)$$

Resultant displacement from the principle of superposition is $\Rightarrow$

$$y = y_1 + y_2$$

$$y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t \cos \phi + \sin \phi \cos \omega t)$$

$$y = \sin \omega t (a_1 + a_2 \cos \phi) + \cos \omega t (a_2 \sin \phi) \quad - (3)$$

$$\text{Let } a_1 + a_2 \cos \phi = A \cos \theta \quad - (4)$$

$$a_2 \sin \phi = A \sin \theta \quad - (5)$$

where A and θ are new constants,

so from (3)

$$y = \sin \omega t (A \cos \theta) + \cos \omega t (A \sin \theta)$$

$$\underline{y = A \sin(\omega t + \theta)} \quad - (6)$$

This is the equation of resultant wave having amplitude A and phase θ .

Squaring and adding eq. (4) & (5), we get.

$$A^2 (\cos^2 \theta + \sin^2 \theta) = (a_1 + a_2 \cos \phi)^2 + (a_2 \sin \phi)^2$$

$$A^2 = a_1^2 + a_2^2 \cos \phi + 2a_1 a_2 \cos \phi + a_2^2 \sin^2 \phi$$

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

Resultant intensity $\propto (\text{amplitude})^2$

$$I_R \propto A^2$$

Considering proportionality constant as 1.

$$I_R = A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

If I_1 and I_2 are intensities of individual superposing waves :-

$$I_1 = a_1^2 \quad I_2 = a_2^2$$

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

Conditions of constructive interference
for I_R to be maximum

$$\cos \phi = 1$$

$$\text{or } \phi = 0, 2\pi, 4\pi, \dots$$

$$\phi = 2n\pi \quad n = 0, 1, 2, 3, \dots$$

So, maximum intensity (Bright fringe)

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= a_1^2 + a_2^2 + 2a_1 a_2 = (a_1 + a_2)^2$$

Condition of destructive interference
for I_R to be minimum

$$\cos \phi = -1$$

$$\phi = \pi, 3\pi, 5\pi \dots$$

$$\phi = (2n-1)\pi$$

$$n = 1, 2, 3 \dots$$

So, minimum intensity (Dark fringe)

$$\begin{aligned} I_{\min} &= I_1 + I_2 - 2\sqrt{I_1 I_2} \\ &= a_1^2 + a_2^2 - 2a_1 a_2 \\ &= (a_1 - a_2)^2 \end{aligned}$$

Q1 If amplitudes of superposing waves are equal
i.e. $a_1 = a_2 = a$

$$\begin{aligned} I_R &= a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi \\ &= a^2 + a^2 + 2a^2 \cos \phi \\ &= 2a^2 + 2a^2 \cos \phi \\ &= 2a^2 (1 + \cos \phi) \end{aligned}$$

$$\begin{aligned} I_{\max} &= (a_1 + a_2)^2 \\ &= (a + a)^2 = (2a)^2 = \underline{\underline{4a^2}} \end{aligned}$$

$$I_{\min} = (a_1 - a_2)^2 = (a - a)^2 = \underline{\underline{0}}$$

Q2 If in an interference pattern the ratio b/w maximum and minimum intensities is 36:1, find the ratio b/w the amplitudes and intensities of two superposing waves.

→ $\frac{I_{\max}}{I_{\min}} = \frac{36}{1}$ Let two waves of amplitudes a_1 & a_2 get superposed.

$$\frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{36}{1} = \frac{(6)^2}{(1)^2}$$

$$\frac{a_1}{a_2} = \frac{6}{1}$$

$$\text{and } \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{36}{1} = \underline{\underline{36}}$$

Q3 Two coherent waves of amplitude 4 and 2 units get superposed, if phase difference of the resultant wave is δ find resultant intensity, ratio of max. and min. intensity

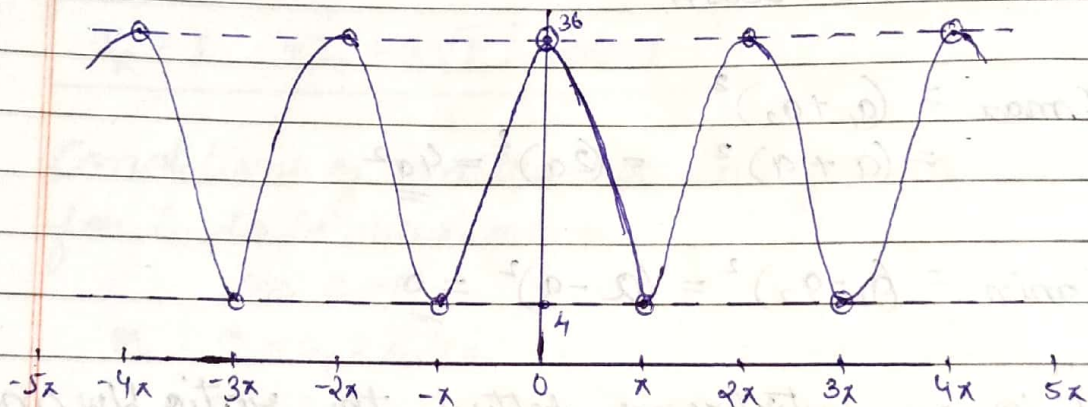
$$\begin{aligned} \rightarrow I_R &= 16 + 4 + 16 \cos \delta \\ &= 20 + 16 \cos \delta \end{aligned}$$

$$I_{\max} = 16 + 4 + 16 = 36$$

$$I_{\min} = 16 + 4 - 16 = 4$$

$$\frac{I_{\max}}{I_{\min}} = \frac{36}{4} = \frac{9}{1} = 9:1$$

Also plot the intensity distribution curve showing the variation of intensity with phase difference



Q4 Two coherent waves having intensity in the ratio α interference prove that

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{\alpha}}{\alpha + 1}$$

$$\rightarrow \frac{I_1}{I_2} = \alpha = \frac{a_1^2}{a_2^2} \Rightarrow a_1^2 = \alpha a_2^2 \text{ or } a_1 = \sqrt{\alpha} a_2$$

$$\begin{aligned} \text{LHS} &= \frac{a_1^2 + a_2^2 + 2a_1a_2 - a_1^2 - a_2^2 + 2a_1a_2}{a_1^2 + a_2^2 + 2a_1a_2 + a_1^2 + a_2^2 - 2a_1a_2} = \frac{4a_1a_2}{2a_1^2 + 2a_2^2} \\ &= \frac{2a_1a_2}{a_1^2 + a_2^2} = \frac{2\sqrt{\alpha}a_2^2 a_2}{\alpha a_2^2 + a_2^2} = \frac{2\sqrt{\alpha}}{1 + \alpha} \end{aligned}$$

Q4 Two identical coherent waves produce interference pattern, find the intensity at the centre of a bright fringe, also find the intensity at a point one quarter of the distance b/w two fringes from the centre, find the ratio of these two intensities.

→ $I_{\text{centre of bright fringe}} = ?$

for bright fringe $\cos \phi = 1$

$$\phi = 0, 2\pi, 4\pi, \dots$$

given $a_1 = a_2 = a$

$$\begin{aligned} I_{R1} &= a_1^2 + a_2^2 + 2a_1 a_2 \\ &= a^2 + a^2 + 2a^2 = 4a^2 \end{aligned}$$

$I_{1/4}$ of distance b/w two fringes

Phase diff b/w two fringes $= 2\pi$

$$\frac{1}{4} \text{ of this } \phi = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\begin{aligned} I_{R2} &= a^2 + a^2 + 2a_1 a_2 \cos \frac{\pi}{2} \\ &= 2a^2 \end{aligned}$$

$$\frac{I_{R1}}{I_{R2}} = \frac{4a^2}{2a^2} = 2:1$$

Average intensity I_{av} and law of conservation of energy

$$\begin{aligned}
 I_{av} &= \frac{\int_0^{2\pi} I_R d\phi}{\int_0^{2\pi} d\phi} = \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi) d\phi}{\int_0^{2\pi} d\phi} \\
 &= \left[a_1^2 \phi + a_2^2 \phi + 2a_1 a_2 \sin\phi \right]_0^{2\pi} \\
 &= \frac{a_1^2 (2\pi) + a_2^2 (2\pi) + 2a_1 a_2 (0)}{2\pi} \\
 &= a_1^2 + a_2^2 = \underline{\underline{I_1 + I_2}}
 \end{aligned}$$

Average intensity is defined as the average of maximum and minimum intensities

of the resultant wave.
As the phase of the wave varies from 0 to 2π

$$I_{av} = \frac{\int_0^{2\pi} I_R d\phi}{\int_0^{2\pi} d\phi}$$

It means that I_{av} is equal to sum of intensities of individual superposing waves, this means that total energy remain conserved in the phenomenon of interference.

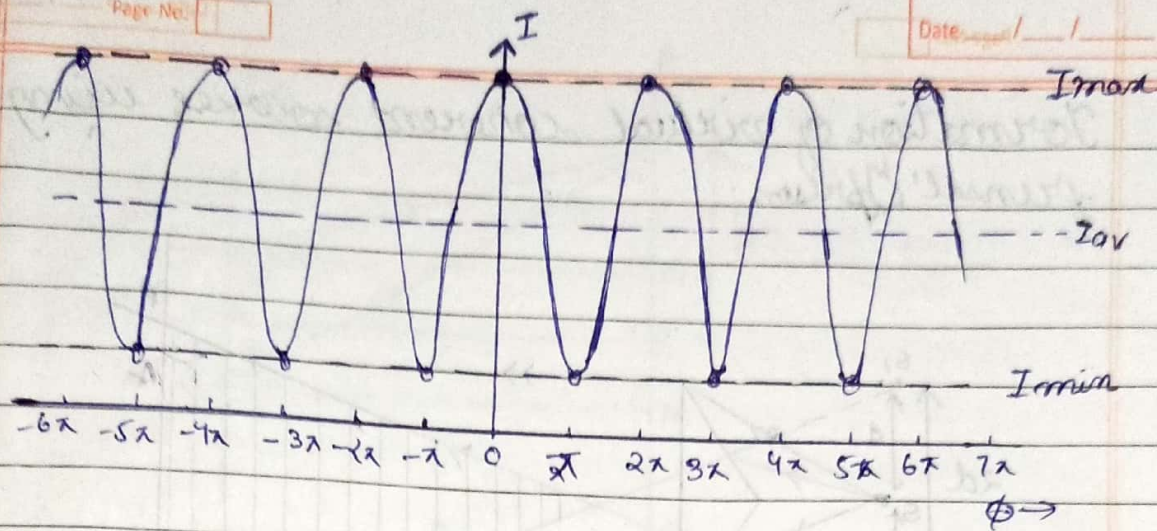
Intensity distribution curve (I versus phase diff)

If $a_1 \neq a_2$

$$I_R = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi$$

$$I_{max} = (a_1 + a_2)^2 \quad \phi = 0, 2\pi, 4\pi, \dots$$

$$I_{min} = (a_1 - a_2)^2 \quad \phi = \pi, 3\pi, 5\pi, \dots$$



If $a_1 = a_2 = a$

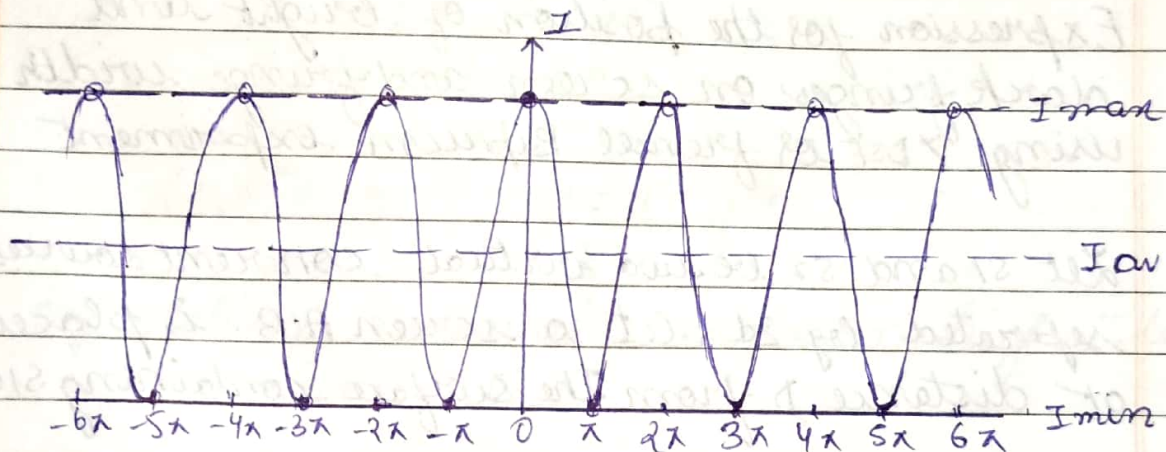
$$I_{max} = 4a^2$$

$$\phi = 0, 2\pi, \dots$$

$$I_{min} = 0$$

$$\phi = \pi, 3\pi, \dots$$

$$I_{av} = 2a^2$$



Classification of interference

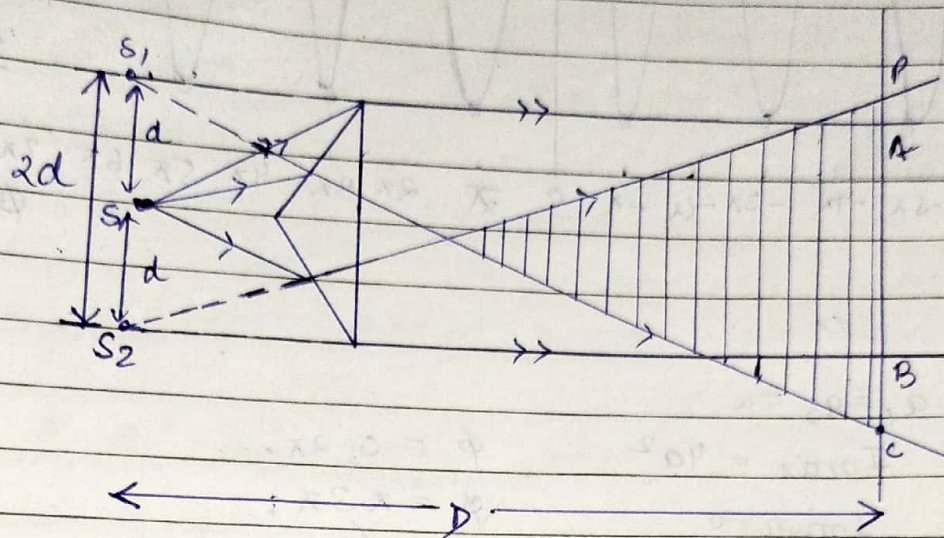
i) Division of wavefront

In this method incident wavefront is divided into two parts by reflection, refraction or diffraction, here narrow source of light is used.

ex - Young's double slit exp. and Fresnel's biprism exp.

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Formation of virtual coherent sources using Fresnel's biprism.



Expression for the position of bright and dark fringes on screen and fringe width using DSE or Fresnel Biprism experiment

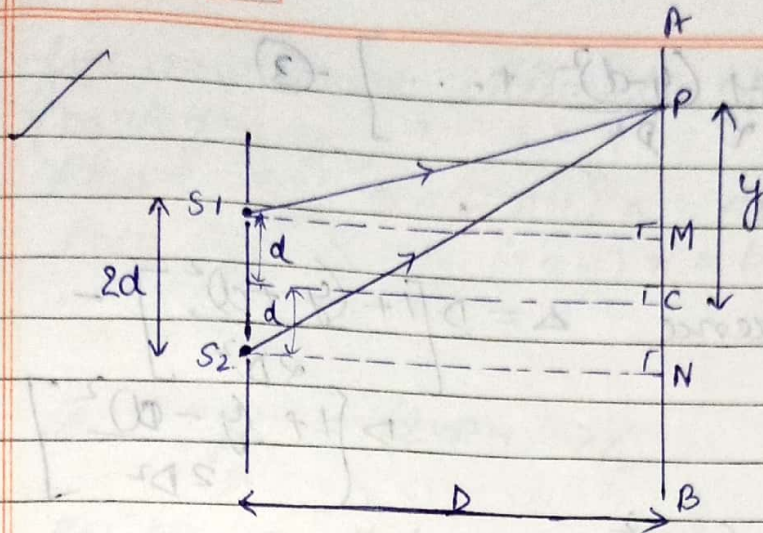
Let S_1 and S_2 be two virtual coherent sources separated by $2d$, let a screen AB is placed at distance D from the surface containing S_1 and S_2

If light rays of wavelength reach at point P on screen,

Path difference b/w the light rays from S_1 and S_2 that reach at point P on screen

$$\Delta = S_2P - S_1P \quad \text{--- (1)}$$

Let O be the mid point of $S_1 S_2$ then draw $S_1M \perp S_1M \perp AB$, $S_2N \perp AB$, $OC \perp AB$



Path difference $\Delta = S_2P - S_1P$ — (1)
 from right angled ΔS_2PN — (using Pythagoras theorem)

$$\begin{aligned}(S_2P)^2 &= (S_2N)^2 + (PN)^2 \\ &= D^2 + (y+d)^2 \\ &= D^2 \left[1 + \frac{(y+d)^2}{D^2} \right]\end{aligned}$$

$$(S_2P)^2 = D^2 \left[1 + \frac{(y+d)^2}{D^2} \right]^{1/2}$$

expanding binomially neglecting higher terms.
 $(y+d) \ll D$.

$$S_2P = D \left[1 + \frac{1}{2} \frac{(y+d)^2}{D^2} + \dots \right] \quad \text{--- (2)}$$

from right angled ΔS_1PM ,

$$(S_1P)^2 = D^2 + (y-d)^2$$

$$S_1P = D \left[1 + \frac{(y-d)^2}{D^2} \right]$$

similarly expanding binomially
 $(y-d) \ll D$; we get

first term

$$S.P = D \left[1 + \frac{1}{2} \frac{(y-d)^2}{D^2} + \dots \right] \quad (3)$$

so using eq. (1)

$$\text{Path difference } \Delta = D \left[1 + \frac{(y+d)^2}{2D^2} \right] - D \left[1 + \frac{(y-d)^2}{2D^2} \right]$$

$$\Delta = \frac{(y+d)^2}{2D} - \frac{(y-d)^2}{2D} = \frac{2yd}{D} \quad (4)$$

for maximum intensity at point P on screen (bright fringes)

$$\text{Phase diff } \phi = 2n\pi \quad n=0, 1, 2, \dots$$

$$\text{Path diff } \Delta = \frac{\lambda}{2\pi} \times 2n\pi = n\lambda \quad n=0, 1, \dots$$

$$\text{so, } \frac{2yd}{D} = n\lambda$$

Position of n^{th} bright fringe from the centre of screen

$$y_n = \frac{nD\lambda}{2a} \quad n=0, 1, 2, 3$$

for $n=0$, $y_0=0$ [zero order fringe at centre of screen]

for minimum intensity at point P on screen
(Dark fringe) $\rightarrow$

Phase diff $\phi = (2n-1)\pi$ $n=1, 2, \dots$

Path diff $\Delta = \frac{\lambda}{2\pi} (2n-1)\pi = (2n-1)\frac{\lambda}{2} = (2n-1)\lambda/2$

$n=1, 2, 3, \dots$

So, $\frac{2yd}{D} = (2n-1)\lambda/2$

Position of n^{th} dark fringe from centre $y'_n = (2n-1)\frac{\lambda D}{4d}$ $n=1, 2, 3$

Inter

Fringe width (β)

It is the distance b/w two consecutive bright fringes or two consecutive dark fringes on the screen.

If y_{n+1} and y_n be the position of n^{th} and $(n+1)^{\text{th}}$ bright fringes on screen, then $\beta = y_{n+1} - y_n$
 $= \frac{(n+1)D\lambda}{2d} - \frac{nD\lambda}{2d}$

$\beta = \frac{D\lambda}{2d}$

$2d$

If y'_{n+1} and y'_n be the position of n^{th} & $(n+1)^{\text{th}}$ dark fringes, $\beta = y'_{n+1} - y'_n = (2(n+1)-1)\frac{D\lambda}{4d} - (2n-1)\frac{D\lambda}{4d}$

$\beta = \frac{D\lambda}{2d}$

Fringe width, $\beta \propto D$

$$\beta \propto \lambda$$

$$\beta \propto \frac{1}{2d}$$

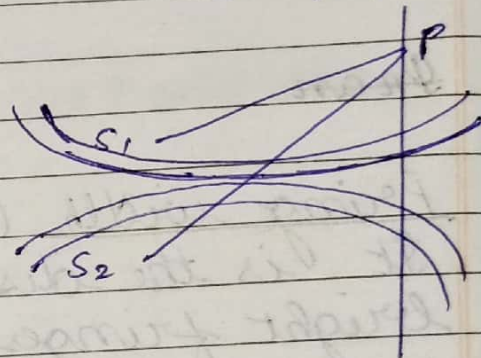
$\beta \neq n$ (order of fringe)

Shape of fringes in YDSE or Biprism expt
for max intensity on screen $\Delta = S_2P - S_1P = n\lambda$
 $n = 0, 1, 2, \dots$

for min intensity on screen $\Delta = S_2P - S_1P = (2n-1)\lambda/2$
 $n = 1, 2, 3, \dots$

$S_2P - S_1P = \text{constant}$
This is eqn. of hyperbola.

$$e = \sqrt{1 + \frac{b^2}{a^2}} > 1 \approx 10^6$$



In these experiment the shape of the fringes is hyperbolic but due to large value of e the fringes on the screen appear to be parallel straight, equidistant bright and dark lines.

Calculation of $2d$ in biprism experiment

1.) Deviation method

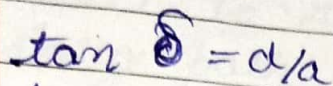
α = angle of biprism

a = distance b/w slits

μ = refractive index of biprism

(should be acute always)

and biprism



As $\frac{d}{a}$ is very small, $\tan \delta = \delta$

$$\delta = (n-1)\alpha$$

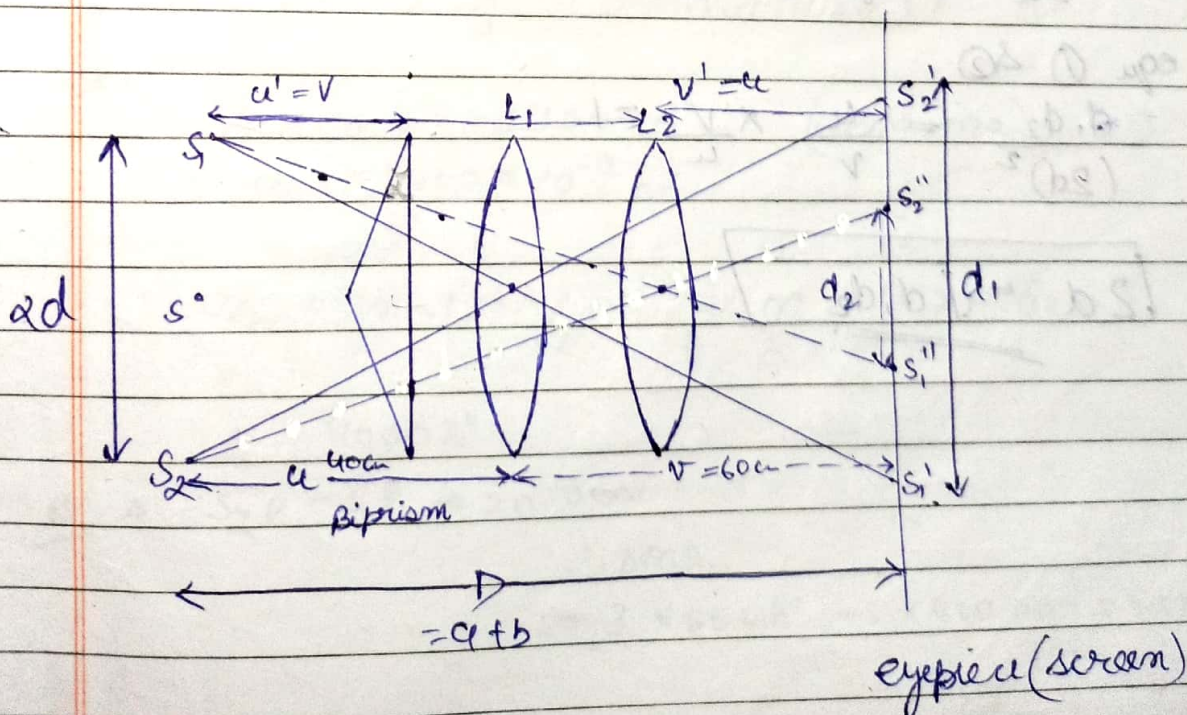
$$\frac{d}{a} = (n-1) \alpha$$

$$\underline{d = a(u-1) \propto}$$

$$2d = 2a(u-1) \propto$$

Displacement method

A convex lens is placed b/w biprism and screen (eyepiece)



u = object distance

v = image distance

d_1 = distance b/w images of S_1 and S_2

on screen for I position of lens (L_1)

d_2 = distance b/w images of S_1 and S_2 on screen for II position of lens (L_2)

Magnification formula

$$m = \frac{H_{\text{image}}}{H_{\text{object}}} = \frac{v}{u}$$

$$\boxed{\frac{d_1}{2d} = \frac{v}{u}} \quad \text{--- (1)}$$

for conjugate position of lens (L_2)

d_2 = distance between image of S_1 and S_2 on screen for II position of lens (L_2)

$$u' = v$$

$$v' = u$$

$$m = \frac{d_2}{2d} = \frac{v'}{u'}$$

$$\frac{d_2}{2d} = \frac{u}{v} \quad \text{--- (2)}$$

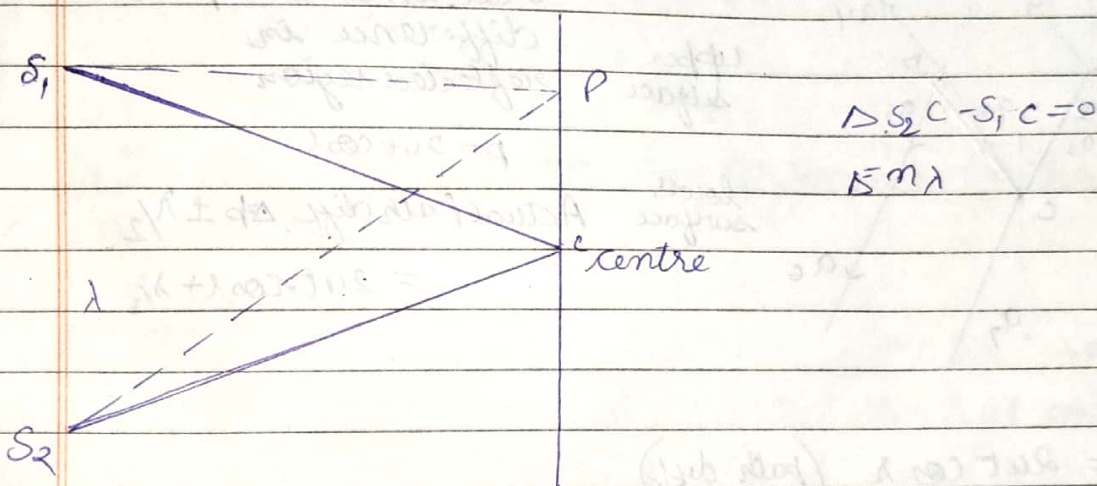
eqn (1) & (2)

$$\frac{d_1 d_2}{(2d)^2} = \frac{u}{v} \times \frac{v}{u} = 1$$

$$\boxed{2d = \sqrt{d_1 d_2}}$$

Fringes in white light in YDSE or Biprism exp^t (4000Å - 7000Å)

When white light is used in these experiment at centre (0 order fringe) colour will be white as we go above or below the centre fringes in increasing order of wavelength i.e. violet to red are seen but at a point of large path difference on the screen only uniform illumination is seen



for $n=0$ $\Delta = 0$ (maxima)
 $\Delta = (2n-1) \frac{\lambda}{2}$ (minima)

ex^t $\Delta = n\lambda = 1 \times 4000 \text{ Å}$ (maxima)
 screen = $4000 \times 10^{-8} \text{ cm}$

$\Delta = (2n-1) \frac{\lambda}{2}$ $n=1$ $\lambda = 8000 \text{ Å}$

$\Delta = 4000 \text{ Å}$

$\Delta = S_2P - S_1P \rightarrow 20,000$

$\therefore \Delta = n\lambda$

$= 3 \times 666 \text{ Å} = 5 \times 4000 \text{ Å} = 4 \times 5000 \text{ Å}$

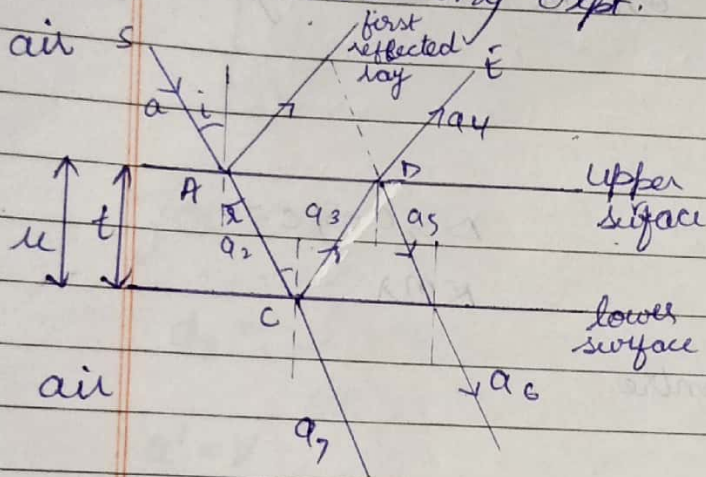
Interference due to division of amplitude

- 1) Partial reflection and partial refraction.
- 2) Broad source of light is used.

ex → Interference in parallel sided thin film.

Interference in wedge shaped thin film

Newton's ring expt.



Mathematical path difference in reflected region

$$p = 2ut \cos i$$

Actual path diff. $\Delta p \pm \lambda/2$
 $= 2ut \cos i \pm \lambda/2$

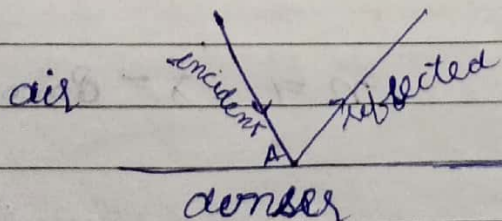
$$p = 2ut \cos i \text{ (path diff)}$$

① $SA + AB$

② $SA + AC + DE$

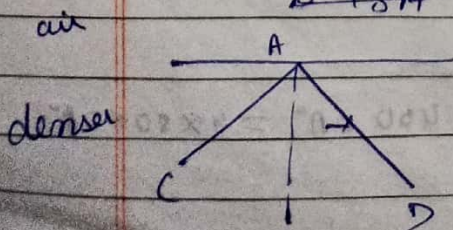
Path diff b/w
 coherent rays
 in reflected region
 $\lambda = \text{angle of reflection}$

Stokes law



Additional path change of $\lambda/2$

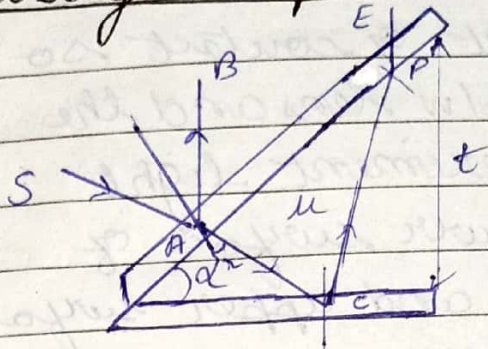
$$\Delta + SA - \Delta = SA + SB \pm \lambda/2$$



No additional path difference

According to this law whenever a light ray is reflected from the surface of denser medium additional path change of $\lambda/2$ take place
 or phase change of π

Wedge shaped thin film

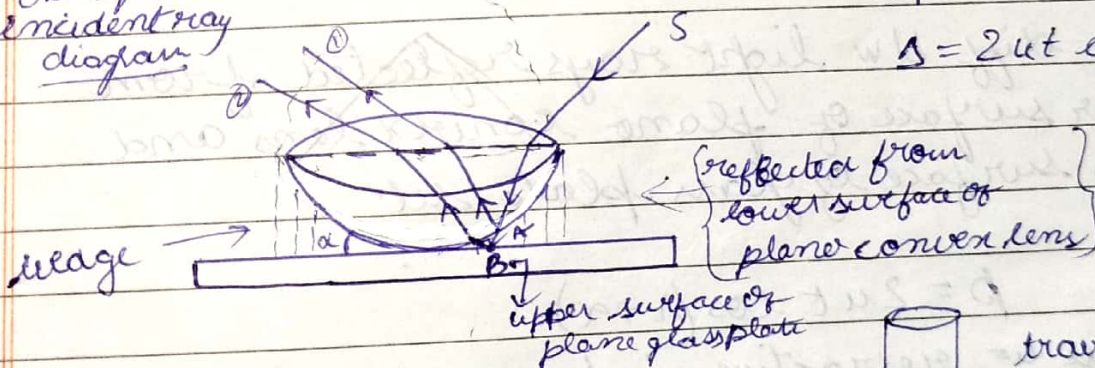


$\alpha = \text{angle of wedge}$

$$\Delta = 2ut \cos(r + \alpha) \pm \lambda/2$$

Newton's ring expt (interference by division of amplitude)

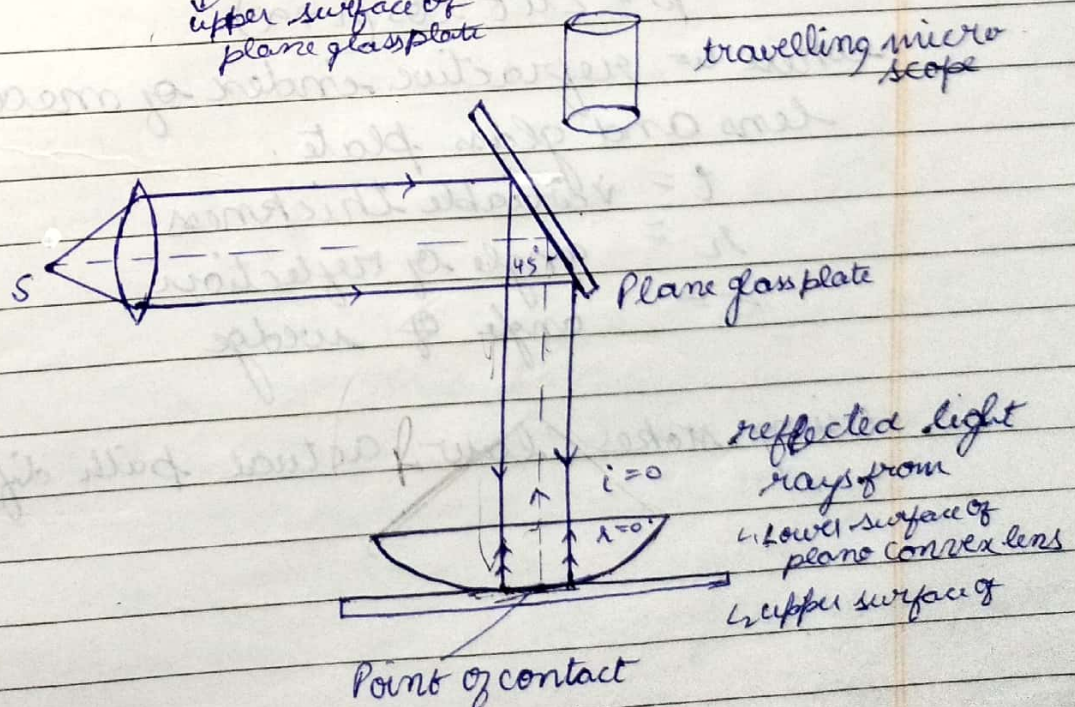
oblique incident ray diagram



$$p = 2ut \cos(r + \alpha)$$

$$\Delta = 2ut \cos(r + \alpha) \pm \lambda/2$$

Ray diagram expt set up



In Newton's ring exp. a plano convex lens of large radius of curvature is placed on a plane glass plate so that at point of contact of lens and the glass plate thickness is zero and thickness increases as we go away from the point of contact so a wedge is forming b/w lens and the glass plate. In this experiment light rays reflected from lower surface of ~~the~~ plano convex lens and upper surface of plane glass plate ^{get superposed} interfere and interference pattern in the form of concentric bright and dark fringe with dark centre is form.

The air film in b/w lens and the glass plate is of variable thickness.

Path diff b/w light rays reflected from lower surface of plano convex lens and upper surface of glass plate is-

$$p = 2ut \cos(r + \alpha)$$

where u = refractive index of medium ~~in~~ b/w lens and glass plate.

t = variable thickness

r = angle of reflection

α = angle of wedge

using Stokes law, actual path diff is,

$$\Delta = 2ut \cos(r + \alpha) + \lambda/2 \quad - (1)$$

here $i = r = 0$ (normal incidence)

As radius of curvature of lens is large, α is very small so $\cos \alpha \approx 1$ so,

$$\Delta = 2ut + \lambda/2 \quad - (2)$$

Condition of maxima bright

$$\Delta = n\lambda$$

$$n = 0, 1, 2, \dots$$

using eq (2),

$$2ut + \lambda/2 = n\lambda$$

or

$$2ut = (2n-1) \lambda/2 \quad \left. \begin{array}{l} n = 1, 2, 3, \dots \\ (3) \end{array} \right\}$$

Condition of minima (Dark rings)

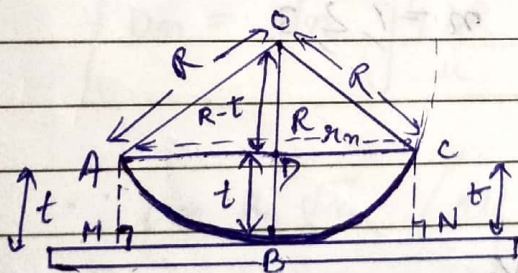
$$\Delta = (2n+1) \lambda/2$$

$$n = 0, 1, 2, \dots$$

using (2)

$$2ut + \lambda/2 = (2n+1) \lambda/2$$

$$(2ut = n\lambda) \quad - (4) \quad n = 0, 1, 2, \dots$$



If lens ABC is placed on glass plate,

R = radius of curvature of plano convex lens

r_n = radius of n^{th} ring passing through point A & C on lens

t = $\perp$ distance from lens to glass plate from point A & C

Using Pythagoras Theorem.

$$(OC)^2 = (OD)^2 + (DC)^2$$

$$R^2 = (R-t)^2 + (\lambda n)^2$$

$$R^2 = R^2 - 2Rt + t^2 + \lambda n^2$$

$$\lambda n^2 = 2Rt - t^2$$

As t is very small (due to small value of α) neglecting t^2 , we get -

$$\therefore \lambda n^2 \approx 2Rt$$

$$\text{or } \boxed{t = \frac{\lambda n^2}{2R}} \quad \text{--- (5)}$$

If D_n is diameter of n th rings
then $\lambda n = \frac{D_n}{2}$

$$t = \frac{D_n^2}{8R} \quad \text{--- (6)}$$

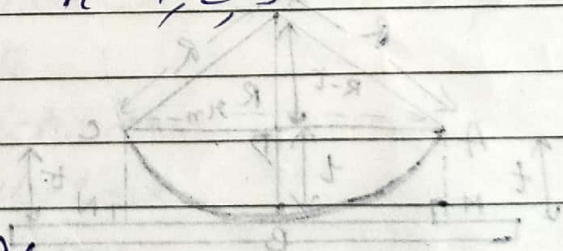
Diameter of bright rings
using eq (5) & (6);

$$2ut = (2n-1) \frac{\lambda}{2} \quad n = 1, 2, 3$$

Put $t = \frac{D_n^2}{8R}$, then

$$2u \left(\frac{D_n^2}{8R} \right) = (2n-1) \frac{\lambda}{2}$$

$$\boxed{D_n^2 = (2n-1) \left(\frac{2R\lambda}{u} \right)}$$



Point of contact is dark (on screen)

Diameter of n th bright ring

$$D_n = \sqrt{2n-1} \sqrt{\frac{2\lambda R}{\mu}} \quad n=1, 2, \dots$$

$$D_n \propto \sqrt{2n-1}$$

Diameter $\propto$ square root of odd natural nos.

Diameter of dark rings using eqn. (4) & (6)

$$2\mu t = n\lambda \quad \text{ant } t = \frac{D_n^2}{8R}$$

$$2\mu \left(\frac{D_n^2}{8R} \right) = n\lambda$$

$$D_n^2 = \frac{8R n \lambda}{2\mu}$$

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$D_n = \sqrt{n} \sqrt{\frac{4\lambda R}{\mu}} \quad n=1, 2, 3$$

$$\therefore D_n \propto \sqrt{n} \quad (\text{all natural nos.})$$

Determination of λ in Newton's ring expt

Diameter of n th dark ring is given by $D_n^2 = \frac{4n\lambda R}{\mu}$

then diameter of $(n+b)$ th dark ring is given by $D_{n+b}^2 = \frac{4(n+b)\lambda R}{\mu}$

$$\text{So, } D_{n+p}^2 - D_n^2 = \frac{4pR}{\mu}$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

for air as medium b/w lens & glass plate $\mu = 1$

$$\text{So, } \lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \text{ air}$$

determination of μ in Newton's ring expt.

Diameter of n^{th} dark ring is given by $D_n^2 = \frac{4nR}{\mu}$

$$\lambda \quad " \quad (n+p)^{\text{th}} \quad " \quad " \quad " \quad " \quad D_{n+p}^2 = \frac{4(n+p)R}{\mu}$$

$$D_{n+p}^2 - D_n^2 = \frac{4pR}{\mu}$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \mu$$

$$\mu_{\text{medium}} = \frac{4pR\lambda}{[D_{n+p}^2 - D_n^2]_{\text{medium}}} \quad \text{--- (1)}$$

for air as medium b/w lens & glass plate $\mu = 1$
 so $1 = \frac{[D_{n+p}^2 - D_n^2]_{\text{air}}}{4pR} \quad \text{--- (2)}$

Dividing (1) by (2)

$$\boxed{\mu_{\text{medium}} = \frac{[D_{n+p}^2 - D_n^2]_{\text{air}}}{[D_{n+p}^2 - D_n^2]_{\text{medium}}}}$$

Page No. Date / /

Why centre is dark in Newton's ring exp? (reflected light)

$$\Delta = 2ut \cos(r + \alpha) + \lambda/2$$
$$\cos(r + \alpha) = \cos \alpha \approx 1$$

$$\therefore \Delta = 2ut + \lambda/2 \rightarrow \text{Stoke's law}$$

At centre $t = 0$

$$\Delta = \lambda/2 \quad (\text{condition of minima})$$

Why pattern is circular $\rightarrow$

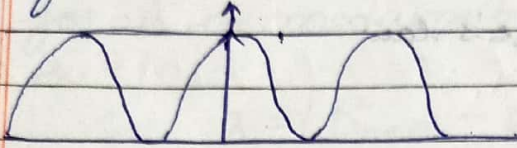
In Newton's ring exp. the locus of points of constant thickness lie in a circle so fringes are circular in shape.

$$\Delta = 2ut \cos(r + \alpha) + \lambda/2$$
$$\text{at } t=0, \cos(r + \alpha) = 1$$
$$\Delta = 2ut + \lambda/2$$
$$\text{at centre } t = 0$$
$$\Delta = \lambda/2$$

Diffraction

Interference

superposition of waves from coherent ^{sources} waves



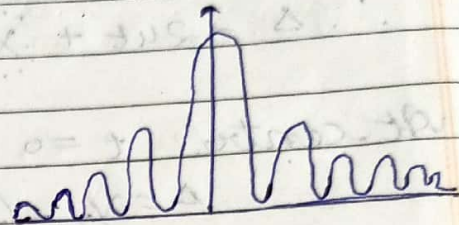
All maxima are same

All minima are perfectly dark

Distance b/w fringes is constant

Diffraction

Bending of light



Minima are not perfectly dark

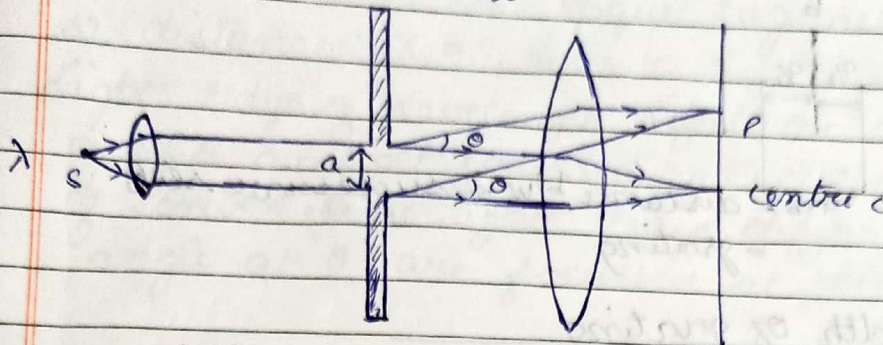
Fresnel's diffraction

- 1) Near field pattern
- 2) Spherical or cylindrical wavefronts
- 3) No lens are required

Fraunhofer diff.

- 1) Far field pattern
- 2) Plane wavefronts
- 3) Lens are required

Fraunhofer diffraction due to single slit.



a = width of slit

θ = angle of diffraction

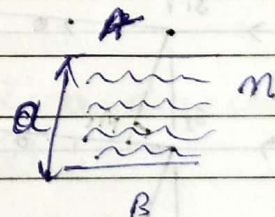
λ = wavelength of light

Resultant amplitude due to superposition of waves from single slit.

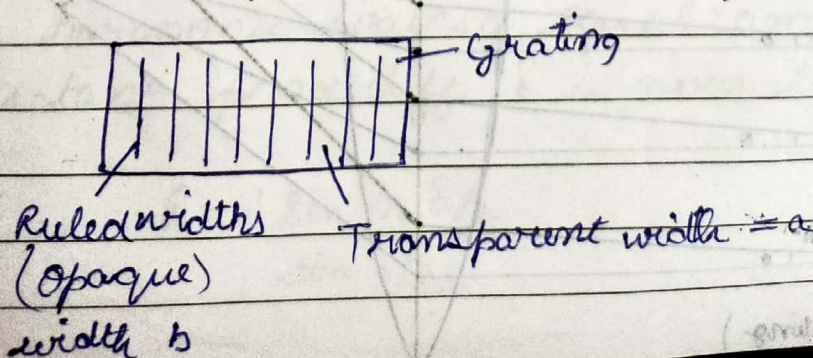
$$R = \frac{A \sin \alpha}{\alpha} \quad \text{where } \alpha = \frac{\pi a \sin \theta}{\lambda}$$

Resultant amplitude due to superposition of n harmonic waves of equal amplitude and constant phase diff δ -

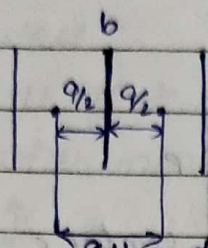
$$R = \frac{A \sin n\delta/2}{\sin \delta/2}$$



Diffraction grating (N slit surface)



alternate transparent and opaque regions

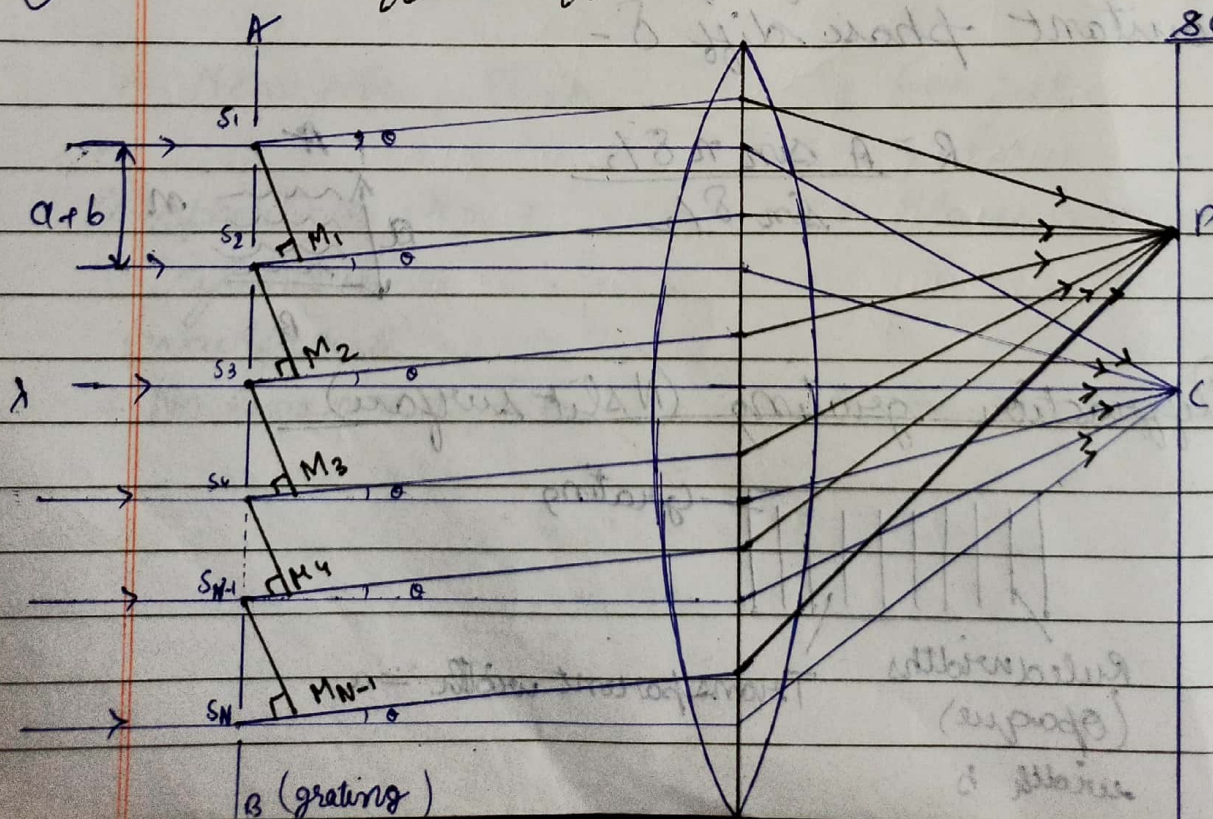


$a+b$ = distance b/w 2 successive slits
= grating

$a+b$ = width of grating
No. of lines on grating

An arrangement consisting of fine parallel equidistant alternate transparent and opaque regions on the plane of glass plate with the help of diamond point ^{pen} is called ~~point~~ diffraction grating

Fraunhofer diffraction due to N-slits (grating)

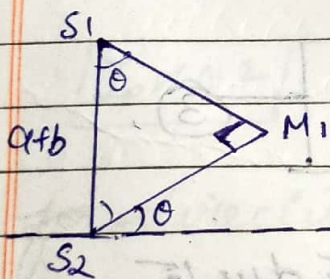


Let AB be a plane transmission grating consisting of N slits $S_1, S_2, \dots, S_N$ each of width a , and alternate opaque regions of width b , so, distance $S_1 S_2 = S_2 S_3 = \dots = S_{N-1} S_N = a + b$. Light rays of wavelength λ fall on centre $S_1, S_2, \dots, S_N$ of slits and get focussed on screen, with the help of convex lens light rays diffracting at an angle at ' θ ' are focussed at point P on the screen.

δ = Phase diff b/w waves from successive slits.

To find the path diff. b/w the wave from two successive slit draw $S_2 M_2 \perp S_2 P$, $S_2 M_2 \perp S_3 P$, $S_{N-1} M_{N-1} \perp S_{N-1} P$.

So path diff b/w waves from two successive slits $\Delta = S_2 M_1 = S_3 M_2 = \dots = S_N M_{N-1}$



$$\Delta = S_2 M_1 = (a+b) \sin \theta$$

So phase diff. b/w waves from two successive slits

$$\delta = \frac{2\pi}{\lambda} \Delta = \frac{2\pi}{\lambda} (a+b) \sin \theta \quad \text{--- (1)}$$

As resultant amplitude R due to superposition of n harmonic waves of equal amplitude and constant phase diff δ is given by -

$$R = \frac{a' \sin n \delta / 2}{\sin \delta / 2}$$

Here, $a' =$ amplitude of wave from each side $= A \sin \alpha$

where $\alpha = \frac{\pi a \sin \theta}{\lambda}$

$n = N$ (No. of slit)

$\beta = \frac{\pi (a+b) \sin \theta}{\lambda} = \beta$ (from ①)

So resultant amplitude due to diffraction from grating

$$R = A \frac{\sin \alpha}{\alpha} \frac{\sin N\beta}{\sin \beta} \quad \text{--- ②}$$

where $\alpha = \frac{\pi a \sin \theta}{\lambda}$

$\beta = \frac{\pi (a+b) \sin \theta}{\lambda}$

So intensity due to diffracting from grating

$I = R^2$

or

$$I = \frac{A^2 \sin^2 \alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2 \beta} \quad \text{--- ③}$$

intensity due to diffraction of light from single slit

intensity due to interference b/w waves from N -slits

Directions of principle maxima
 for intensity to be maximum, $\sin \beta \rightarrow 0$
 i.e., $\beta = \pm n\pi$ $n = 0, 1, 2, 3, \dots$

for $\beta = \pm n\pi$

$$\frac{\sin N\beta}{\sin \beta} = \frac{0}{0} \quad \left\{ \text{indeterminate form} \right\}$$

Using 2-Hospital rule

$$\lim_{\beta \rightarrow n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \rightarrow n\pi} \frac{\frac{d(\sin N\beta)}{d\beta}}{\frac{d(\sin \beta)}{d\beta}} = \lim_{\beta \rightarrow n\pi} \frac{N(\cos N\beta)}{(\cos \beta)} = N$$

$$\text{So for } \beta = \pm n\pi, \quad \frac{\sin^2 N\beta}{\sin^2 \beta} = N^2$$

So intensity of principle maxima

$$I_p = A^2 \frac{\sin^2 \alpha}{\alpha^2} N^2$$

for $\beta = \pm n\pi$

$$\boxed{I_p \propto N^2}$$

for directions of principal maxima

$$\beta = \pm n\pi \quad n = 0, 1, 2, \dots$$

$$\text{or, } \frac{2\pi}{\lambda} (a+b) \sin \theta = \pm n\pi$$

$$\boxed{(a+b) \sin \theta = n\lambda} \quad - (4) \quad n = 0, 1, \dots$$

Direction of minima

$$I = A^2 \frac{\sin^2 \alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2 \beta}$$

for intensity to be minimum,

$$\sin N\beta = 0 \quad \text{but } \sin \beta \neq 0$$

$$N\beta = \pm m\pi$$

$$N \times \frac{(a+b)}{\lambda} \sin \theta = \pm m\pi$$

$$\boxed{N(a+b) \sin \theta = \pm m\lambda}$$

$$(a+b) \sin \theta = \frac{m}{N} \lambda$$

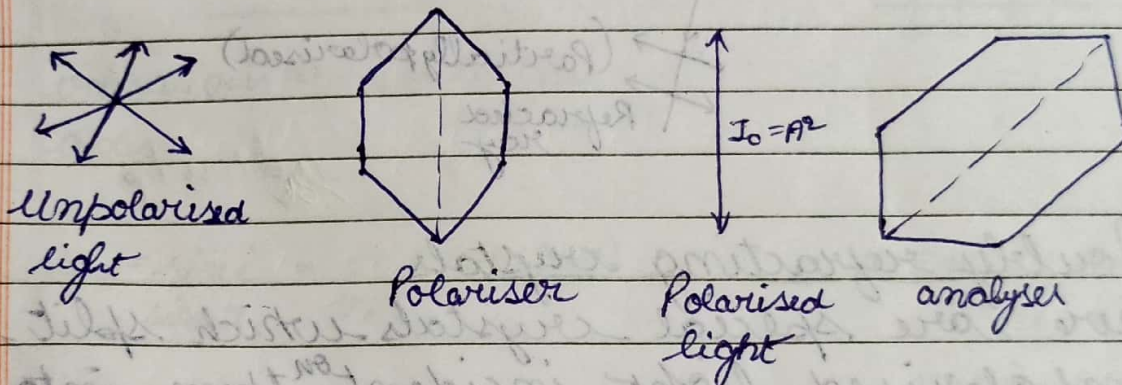
$$m \neq 0, N, 2N, \dots$$

$$m = 1, 2, \dots, \frac{N-1}{2}, \frac{N+1}{2}, \dots, 2N-1$$

$$\boxed{I \propto \sin^2 \theta}$$

Polarisation

- The phenomenon of polarisation explains the transverse wave nature of light. When unpolarised light is passed through polariser (ex - tourmaline crystal, nicole prism) the vibration of light are confined in a single direction and light is said to be polarised light. If another crystal ^(analyser) is placed against in front of the polarised light, the intensity of emergent light varies with angle b/w the planes of polariser and analyser.

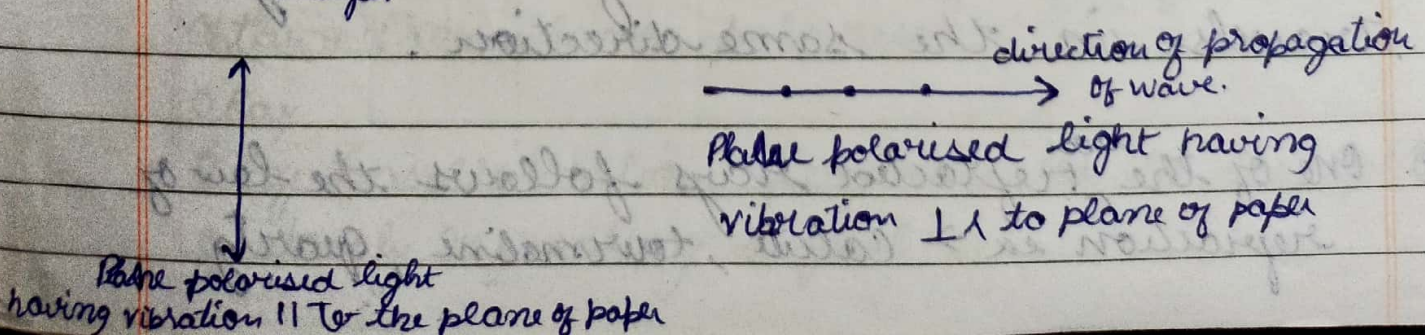


Law of Malus

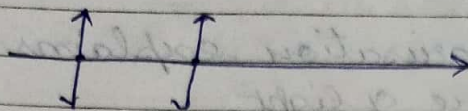
According to this law the intensity of light coming through analyser is directly proportional to the square of cosine of angle b/w the planes of polariser and analyser.

$$I_{\text{analyser}} \propto \cos^2 \theta$$

$$I_{\text{analyser}} = I_0 \cos^2 \theta = A^2 \cos^2 \theta$$

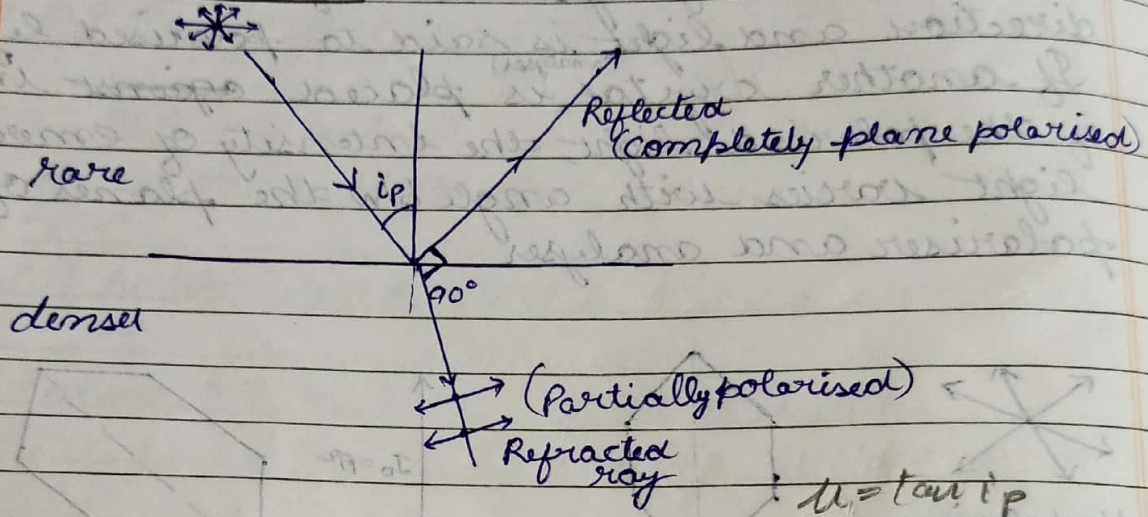


Partially polarised light



Having vibration in more than one direction.

Brewster's law



Doubly refracting crystals

There are special crystals which split the unpolarised light incident on them into two refracting rays are called doubly refracting crystals. There are of two types

① Uniaxial

② Biaxial

Uniaxial

① In such crystal there is a single direction in which the two refracted rays travel ^{with the} same direction.

② One of the ^{two} refracted rays follows the law of refraction ex → Calcite, tourmaline, quartz.

Biaxial

- ① In such crystals there are two direction in which the two refracted rays travels with same velocity
- ② None of the refracted follows the laws of refraction ex → mica, aragonite etc. Topaz

Optic axis

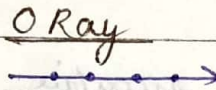
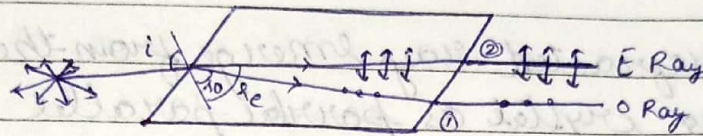
Optic axis is the direction in doubly refracting crystal in which all the refracted rays travels with the same velocity.

- this direction is identified by ~~white~~ drawing a ~~45°~~ axis from blunt corner making equal angle with all the edges meeting there

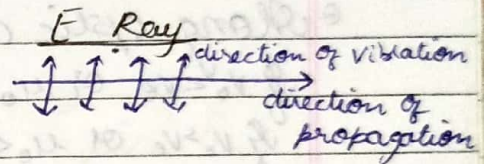
Double refraction

When unpolarised light ray falls on uniaxial doubly refracting crystal, it splits into two refracted rays, this phenomenon of splitting is called double refraction.

One of the refracted ray is called ordinary ray (O Ray) and second refracted ray is called extra ordinary ray (E Ray).



vibrations are $\perp$ to optic axis



$$\mu_o = \frac{\sin i}{\sin r_o} = \frac{c}{v_o} \text{ (constant)}$$

$$\mu_e = \frac{\sin i}{\sin r_e} \neq \text{constant}$$

varies with direction

If $v_o < v_e$ or $\mu_o > \mu_e \rightarrow$ uniaxial negative crystal
ex $\rightarrow$ calcite

If $v_o > v_e$ or $\mu_o < \mu_e \rightarrow$ uniaxial positive crystal
ex $\rightarrow$ quartz.

Characteristics

On double refraction both of the refracted rays obtained are plane polarised.

O Ray

1. O Ray has vibration perpendicular to the optic axis of doubly refracting crystal.

- The refracting index of O ray remain constant and it follows the laws of refraction.

E Ray

- E Ray has vibration parallel to the optic axis of doubly refracting crystal.
- The refractive index of E ray varies with direction.

• The two refracted rays emerge from the doubly refracting crystal as parallel ray.

• Along optic axis, $v_o = v_e$

If $v_o < v_e$ or $\mu_o > \mu_e \rightarrow$ uniaxial negative crystal, ex - calcite

If $v_o > v_e$ or $\mu_o < \mu_e \rightarrow$ uniaxial positive crystal, ex - quartz

Superposition of two plane polarised light waves having $\perp$ directions vibrations

When plane polarised light of amplitude A falls on a thin calcite plate at an angle θ with the optic axis, it split into two components $a \cos \theta$ and $a \sin \theta$. As E ray has vibrations parallel to optic axis, its amplitude is $a \cos \theta$ and $a \sin \theta$

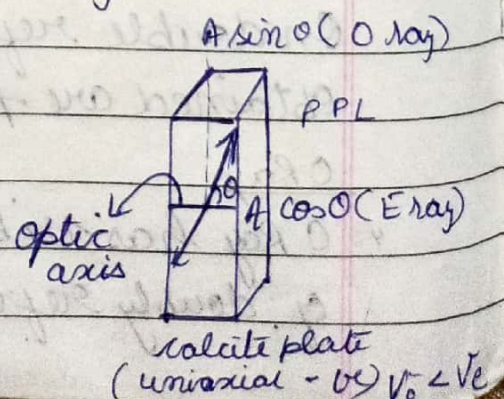
is the amplitude for O ray

In calcite crystal v_e is greater than v_o so a phase diff. of δ is introduced b/w the two waves. so displacement eq. of E ray and O ray are $\rightarrow$

$$X = A \cos \theta \sin(\omega t + \delta) \quad \text{--- (1)}$$

$$Y = A \sin \theta \sin \omega t \quad \text{--- (2)}$$

where $A \cos \theta = a$
 $A \sin \theta = b$ } $\rightarrow$ are amplitudes of E ray and O ray



then

$$X = a \sin(\omega t + \delta) \quad \text{and } Y = b \sin \omega t$$

$$X = a(\sin \omega t \cos \delta + \cos \omega t \sin \delta) \quad - (4)$$

$$\text{As } \sin \omega t = \frac{Y}{b}$$

$$\cos \omega t = \sqrt{1 - \sin^2 \omega t} = \sqrt{1 - \frac{Y^2}{b^2}}$$

$$X = a \left[\frac{Y}{b} \cos \delta + \sqrt{1 - \frac{Y^2}{b^2}} \sin \delta \right]$$

$$\frac{X}{a} = \frac{Y}{b} \cos \delta + \sqrt{1 - \frac{Y^2}{b^2}} \sin \delta$$

$$\frac{X}{a} - \frac{Y}{b} \cos \delta = \sqrt{1 - \frac{Y^2}{b^2}} \sin \delta$$

Squaring both sides

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} \cos^2 \delta - \frac{2XY}{ab} \cos \delta = \left(1 - \frac{Y^2}{b^2}\right) \sin^2 \delta$$

$$\boxed{\frac{X^2}{a^2} + \frac{Y^2}{b^2} - \frac{2XY}{ab} \cos \delta = \sin^2 \delta} \quad - (5)$$

This is the equation of oblique ellipse, so resultant light vector is elliptically polarised

Case 1 Phase diff $\delta = 0, 2\pi, \dots, 2n\pi$

$$\cos \delta = 1 \quad \sin^2 \delta = 0$$

$$\text{then eq (5)} \quad \frac{X^2}{a^2} + \frac{Y^2}{b^2} - \frac{2XY}{ab} = 0$$

$$\left(\frac{X}{a} - \frac{Y}{b}\right)^2 = 0$$

$$\frac{Y}{b} = \frac{X}{a} \quad \text{or} \quad \boxed{Y = \frac{bX}{a}}$$

This is eq of straight line so resultant light is plane polarised.

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Case II Phase diff $\delta = \pi, 3\pi, 5\pi, \dots, (2n-1)\pi$

$$\cos \delta = -1 \quad \sin^2 \delta = 0$$

then eq (5) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$

$$\left(\frac{x}{a} + \frac{y}{b} \right)^2 = 0$$

$$\frac{y}{b} = -\frac{x}{a} \quad \text{or} \quad \boxed{y = -\frac{bx}{a}}$$

This is eqn of straight line so resultant light is plane polarised

Case III $\rightarrow \delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots, (2n-1)\frac{\pi}{2}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

eq of ellipse (elliptically polarised)

Case IV : $\delta = \frac{\pi}{2}, \frac{3\pi}{2}, \dots, (2n-1)\frac{\pi}{2}$

Also, $a = b$

$$A \cos \theta = A \sin \theta \quad \text{i.e.} \quad \theta = 45^\circ$$

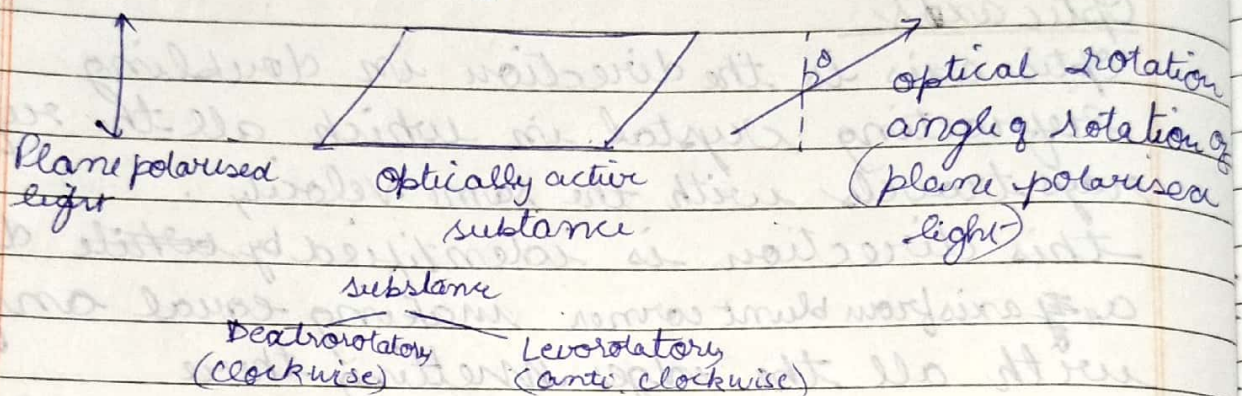
then,

$$\boxed{x^2 + y^2 = a^2}$$

eq. of circle (circularly polarised)

Rotatory polarisation or optical activity

When plane polarised light is passed through optically active substance the plane of polarisation of light is rotated by certain angle. This phenomenon is called rotatory polarisation, and the property of this optically active substance is called optical activity.



Optical rotation

The angle by which the plane of polarisation of plane pol light is rotated when passed through optically active substance is called optical rotation measured in degrees.

Biot's laws of plane polarised light

Optical rotation depends on

- ①. $\theta \propto l$ (length transversed in optically active)
- ②. $\theta \propto c$ (concentration or density = mass/volume)
- ③. $\theta \propto \frac{1}{\lambda^2}$ (rotatory dispersion) θ_{red} is least
 θ_{violet} is max
- ④ If plane polarised light passes through more than one optically active substance then
 $\theta = \theta_1 + \theta_2 + \dots + \theta_n$
 $\theta = +ve$ for dextro
 $\theta = -ve$ for laevo

Specific rotation $\rightarrow (S)$

Specific rotation of a substance at a given temp. for a particular wavelength of light is equal to optical ~~rotation~~ rotation if length and concentration of optically active substance are unity.

If $l = 1 \text{ dm}$

$c = 1 \text{ gm/cc}$

$S = 0$

$=$

$0 \propto l c$

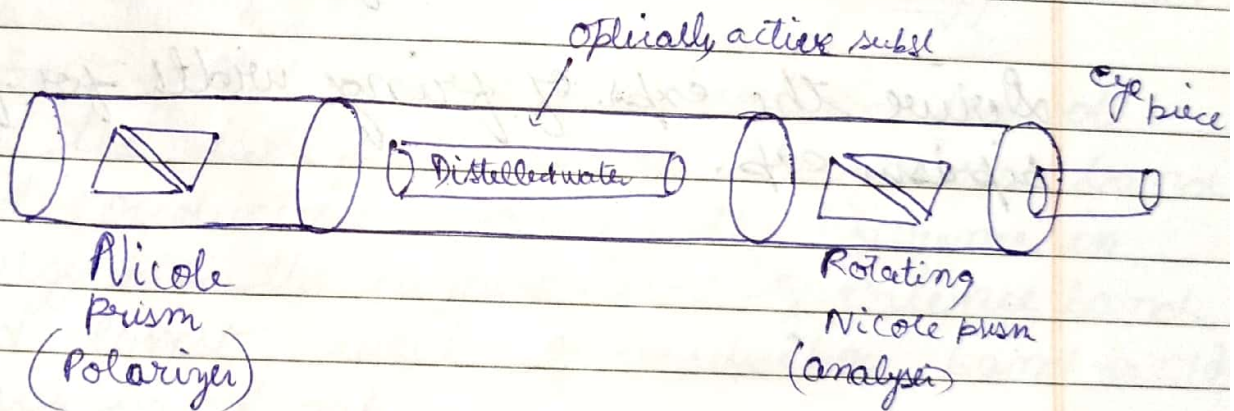
$0 \propto S l c$

$S = 0 - \text{deg.}$

$\text{dm} - l c \rightarrow \text{gm/cc}$

$S \neq 0, l, c$

Biquartz Polarimeter.



Solid State Physics

Energy bands in solids

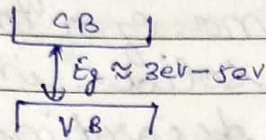
In case of single isolated atoms there are discrete energy levels which are acted upon by forces within the atom. When large no. of atoms are brought together to form a solid interactions b/w neighbouring atoms occur, according to Pauli's exclusion principle no two e^- s can have the same energy so splitting of a energy level into sub energy levels takes place. The difference of energy b/w two sub levels is of the order of 10^{-40} eV. The combination of such sub energy levels to form a band is called energy band.

Highest filled band is called valence band and the level just above the valence band is called conduction band, the difference in energy b/w the highest level of valence band and lowest level of conduction band is called energy band gap.

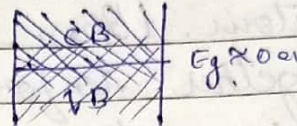
Classification of solids

Insulator	Conductor	semiconductors
1. Do not allow current to pass through them ex - Plastic, Rubber, mica etc	1. Allow current to pass through them easily ex $\rightarrow$ Cu, Al	1. Allow current to pass partially through them ex $\rightarrow$ Ge, Si
2. Resistivity ρ is very high, i.e., about $10^7 \Omega m$ - $10^{10} \Omega m$	2. Resistivity is very low about $10^{-6} - 10^{-8} \Omega m$	2. Resistivity more than conductor but less than insulator order $10^{-2} - 10^5 \Omega m$
3. Conductivity σ is very low	3. σ is very high	3. σ is less than conductor more than insulator

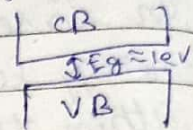
4. σ is independent of temperature
 5. Energy band gap is of order 3-5 eV



4. σ is inversely proportional to temp
 $\sigma \propto \frac{1}{temp}$
 5. Energy band gap is 0 eV



4. $\sigma \propto temp$
 5. $E_g \approx 1 eV$



* At absolute 0 semiconductor behaves like insulator

Classification of semiconductor

Intrinsic semiconductor

- ① Pure semiconductor
- ② σ is low
ex: Ge, Si

Extrinsic semiconductor

- ① Addition of impurities to
- ② increase the σ
 σ is high

Impurities $\rightarrow$ Dopants

Acceptor impurities $\rightarrow$

Trivalent

Group 13

ex $\rightarrow$ Al, B, Ga, In, Tl

Pentavalent

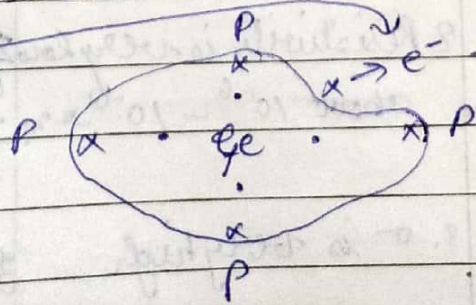
Group 15

ex: As, Sb, P

Donor impurities $\leftarrow$

N type semi

Pure semiconductor + Pentavalent impurity

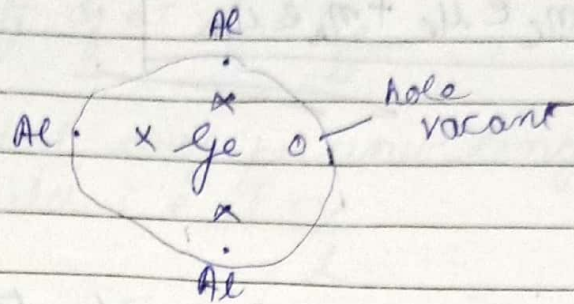


$e^- \rightarrow$ negatively charged

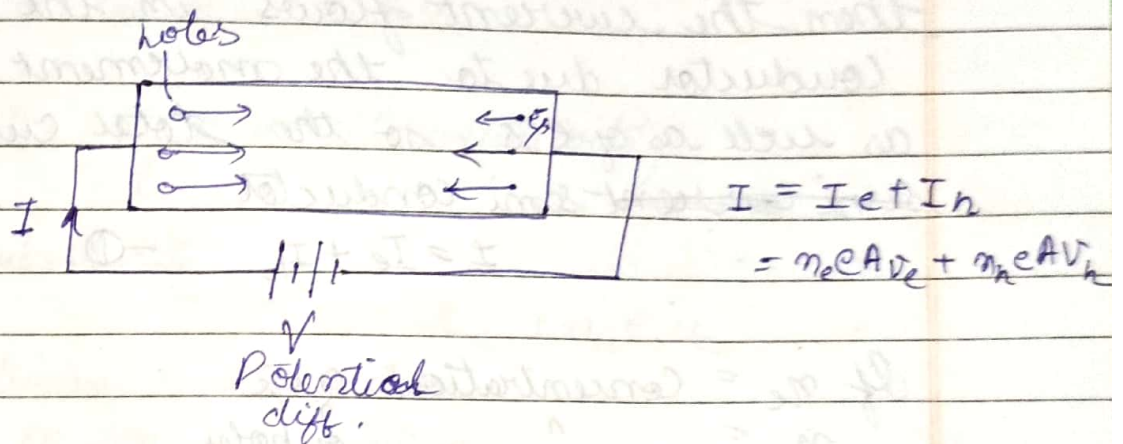
concentration of free e^- get increased

P-type semiconductor

Pure semiconductor + Trivalent impurity



Conduction in semiconductors



n_e = Concentration of e^- s

$v_e \rightarrow$ drift velocity of e^- s

n_h = concentration of holes

$v_h \rightarrow$ drift velocity of holes

Concentration $\rightarrow \frac{\text{no of } e^- \text{ or holes}}{\text{Volume}}$

Conductivity of semiconductors

$$I = n_e e A v_e + n_h e A v_h$$

$$V = IR = [n_e e A v_e + n_h e A v_h] R$$

$$V = [n_e e A v_e + n_h e A v_h] \frac{e l}{A}$$

$$E \frac{V}{l} = n_e e v_e + n_h e v_h$$

$$E = [n_e e \mu_e + n_h e \mu_h] E$$

$$\left(\mu = \frac{v_d}{E} \right) v = \mu E$$

$$\sigma = \frac{1}{\rho} = n_e e \mu_e + n_h e \mu_h$$

$$\boxed{\sigma = n_e e \mu_e + n_h e \mu_h}$$

Let us consider a semiconductor material across which potential diff V is applied then the current flows in the semiconductor due to the movement of holes as well as of e^- s so the total current ^{flow through} ~~in~~ ~~semi conductor~~ ~~semi conductor~~

$$I = I_e + I_h \quad \text{--- (1)}$$

If n_e = concentration of e^- s

n_h = " " of holes

v_e, v_h = drift velocities of e^- s and holes

e = charge of each e^- or hole

A = Area of cross section of semiconductor

then,

$$I = n_e e A v_e + n_h e A v_h$$

If ρ is resistivity offered by semiconductor of length l and area A . then resistance $R = \frac{\rho l}{A}$

the potential diff V across the semiconductor

$$V = IR$$

$$V = [n_e e A v_e + n_h e A v_h] \frac{\rho l}{A}$$

$$\frac{V}{l} = (n_e e v_e + n_h e v_h) \rho$$

so drift velocity per unit electric field is termed as mobility,
 so mobility of e^- $\mu_e = \frac{v_e}{E}$, $\mu_n = \frac{v_n}{E}$

Also potential drop per unit length is equal to electric field i.e. $E = \frac{V}{l}$

$$E = (n_e e \mu_e E + n_n e \mu_n E) \rho$$

$$\frac{1}{\rho} = n_e e \mu_e + n_n e \mu_n$$

so $\frac{1}{\rho} = \sigma$ (Conductivity)

conductivity

$$\sigma = n_e e \mu_e + n_n e \mu_n$$

for intrinsic semiconductors,

$$n_e = n_n = n_i \text{ — intrinsic charge carrier concentration}$$

so

$$\sigma = n_i e (\mu_e + \mu_n)$$

for N-type semiconductors

$$n_e \gg n_n$$

so $\sigma \approx \sigma_e$

$$\sigma \approx n_e e \mu_e$$

for P-type semiconductor

$$n_n \gg n_e$$

so $\sigma \approx \sigma_h$

$$\sigma \approx n_n e \mu_n$$

Law of mass action

$$n_e \times n_h = (n_i)^2$$

Fermi dirac statisticsFermi function

$$F(E) = \frac{1}{1 + e^{(E - E_F)/KT}}$$

$\downarrow$
 0 to 1
 (probability)

E_F = Fermi energy in eV
 K = Boltzmann constant

particles having half spin

As e^- s are fermi ions fermi dirac statistics is applied to find the probability of presence of e^- in a particular energy level having energy E at temp. T K

Case 1

If $E = E_F$ at $T > 0$ K

$$F(E) = \frac{1}{1 + e^{0/KT}} = \frac{1}{1 + 1} = \frac{1}{2}$$

Probability Prob. of finding e^- in conduction band is $1/2$

Case 2: If $E > E_F$ at $T = 0$ K

$$F(E) = \frac{1}{1 + e^{+\infty}} = \frac{1}{1 + \infty} = \frac{1}{\infty} = 0$$

Probability of finding e^- in conduction band is 0.

Case 3If $E < E_F$ at $T = 0K$

$$F(E) = \frac{1}{1 + e^{-\frac{E - E_F}{kT}}} = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + \frac{1}{e^{\infty}}} = \frac{1}{1 + \frac{1}{\infty}} = \frac{1}{1 + 0} = 1$$

so, probability of finding e^- in VB is 1.Fermi energy level

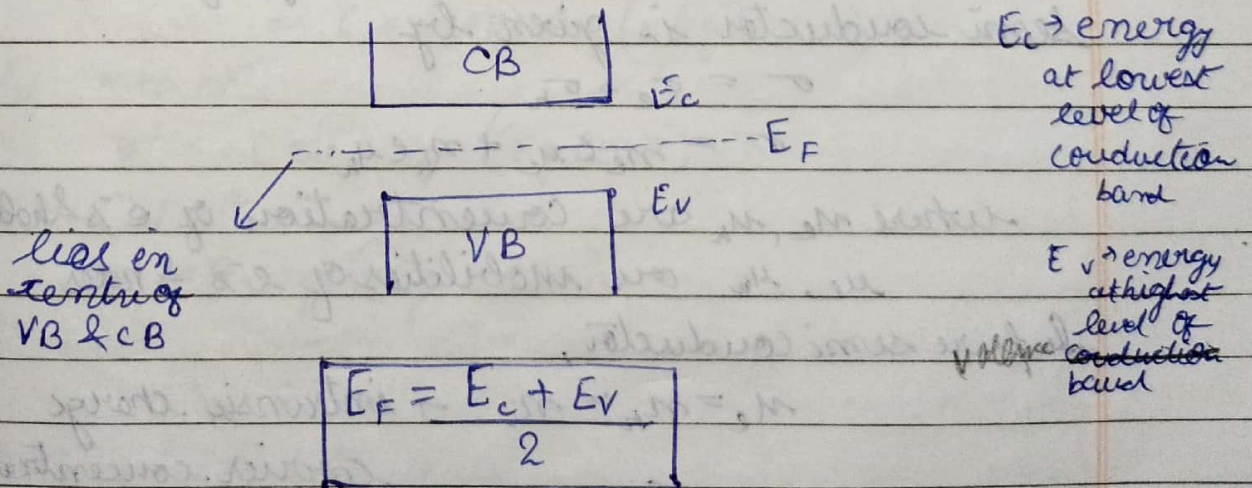
Fermi energy level is the energy level in the solid above which all the ~~bands~~ ^{levels} are empty and below which all the levels are ~~completely~~ filled at 0K

Fermi energy

It is defined as the maximum energy that the e^- ^{comp} have at 0K

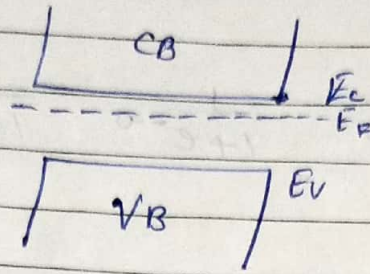
Position of fermi level in semi conductorIntrinsic semi conductor

$$n_e = n_h$$



N type Semiconductor

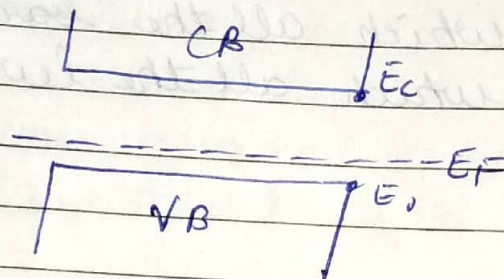
$$n_e \gg n_h$$



Fermi level lies just below the CB

P type semiconductor

$$n_h \gg n_e$$



Fermi level lies just above the VB

Variation of conductivity of semiconductor with temperature

In intrinsic semiconductor there are e^- as well as holes so conductivity of intrinsic semiconductor is given by

$$\sigma = \sigma_e + \sigma_h$$

$$\sigma = n_e e \mu_e + n_h e \mu_h$$

where n_e, n_h are concentration of e^- & holes
 μ_e, μ_h are mobilities of e^- & holes

In pure semiconductor,

$$n_e = n_h = n_i \rightarrow \text{intrinsic charge carrier concentration}$$

$$\therefore \sigma = n_i e (\mu_e + \mu_h)$$

The mobilities μ_e and μ_h depends on temp. T according to the relation

$$\mu_e \propto \frac{1}{T^{3/2}}, \quad \mu_h \propto \frac{1}{T^{3/2}}$$

$$\text{or } \mu_e = \alpha T^{-3/2}$$

$$\mu_h = \beta T^{-3/2} \quad \text{--- (3)}$$

where α & β are constants.

So eqn. (3) becomes

$$\sigma = n_i e (\alpha + \beta) T^{-3/2} \quad \text{--- (4)}$$

The concentration n_i is dependent on temp T according to relation.

$$n_i = CT^{3/2} e^{-E_g/2KT} \quad \text{--- (5)}$$

where $C = 2 \left(\frac{2\pi k}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4}$ effective mass
 $= 4.83 \times 10^{21}$ (constant term)

E_g = Energy band gap in eV

k = Boltzmann Constant

using eq. (4) & (5),

$$\begin{aligned} \sigma &= CT^{3/2} e^{-E_g/2KT} e(\alpha + \beta) T^{-3/2} \\ &= C e(\alpha + \beta) \exp(-E_g/2KT) \end{aligned}$$

Let $\alpha + \beta = r$

$$\sigma = C e r \exp(-E_g/2KT)$$

Let $C e r = A$

$$\sigma = A \exp(-E_g/2KT)$$

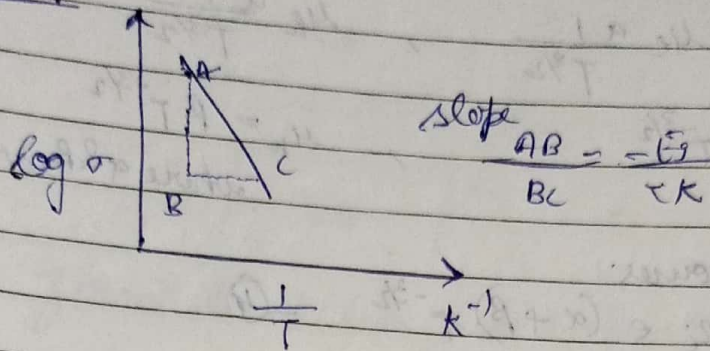
At $T = \infty$,

$$A = \sigma_{\infty}$$

Taking log on both sides

$$\log \sigma = \log \sigma_{\infty} - \frac{E_g}{2KT} \quad \text{--- (6)}$$

So conductivity varies directly with temp in semiconductors.
graph.



At $T = \infty$, $\sigma = \sigma_{\infty}$

The graph is plotted b/w $\log \sigma$ and $1/T$ on extra plotting the graph we get the value of conductivity at infinite.
The slope of the graph gives the value $-E_g/2K$

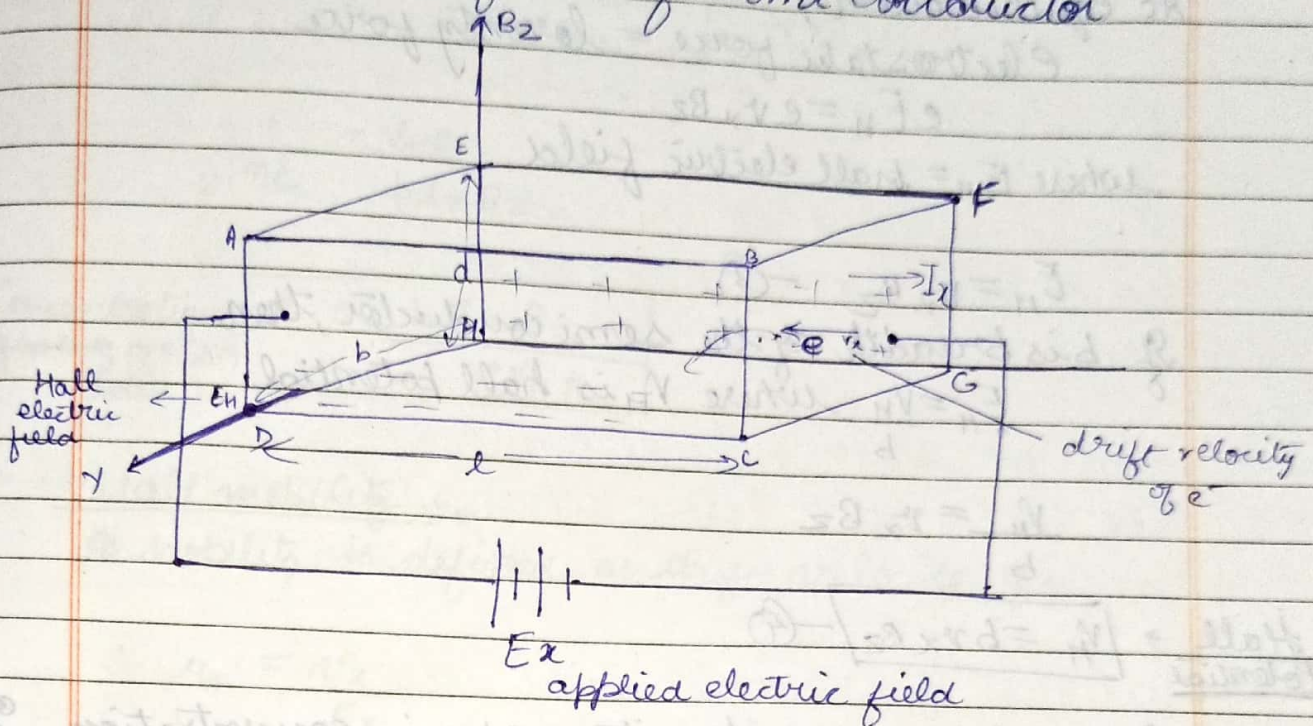
Hall effect

According to this effect when a conductor or semiconductor is placed in current is made to flow through a conductor or a semiconductor and magnetic field is applied in the direction $\perp$ to current and electric field is developed in the material in the direction $\perp$ to both current and magnetic field, the effect is called Hall effect and the voltage so developed is called Hall voltage (V_H).

Uses

This effect is used to find the concentration or density of charge carrier in the conductor or semiconductor.

It is used to find ^{the type} of semiconductor



Let us consider a n type semiconductor placed in magnetic field B_z in z direction, let current I_x flows in x direction when electric field E_x is applied ~~across~~ across the semiconductor, then e^- s drift with velocity v_x in direction opposite to current then Lorentz force act on e^- s, direction of which is determined by Fleming left-hand rule

Lorentz force acting on e^- s

$$F_L = q(\vec{v} \times \vec{B}) =$$

$$= e(v_x B_z) \quad \text{--- (1)}$$

due to Lorentz force e^- s start accumulating ^{on} one face of semiconductor. To stop the accumulation of e^- s electrostatic force acts in direction opposite to it

Electrostatic force $F_E = qE$

$$= eE_H \quad \text{--- (2)}$$

At equilibrium,
Electrostatic force = Lorentz force
$$eE_H = ev_x B_z$$

where E_H = Hall electric field

$E_H = v_x B_z$ — (3)
If b is breadth of the semiconductor, then
$$E_H = \frac{V_H}{b}$$
 where V_H is Hall potential.

$$\frac{V_H}{b} = v_x B_z$$

Hall Potential $\Rightarrow \boxed{V_H = b v_x B_z}$ — (4)

If J_x is current density and n is concentration of e^- s in semiconductor,

$$J_x = ne v_x$$

$$v_x = \frac{J_x}{ne}$$
 — (5)

The quantity $\frac{1}{ne}$ is called Hall coefficient R_H

$$R_H = +ve \rightarrow \text{P-type semiconductor}$$

$$R_H = -ve \rightarrow \text{N-type semiconductor}$$

So, $\boxed{v_x = J_x R_H}$ — (6)
from (3) & (6)

$$\boxed{E_H = J_x R_H B_z}$$

$$\boxed{R_H = \frac{E_H}{J_x B_z}} \quad -7$$

Hall Coefficient

$$\boxed{R_H = \frac{V_H/b}{J_x / \text{Area} B_z}}$$

here Area $A = b \times d$

$$R_H = \frac{V_H A}{b I_x B_z} \quad \text{--- (5)}$$

$$\frac{1}{ne} = \frac{V_H A}{b I_x B_z}$$

Concentration of electrons holes	$n = \frac{b I_x B_z}{e V_H A}$
-------------------------------------	---------------------------------

Hall mobility μ_H

As mobility is defined as drift velocity of e-

$$\mu_H = \frac{v_x}{E_x}$$

from eq (6) $v_x = J_x R_H$

or,

$$\mu_H = \frac{J_x}{E_x} R_H$$

As $\vec{J} = \sigma \vec{E}$ where σ is conductivity

$$\therefore \mu_H = \sigma R_H$$

Superconductivity

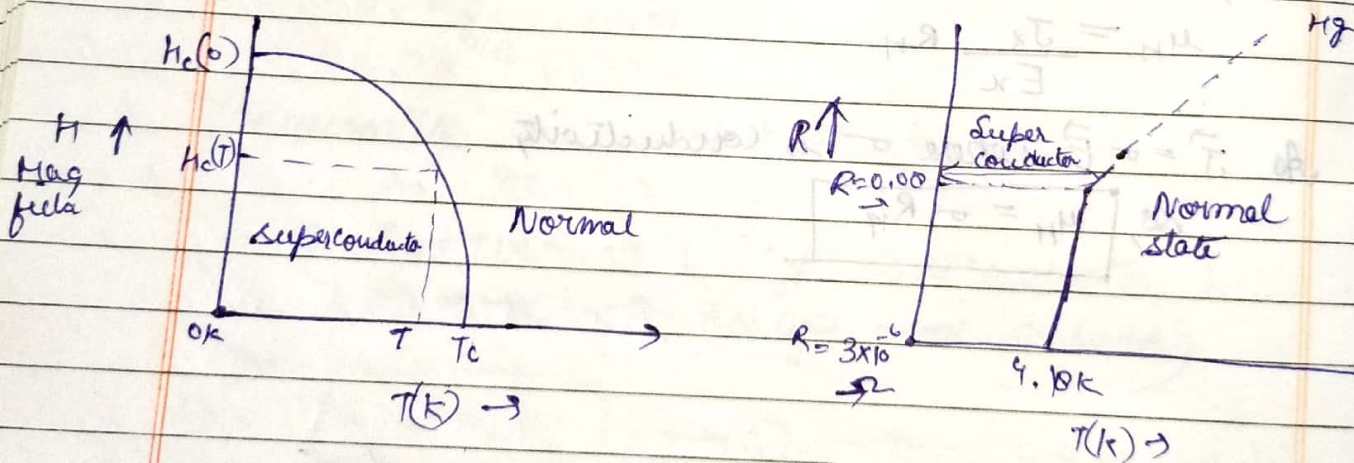
The phenomena of complete and sudden disappearance of electrical resistance when temp. across the material is ~~used~~ reduced to critical temp. is called superconductivity and such materials are called superconductors.

Critical or transition temperature (T_c)

The temp. at or below which the material ~~behaves~~ behaves like a superconductor is called T_c .

Variation of magnetic field with temperature

For superconductors the graph plotted b/w magnetic field and temp. is of parabolic nature as shown in the figure.



$$H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right] \quad \left\{ \text{temp } T \text{ should be in K} \right\}$$

where

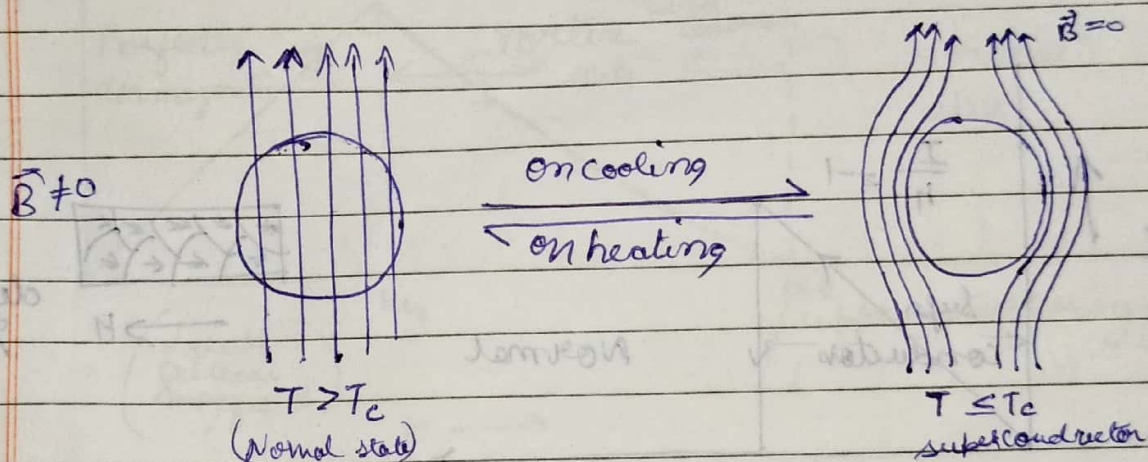
$H_c(T)$ = Critical mag. field at T K

$H_c(0)$ = " " " " 0K

T_c = Critical temp in Kelvin

Meissner effect

According to this effect when a material is placed in magnetic field, it is found that magnetic lines of force pass through the interior of the material above critical temp but when material is cooled and temp. is at or below T_c , the magnetic lines of force are pushed out of the material and the material behaves like a superconductor. The phenomenon of expulsion of magnetic lines of force from the interior of the material below critical temp is called Meissner effect. It is a reversible effect.



If I is the intensity of magnetisation of the material placed in magnetic field H then
Magnetic induction $\vec{B} = \mu_0(\vec{I} + \vec{H})$

As $\vec{B} = 0$ for $T \leq T_c$

$$0 = \mu_0(\vec{I} + \vec{H})$$

$$0 = \vec{I} + \vec{H}$$

$$\mu_0$$

$$\vec{I} = -\vec{H}$$

Magnetic susceptibility

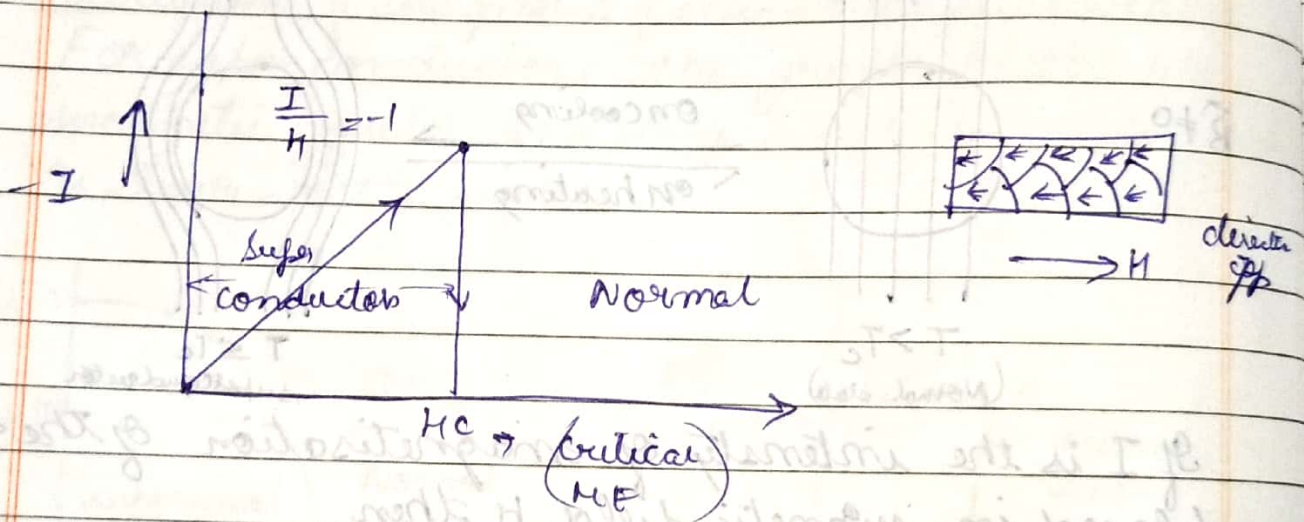
$$\chi_m = \frac{I}{H} = -1$$

∴ semiconductors are perfectly diamagnetic

Types of superconductors

Type I superconductors

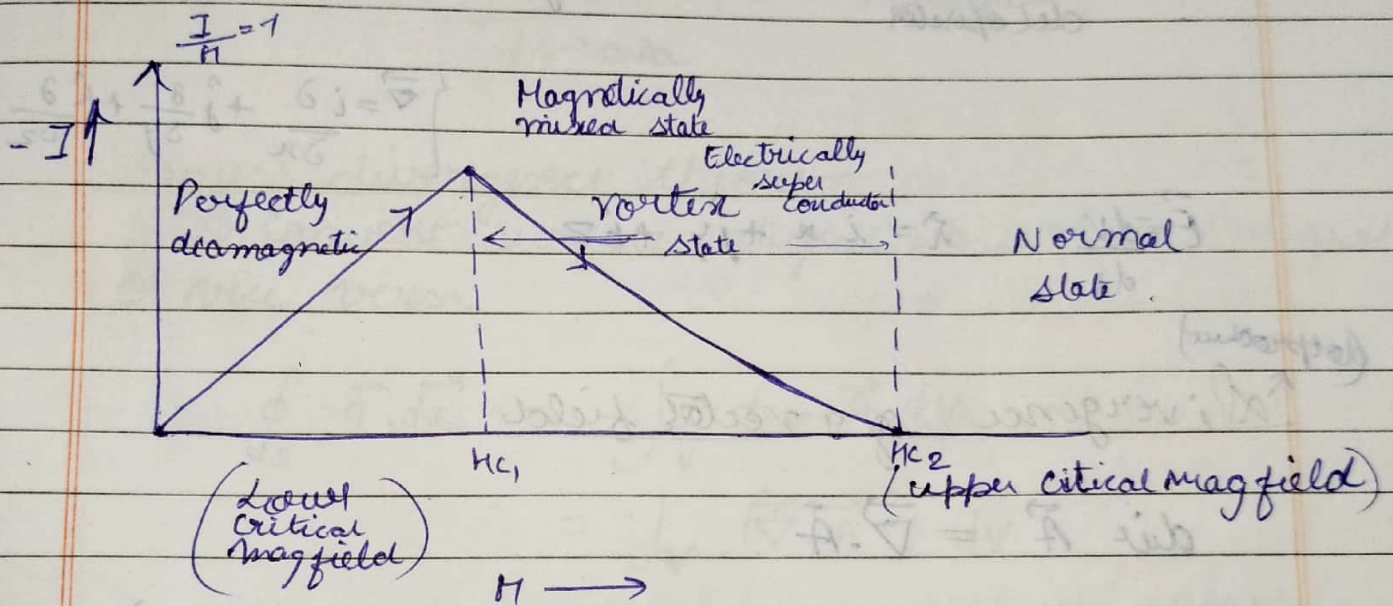
1. They are old soft superconductors
2. They exhibit Meissner effect completely
3. The critical magnetic field (H_c) is very low that is of the order of 0.1 T.
4. So such superconductors cannot be used in high field superconducting magnet.
5. ex tin, lead, mercury



Type II superconductor

1. They are also old hard superconductors
2. They exhibit Meissner effect incompletely i.e. upto lower critical magnetic field H_{c1} , material is perfectly diamagnetic but as magnetic field is increased above H_{c1} , magnetisation (I) ~~the~~ decrease gradually upto H_{c2} (upper critical mag. field), between H_{c1} and H_{c2} , material is in mixed state or vortex state. Above H_{c2} material behaves like a normal conducting material.

3. H_{C2} (upper critical mag field) is about 100 times more than the critical mag field in soft superconductors, they are suitable for use in high field super conducting material magnet.
 ex: Lead indium alloy
 Nb - Zr



Applications

- 1) Maglev trains
- 2) MRI
- 3) Power generators
- 4) Cryton switches

Electromagnetics

It is related with electric field as well as with magnetic field.

Gradient

$$\text{grad } \phi = \underbrace{\vec{\nabla}}_{\text{del operator}} \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$$

$$\left\{ \vec{\nabla} = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right.$$

$$\vec{E} = \frac{dV}{dx} \quad \hat{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

(dot product)

Divergence of a vector field

$$\text{div } \vec{A} = \vec{\nabla} \cdot \vec{A}$$

$$= \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z$$

$$\text{div } \vec{A} = \oint_s \frac{\vec{A} \cdot d\vec{s}}{dV} \quad \left. \vphantom{\oint_s} \right\} \text{surface integral flux}$$

$$\text{div } \vec{A} dV = \oint_s \vec{A} \cdot d\vec{s}$$

Curl of a vector field

$$\text{Curl } \vec{A} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\text{Curl } \vec{A} = \oint \vec{A} \cdot d\vec{l} \quad \left. \begin{array}{l} \text{line integral} \\ d\vec{l} \rightarrow \text{area} \end{array} \right\}$$

Gauss divergence theorem

To convert surface integral to volume integral
or vice versa

$$\begin{aligned} \oint_S \vec{A} \cdot d\vec{s} &= \int_V \text{div } \vec{A} \, dV \\ &= \int_V (\vec{\nabla} \cdot \vec{A}) \, dV \end{aligned}$$

Stokes theorem

To convert line integral into surface integral
or vice versa.

$$\begin{aligned} \oint_C \vec{A} \cdot d\vec{l} &= \int_S \text{curl } \vec{A} \cdot d\vec{s} \\ &= \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{s} \end{aligned}$$

$$q = \int_V \rho \, dv$$

$$I = \int_S \vec{J} \cdot d\vec{s}$$

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charge distributed in volume V , $q = \int_V \rho \, dv$

where ρ = volume charge density

Current $I = -\frac{dq}{dt}$ (Rate of decrease of charge)

Current flowing through the surface $I = \int_S \vec{J} \cdot d\vec{s}$

where $\vec{J}$ = current density

Equation of continuity

Let us consider a conductor of volume V in which charge q is distributed due to which current I flows on the surface then

As current is equal to rate of decrease of charge-

$$I = -\frac{dq}{dt}$$

Charge distributed in volume V , $q = \int_V \rho \, dv$

where ρ is volume charge density

Current flowing on the surface $I = \int_S \vec{J} \cdot d\vec{s}$

$\vec{J}$ = Current density

$$\int_S \vec{J} \cdot d\vec{s} = -\frac{d}{dt} \int_V \rho \, dv$$

Using Gauss divergence theorem on LHS of above eqn

$$\int_V \text{div } \vec{J} \, dv = -\frac{d}{dt} \int_V \rho \, dv$$

$$\int_V \left(\text{div } \vec{J} + \frac{\partial \rho}{\partial t} \right) dv = 0$$

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$$\nabla \cdot \vec{v} \neq 0$$

$$\Rightarrow \boxed{\text{div } \vec{J} + \frac{\partial \rho}{\partial t} = 0} \quad (2)$$

$$\text{div } \boxed{\vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

This law represents the law of conservation of charge in static field (DC circuit)

$$\rho = \text{constant}, \quad \frac{\partial \rho}{\partial t} = 0$$

$$\boxed{\text{div } \vec{J} = 0}$$

for time varying fields $\rho \neq \text{constant}$

$$\text{so } \boxed{\text{div } \vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

Maxwell equation in differential form

Gauss law of electrostatics

Phy. 129 →

It states that the surface integral of electric field across any closed surface is equal to $1/\epsilon_0$ the charge enclosed in volume V .

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$\text{As } q = \int_V \rho \, dv$$

$$\oint_s \vec{E} \cdot d\vec{s} = \frac{\int_V \rho \, dv}{\epsilon_0}$$

Using Gauss. divergence Theorem on LHS.
we get

$$\int_V \text{div } \vec{E} \, dv = \frac{\int_V \rho \, dv}{\epsilon_0}$$

Comparing both sides.

$$\boxed{\text{div } \vec{E} = \frac{\rho}{\epsilon_0}}$$

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

This is Maxwell equation based on Gauss law in Electrostatics.

^{law}
Gauss of Magnetostatics

Physical significance

It states magnetic flux ^(or cross) around any closed surface is equal to 0, because magnetic lines of force are closed curve.

$$\oint_S \vec{B} \cdot d\vec{s} = 0$$

using Gauss div theorem on LHS,
we get

$$\int_V \text{div } \vec{B} \, dV = 0$$

As $dV \neq 0$

$$\boxed{\text{div } \vec{B} = 0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

This is Maxwell eq. based on Gauss law in Magnetostatics

Faraday law of electromagnetic induction

It states that induced emf in the closed circuit is equal to negative rate of change of magnetic flux linked with circuit.

induced emf

$$e = - \frac{d\phi_B}{dt} \quad \text{--- (1)}$$

not in
proof
only to
motivate

$$e = \frac{\text{work done}}{\text{charge}} = \frac{W}{q} = \frac{\int_C \vec{F} \cdot d\vec{\ell}}{q} = \frac{\int_C q \vec{E} \cdot d\vec{\ell}}{q} = \int_C \vec{E} \cdot d\vec{\ell}$$

electric field effect

As emf e is equal to line integral of $\vec{E}$

$$e = \int_C \vec{E} \cdot d\vec{\ell} \quad \text{--- (2)}$$

$$\text{Magnetic flux } \phi_B = \int_S \vec{B} \cdot d\vec{s} \quad \text{--- (3)}$$

So equ. (1) is

$$\int_C \vec{E} \cdot d\vec{\ell} = - \frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{s}$$

$$\int_C \vec{E} \cdot d\vec{\ell} = - \oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Using Stokes theorem on LHS,

$$\oint_S \text{curl } \vec{E} \cdot d\vec{s} = - \oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Comparing both sides

$$\boxed{\text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t}}$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

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This is Maxwell's equation based on Faraday's law of Electromagnetic induction.

Ampere's circuital law
Ampere law in integral form

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\oint \vec{B} = \mu_0 \vec{H}$$

/

magnetic induction magnetic field

$$\Rightarrow \oint \vec{H} \cdot d\vec{l} = I$$
$$= \int \vec{J} \cdot d\vec{s}$$

$$\boxed{\text{curl } \vec{H} = \vec{J}}$$

It states that line integral of magnetic induction along a closed curve is equal to μ_0 times the current enclosed within the curve.

in differential form $\Rightarrow \text{curl } \vec{H} = \vec{J} \quad \odot$

Taking div on both sides

$$\text{div curl } \vec{H} = \text{div } \vec{J}$$

from vector identity,

$$\boxed{\text{div curl } \vec{H} = 0}$$

So, $\text{div } \vec{J} = 0$

This is eqn. of continuity in static field so we can say that ampere's law is valid only in static field.

So to make ampere's law valid in static field as well as time varying fields an addition term is added on the right hand side of eqn (1) c/d displacement current density.

Displacement current density $\vec{J}_D$,

$$\text{or, } \text{curl } \vec{H} = \vec{J} + \vec{J}_D$$

Taking div on both sides,

$$\text{div curl } \vec{H} = \text{div } \vec{J} + \text{div } \vec{J}_D$$

from vector identity,

$$0 = \text{div } \vec{J} + \text{div } \vec{J}_D$$

$$\text{div } \vec{J}_D = -\text{div } \vec{J}$$

$$\text{from eqn of continuity } \text{div } \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\text{div } \vec{J} = -\frac{\partial \rho}{\partial t} \quad \text{--- (3)}$$

from (2) & (3)

$$\text{div } \vec{J}_D = \frac{\partial \rho}{\partial t} \quad \text{--- (4)}$$

from Maxwell diff. form based on Gauss law in electrostatics,

$$\text{div } \vec{E} = \rho / \epsilon_0$$

$$\rho = \epsilon_0 \text{div } \vec{E} \quad \text{--- (5)}$$

from (4) & (5)

$$\text{div } \vec{J}_D = \frac{\partial}{\partial t} (\epsilon_0 \text{div } \vec{E})$$

displacement
current
density

$$\boxed{\vec{J}_D = \frac{\partial}{\partial t} (\epsilon_0 \vec{E})}$$

so modified ampere law in differential form is

$$\boxed{\begin{aligned} \text{curl } \vec{H} &= \vec{J} + \vec{J}_D \\ \text{curl } \vec{H} &= \vec{J} + \frac{\partial (\epsilon_0 \vec{E})}{\partial t} \end{aligned}}$$

This is maxwell equation in differential form based on modified ampere's law to make it valid in static fields as well as in time varying fields.

It signifies that the magnetic field along a closed curve is equal to ~~sum of~~ sum of conduction current and displacement current.

The time varying electric field is equivalent to current or displacement current.

Maxwell's equations

Integral

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

Differential

$$\text{div } \vec{E} = \frac{\rho}{\epsilon_0}$$

Phy. significance
Gauss law in
Electrostatic

$$\oint_S \vec{B} \cdot d\vec{s} = 0$$

$$\text{div } \vec{B} = 0$$

Gauss law in magnetism

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{s}$$

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

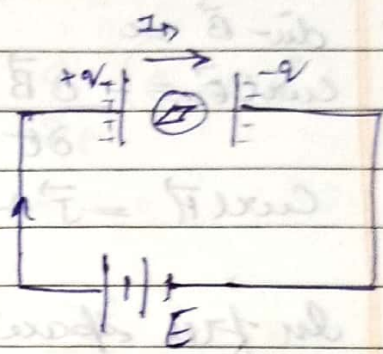
Faraday law of
EM induction

$$\oint_C \vec{H} \cdot d\vec{l} = I + I_D$$

$$\begin{aligned} \text{curl } \vec{H} &= \vec{J} + \vec{J}_D \\ &= \vec{J} + \frac{\partial (\epsilon_0 \vec{E})}{\partial t} \end{aligned}$$

Modified ampere
law

Charging and discharging of capacitor
 During charging of capacitor charge q comes on capacitor plate and after that discharging of capacitor starts, it is found that magnetic needle placed between capacitor plates shows deflection which indicates that there is some current in between the capacitor plates given by -

$$\begin{aligned} \text{displacement current } I_D &= \frac{\partial q}{\partial t} \\ &= \frac{\partial}{\partial t} (\epsilon_0 EA) \\ &= \epsilon_0 A \frac{\partial E}{\partial t} \end{aligned}$$


Gauss law in electrostatics

$$\oint \vec{E} \cdot d\vec{s} = q/\epsilon_0$$

$$EA = q/\epsilon_0$$

$$q = \epsilon_0 EA$$

In case same dielectric is in b/w capacitor plates,

$$I_D = \epsilon A \frac{\partial E}{\partial t}$$

$$\text{as } \frac{\epsilon}{\epsilon_0} = \epsilon_r = K$$

Equation of propagation of electromagnetic wave in free space and to prove that velocity of EM wave in free space is equal to speed of light

Maxwell eq. in diff. form

$$1. \operatorname{div} \vec{E} = \rho / \epsilon_0$$

$$2. \operatorname{div} \vec{B} = 0$$

$$3. \operatorname{curl} \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$4. \operatorname{curl} \vec{H} = \vec{J} + \vec{J}_D = \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

In free space (vacuum)

$\epsilon_0 \rightarrow$ permittivity in free space

$\mu_0 \rightarrow$ permeability " " "

There is no charge source, $\rho = 0$

$$\rho = 0$$

$$\vec{J} = 0$$

volume
charge
density

current
density

$$\vec{B} = \mu_0 \vec{H} \text{ where } \vec{H} = \text{external Magnetic field}$$

In free space, Maxwell diff eq. are

$$\operatorname{div} \vec{E} = 0 \quad \text{--- (1)}$$

$$\operatorname{div} \vec{H} = 0 \quad \text{--- (2)}$$

$$\operatorname{curl} \vec{E} = - \frac{\partial (\mu_0 \vec{H})}{\partial t} \quad \text{--- (3)}$$

$$\operatorname{curl} \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{--- (4)}$$

Taking curl of eq (3) on both sides,

$$\text{curl curl } \vec{E} = -\mu_0 \frac{\partial (\text{curl } \vec{H})}{\partial t}$$

using vector identity on LHS, we get.

$$\boxed{\text{curl curl } \vec{E} = \text{grad}(\text{div } \vec{E}) - \nabla^2 \vec{E}}$$

then,

$$\text{grad}(\text{div } \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial (\text{curl } \vec{H})}{\partial t}$$

using eq (1) & (2), we get

$$\text{grad}(0) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t} \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$-\nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{--- (5)}$$

This is eqn. of electromagnetic wave in terms of electric field vector $\vec{E}$

In terms of $\vec{H}$,

Taking curl of eq (4) on both sides.

$$\text{curl curl } \vec{H} = \epsilon_0 \frac{\partial (\text{curl } \vec{E})}{\partial t}$$

using vector identity on LHS we get.

$$\text{curl curl } \vec{H} = \text{grad}(\text{div } \vec{H}) - \nabla^2 \vec{H}$$

then,

$$\text{grad}(\text{div } \vec{H}) - \nabla^2 \vec{H} = \epsilon_0 \frac{\partial (\text{curl } \vec{E})}{\partial t}$$

using eq ② & ③ we get

$$\text{grad}(0) - \nabla^2 \vec{H} = -\mu_0 \epsilon_0 \frac{\partial}{\partial t} \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} \vec{H} \right)$$

$$-\nabla^2 \vec{H} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}} \quad \text{--- (6)}$$

This is eq of EM wave in terms of $\vec{H}$.

General eq. of wave motion

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{--- (7)}$$

where v = velocity of wave
Comparing eq. (7) with eq (5) or eq (6)

$$\boxed{\frac{1}{v^2} = \mu_0 \epsilon_0}$$

$$v^2 = \frac{1}{\mu_0}$$

$$v^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^8 \text{ m/s} = c //$$

Poynting vector

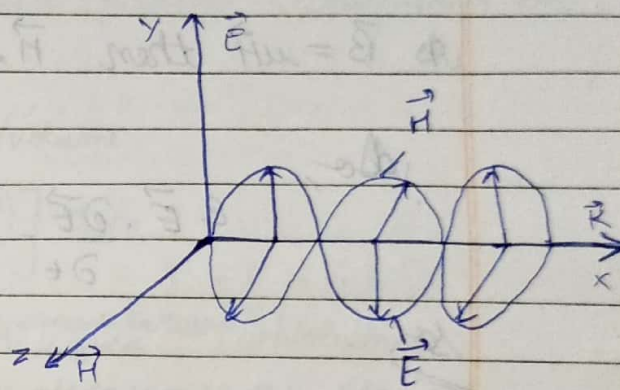
Poynting vector represents the rate of energy flowing through the surface when electromagnetic wave travels from one point to other

$$\vec{P} \text{ or } \vec{S} = \frac{\text{Electromagnetic energy}}{\text{Area} \times \text{time}} = \frac{\text{Power}}{\text{Area}}$$

$$\text{Unit} \rightarrow \text{Watt} / \text{m}^2$$

$$= \frac{VI}{l \times b}$$

$$\vec{S} = \vec{E} \times \vec{H}$$



Poynting theorem

From Maxwell equations based on Faraday law of electromagnetic induction and modified Ampere's law -

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (1)}$$

$$\text{curl } \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (2)}$$

Taking dot product of eq. (1) with $\vec{H}$ and dot product of eq. (2) with $\vec{E}$ we get,

$$\vec{H} \cdot \text{curl } \vec{E} = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} \quad \text{--- (3)}$$

$$\vec{E} \cdot \text{curl } \vec{H} = \vec{E} \cdot \vec{J} + \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \quad \text{--- (4)}$$

from vector identity $\rightarrow \vec{H} \cdot \text{curl } \vec{E} - \vec{E} \cdot \text{curl } \vec{H} = \text{div}(\vec{E} \times \vec{H})$

$$\frac{1}{2} \frac{d(x^2)}{dt} = 2x \frac{dx}{dt}$$

$$\vec{H} \cdot \frac{\partial}{\partial t} \mu \vec{H} = \frac{\partial}{\partial t} \left(\frac{1}{2} \mu \vec{H}^2 \right) = \frac{\partial \mu}{\partial t} \frac{1}{2} \vec{H}^2$$

$$x \frac{dx}{dt} = \frac{1}{2} \frac{d}{dt} x^2$$

Subtracting eq. (4) from eq. (3) and using vector identity on LHS -

$$\vec{H} \cdot \text{curl} \vec{E} - \vec{E} \cdot \text{curl} \vec{H} = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{E} \cdot \vec{J} - \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\text{div}(\vec{E} \times \vec{H}) = -\vec{E} \cdot \vec{J} - \left[\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} + \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \right]$$

As $\vec{B} = \mu \vec{H}$ then $\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} = \vec{H} \cdot \frac{\partial}{\partial t} \mu \vec{H} = \frac{\partial}{\partial t} \left(\frac{1}{2} \mu \vec{H}^2 \right)$

Also,

$$\epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \vec{E}^2 \right) \quad \left(\frac{\partial x^2}{\partial t} = 2x \frac{\partial x}{\partial t} \right)$$

So,

$$\text{div}(\vec{E} \times \vec{H}) = -\vec{E} \cdot \vec{J} - \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mu \vec{H}^2 \right) + \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \vec{E}^2 \right) \right]$$

Taking volume integral on both sides,

$$\int_V \text{div}(\vec{E} \times \vec{H}) dV = - \int_V (\vec{E} \cdot \vec{J}) dV - \int_V \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mu \vec{H}^2 \right) + \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \vec{E}^2 \right) \right] dV$$

↓ Gauss div theorem

$$\int_S (\vec{E} \times \vec{H}) \cdot d\vec{s} = - \int_V (\vec{E} \cdot \vec{J}) dV - \int_V \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mu \vec{H}^2 \right) + \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \vec{E}^2 \right) \right] dV$$

(I) (II)

This eq. represents the conservation of energy when EM wave propagation

$$(I) \int_S (\vec{E} \times \vec{H}) \cdot d\vec{s} = \left(\frac{V}{L} \frac{I}{A} \right) A \Delta t$$

This term represents the rate of electromagnetic energy flowing through the surface when an electromagnetic wave travels.

$$(II) \int_V (\vec{E} \cdot \vec{J}) dV = \left(\frac{V \cdot I}{L \cdot A_{\text{area}}} \right) \times V$$

This term represents the power dissipated in the form of heat.

$$III \int_V \left[\frac{\partial}{\partial t} \left(\frac{1}{2} \mu \vec{H}^2 \right) + \frac{\partial}{\partial t} \left(\frac{\epsilon}{2} \vec{E}^2 \right) \right] dV$$

(Energy stored in terms of magnetic field) Volume (Energy stored in terms of electric field) Volume

It represents the rate of decrease of stored energy when electromagnetic wave travels from one point to another.

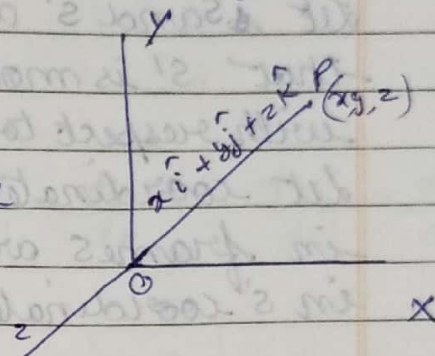
Special theory of relativity

Frame of reference

A coordinate system with respect to which position of motion of the object is specified.

Types of frame of reference

- 1) Inertial frame of reference
- 2) Non inertial frame of reference



Inertial frame of reference

The frames in which Newton's laws (law of inertia) are valid are called inertial frames. They are non accelerated frames.

ex[→] Car moving ^{with a} constant velocity.

Non inertial frame of reference

The frames in which Newton's laws are not valid are called non inertial frames. They are accelerated frames.

ex[→] Weight of a person in a moving lift.

Movement in merry go round.

Earth $\rightarrow$ Inertial $\rightarrow$ for observer on earth

Earth $\rightarrow$ Non inertial $\rightarrow$ for observer in space.

Galilean transformation equations

The equations relating coordinates of two inertial frames when relative velocity is very less than c are old galilean transformation equations.

Let S and S' are two inertial frames such that S' is moving with constant velocity v with respect to frame S in x direction.

Let coordinates of point P for observer in frames are (x, y, z, t) and for observer in S' coordinates of point P are (x', y', z', t')

at time $t = t'$

Position transformation equations are $\rightarrow$

for observer in S'

$$x' = x - vt \quad \text{--- (1)}$$

$$y' = y \quad \text{--- (2)}$$

$$z' = z \quad \text{--- (3)}$$

$$t' = t \quad \text{--- (4)}$$

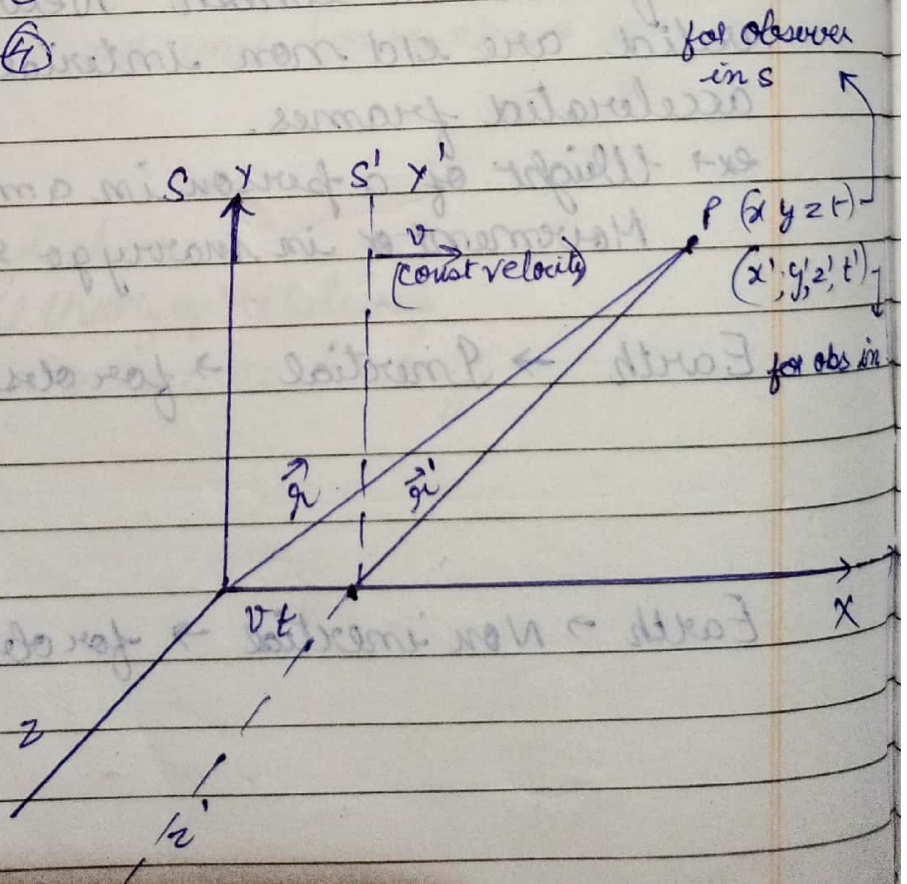
for observer in S

$$x = x' + vt'$$

$$y = y'$$

$$z = z'$$

$$t = t'$$



for velocity transformation equations :-

diff eq (4)

$$dx' = dx - v dt \quad (5) \quad ; \quad dy' = dy \quad (6) \quad ; \quad dz' = dz \quad (7)$$

$$dt' = dt \quad (8)$$

divide (5) by (8) (6) by (8)

so

$$\frac{dx'}{dt'} = \frac{dx}{dt} - \frac{v dt}{dt} \quad , \quad \frac{dy'}{dt'} = \frac{dy}{dt} \quad \frac{dz'}{dt'} = \frac{dz}{dt}$$

$$\boxed{v'_x = v_x - v \quad , \quad v'_y = v_y \quad , \quad v'_z = v_z}$$

(9) (10) (11)

for acceleration transformation eq.

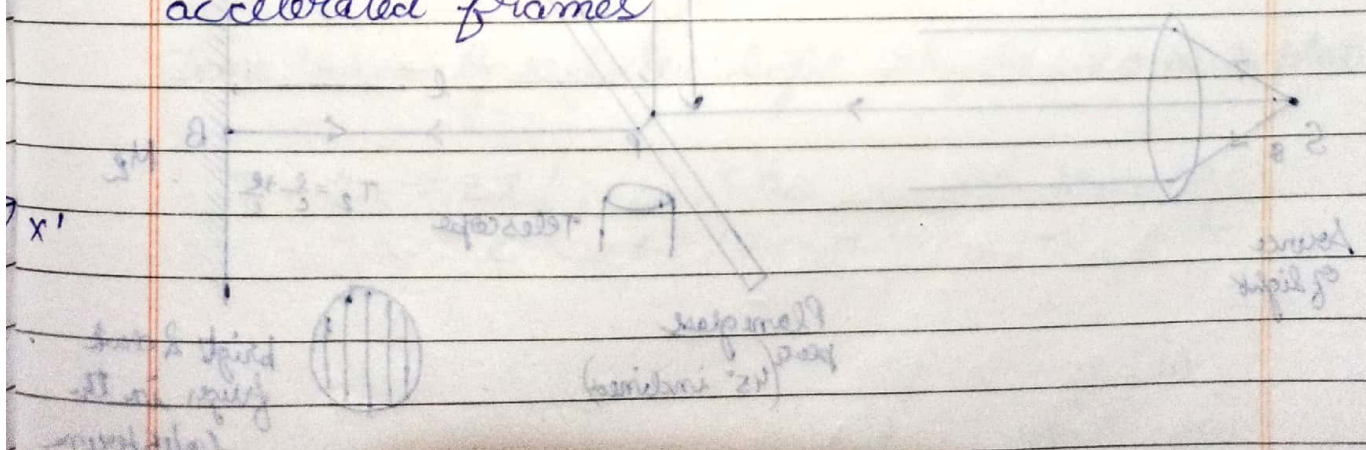
diff eq (9) (10) & (11) w.r.t time

$$\frac{dv'_x}{dt'} = \frac{dv_x}{dt} - 0 \quad ; \quad \frac{dv'_y}{dt'} = \frac{dv_y}{dt} \quad ; \quad \frac{dv'_z}{dt'} = \frac{dv_z}{dt}$$

$$a'_x = a_x \quad , \quad a'_y = a_y \quad , \quad a'_z = a_z$$

(acc. is equal $\therefore$ acc is a hence it is non accelerated frame)

So we can say that inertial frames are non accelerated frames



Ether hypothesis

Contrary,

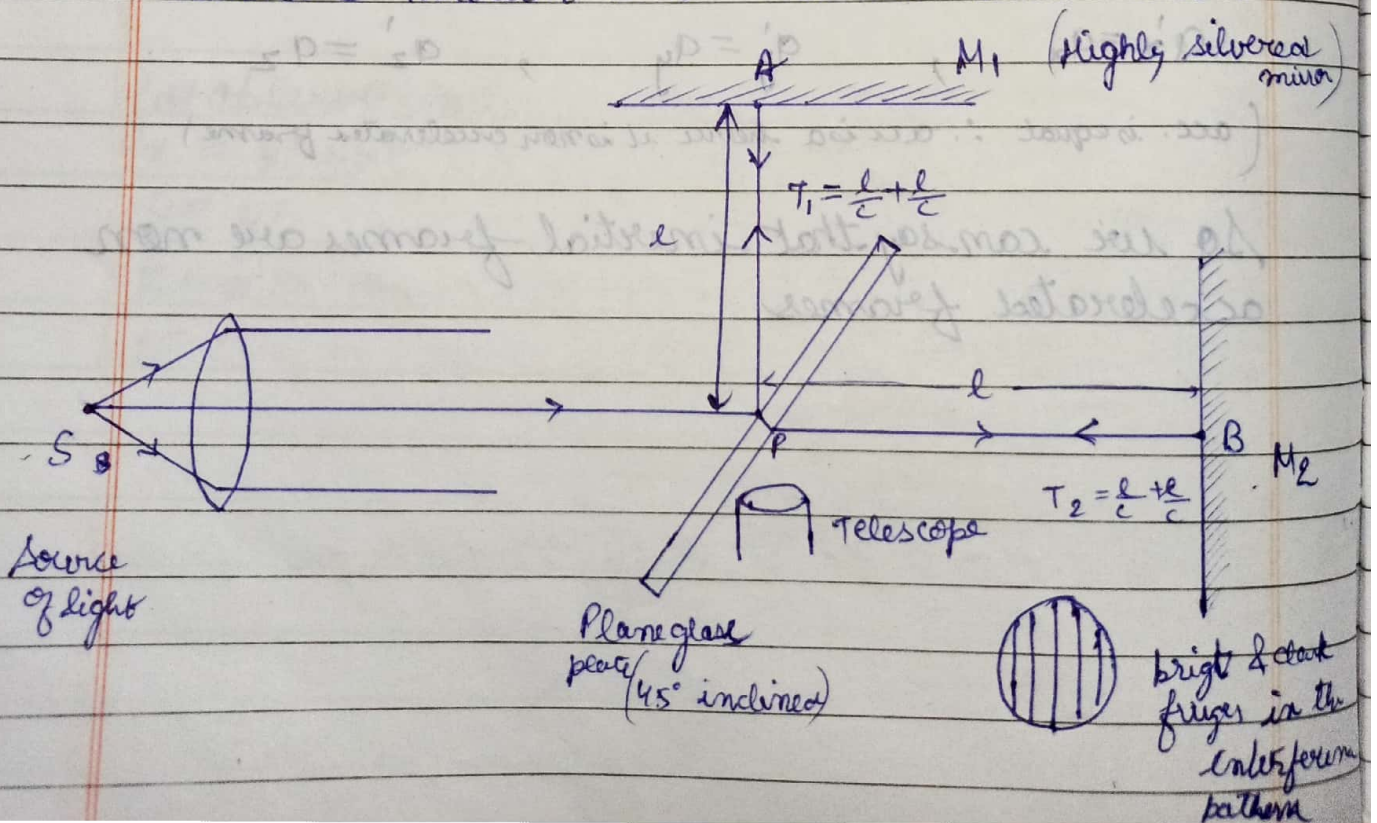
At the beginning of the present, it was assumed that the entire space including earth was filled with a hypothetical medium called ether. It was assumed to be invisible transparent massless, stationary and having very high elasticity and low density. It was also assumed that all the objects move through it without disturbing it.

Michelson

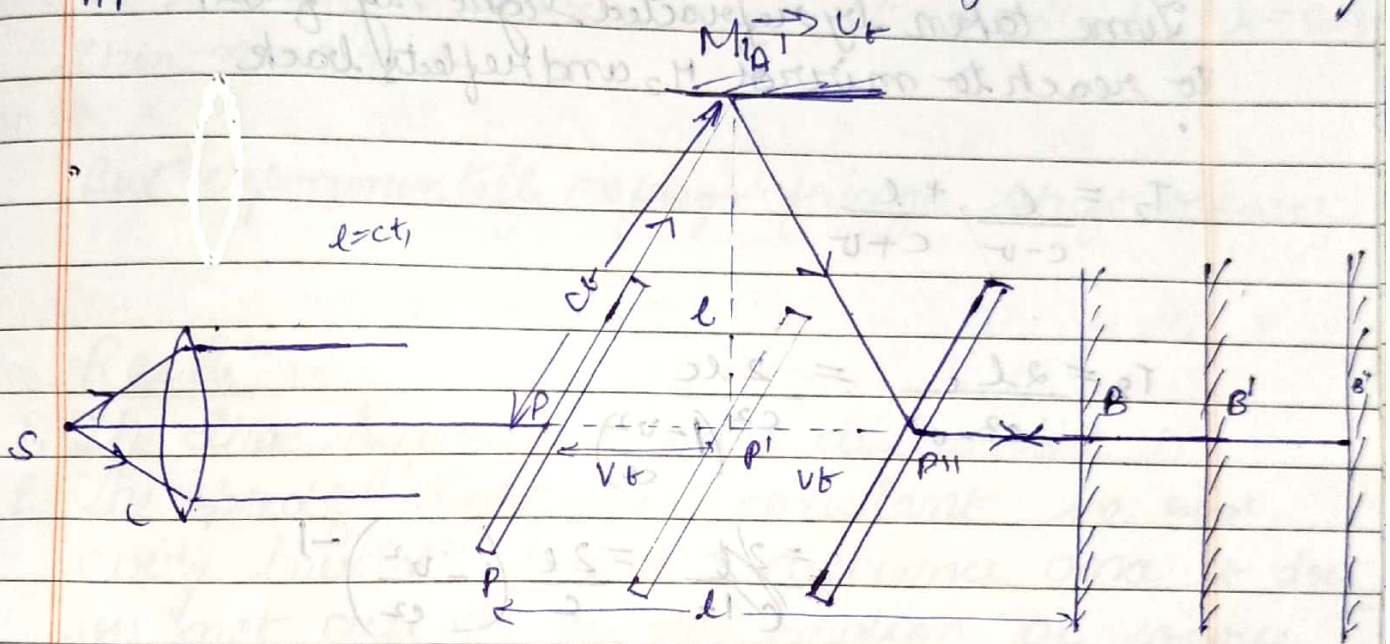
velocity of light $(v) = \sqrt{\frac{E}{P}}$ (elasticity) high
 3×10^{10} (density) low

Michelson Morley Experiment

Object - To confirm the existence of hypothetical medium ether considering relative motion b/w earth and ether.



If apparatus is considered to move with velocity v
 If apparatus is considered to be moving with velocity v in the direction of light the reflected ray travels the distance PA' to reach to mirror M , and reflects back along $A'P'$



from fig $PA' = ct$, $PP' = vt$, $P'A' = l$

Using pythagoras theorem, $(ct)^2 = (vt)^2 + l^2$
 $t^2 (c^2 - v^2) = l^2$

$$t^2 = \frac{l^2}{c^2 - v^2} = \frac{l^2}{c^2(1 - v^2/c^2)}$$

$$t = \frac{l}{c} \left(\frac{1 - v^2/c^2}{c^2} \right)^{-1/2}$$

Time taken by reflected light ray to come to plate P

$$T_1 = 2t = \frac{2l}{c} \left(\frac{1 - v^2/c^2}{c^2} \right)^{-1/2}$$

expanding binomially,

$$T_1 = \frac{2l}{c} \left[1 + \frac{1}{2} \frac{v^2}{c^2} + \dots \right] \quad \text{--- (1)}$$

Time taken by refracted light ray front to reach to mirror M_2 and reflects back

$$T_2 = \frac{l}{c-v} + \frac{l}{c+v}$$

$$T_2 = \frac{2lc}{c^2 - v^2} = \frac{2lc}{c^2 \left(1 - \frac{v^2}{c^2} \right)}$$

$$= \frac{2l}{c} \left(1 - \frac{v^2}{c^2} \right)^{-1}$$

expanding binomially,

$$T_2 = \frac{2l}{c} \left(1 + \frac{v^2}{c^2} + \dots \right) \quad \text{--- (2)}$$

$$\text{Time } \Delta t = T_2 - T_1$$

$$= \frac{2l}{c} \left(1 + \frac{v^2}{c^2} + \dots \right) - \frac{2l}{c} \left(1 + \frac{1}{2} \frac{v^2}{c^2} + \dots \right)$$

$$= \frac{2l}{c} \times \frac{v^2}{2c^2} = \frac{lv^2}{c^3}$$

$$\text{Path change } \Delta = c \Delta t = c \cdot \frac{lv^2}{c^3} = \frac{lv^2}{c^2}$$

$$\text{If path change } \Delta = n\lambda;$$
$$\text{so } n\lambda = \frac{lv^2}{c^2}$$

$$\text{No. of fringes shifted } n = \frac{lv^2}{c^2 \lambda}$$

If apparatus is ~~is~~ rotated by 90° ,
 total fringes shifted $n = \frac{2lv^2}{c^2 \lambda}$

Using
 experimental values

$l = 11 \text{ m}$, $v = 3 \times 10^4 \text{ m/s}$ $c = 3 \times 10^8 \text{ m/s}$ $\lambda = 5000 \text{ \AA}$
 then $n = 0.4$

But experimentally ~~no~~ fringe shift occurs

Results

1. The ether hypothesis was discarded
2. The speed of light is constant in ~~the~~ every inertial frame of reference and it does ~~not~~ depend on the motion of source or observer

Interpretations of negative results of MM expt

i) Ether drag hypothesis

According to this hypothesis the moving earth drags the ether along with it and hence there is no relative motion b/w the earth and the ether and hence no fringe shift occurs.

ii) Lorentz Fitzgerald contraction

A material body moving through ether get contracted in the direction of motion by the factor $\sqrt{1 - \frac{v^2}{c^2}}$ and hence no fringe shift occurs.

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Principle of Constancy of speed of light
The speed of light is constant in every inertial frame of reference and it does not depend on the motion of source or observer.

Einstein postulates of special theory of relativity

1. All the laws of physics have the same mathematical form in every inertial frame of reference.
2. The speed of light is constant in every inertial frame of reference and it does not depend on the motion of source or observer.

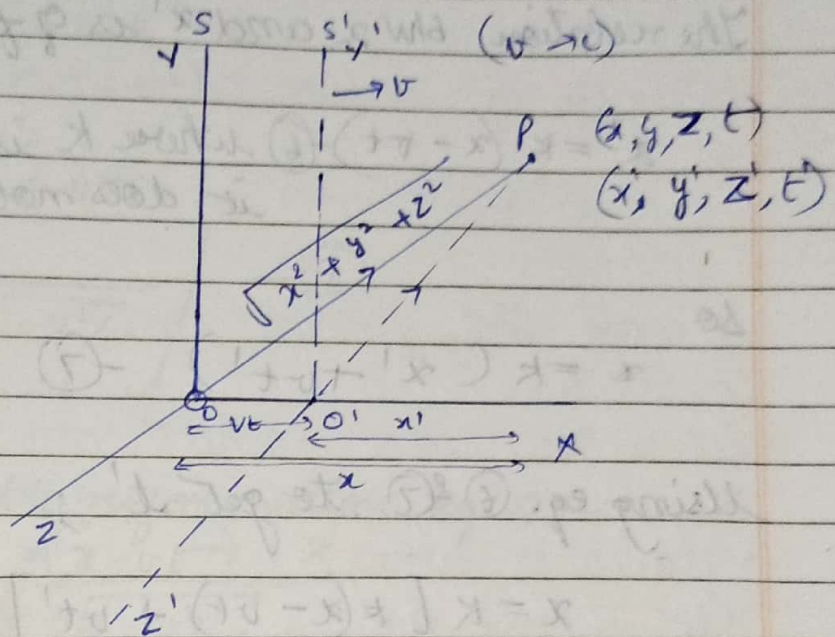
Lorentz transformation equations

The equations relating the coordinates of two inertial frames of reference on the basis of postulates of special theory of relativity are called Lorentz transformation equation.

Let us consider S and S' are two inertial frames such that S' is moving with constant velocity v relative to frame S along x direction. Let O and O' be the origins of frame S and S' that coincide at time $T = T' = 0$.

Let the coordinates of point P for observer in S are (x, y, z, t) and for observer in S' are (x', y', z', t') .

Suppose a light signal is emitted from origin O and O' at time $t=t'=0$



Origins O and O' coincide at time $t=t'=0$

Time taken by light ray emitted from O to reach point P

$$t = \frac{OP}{c} = \frac{\sqrt{x^2 + y^2 + z^2}}{c} \quad \text{or} \quad x^2 + y^2 + z^2 = c^2 t^2 \quad (1)$$

Time taken by light emitted from O' to reach point P

$$t' = \frac{O'P}{c} = \frac{\sqrt{x'^2 + y'^2 + z'^2}}{c} \quad \text{or} \quad x'^2 + y'^2 + z'^2 = c^2 t'^2 \quad (2)$$

from (1) & (2)

$$x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c^2 t'^2 \quad (3)$$

As there is no relative motion along y & z direction

$$y = y' \quad z = z' \quad (4)$$

from (3) & (4)

$$x^2 - c^2 t^2 = x'^2 - c^2 t'^2 \quad -5$$

The relation b/w x and x' is of form -

$$x' = k(x - vt) \quad -6 \text{ where } k \text{ is a constant and it does not depend on } x \text{ \& } t$$

So

$$x = k(x' + vt') \quad -7$$

Using eq. (6) & (7) to get t' ;

$$x = k[k(x - vt) + vt']$$

$$\frac{x}{k} = k(x - vt) + vt'$$

$$vt' = \frac{x}{k} - k(x - vt)$$

$$t' = \frac{x}{vk} - k \left(\frac{x - vt}{v} \right)$$

$$t' = \frac{x}{vk} - \frac{kx}{v} + kt$$

$$t' = k \left[t - \frac{x}{v} \left(1 - \frac{1}{k^2} \right) \right] \quad -8$$

Putting x' and t' in eq 5 we get -

$$\begin{aligned} x^2 - c^2 t^2 &= k^2 (x - vt)^2 - c^2 k^2 \left[t - \frac{x}{v} \left(1 - \frac{1}{k^2} \right) \right]^2 \\ x^2 - c^2 t^2 &= k^2 (x^2 - 2xvt + v^2 t^2) - c^2 k^2 \left[t - \frac{x}{v} \left(1 - \frac{1}{k^2} \right) \right]^2 \end{aligned}$$

Comp. Coefficients of t^2 on both sides -

$$-c^2 = k^2 v^2 - c^2 k^2$$

$$-c^2 = -k^2 (c^2 - v^2)$$

$$k^2 = \frac{c^2}{c^2 - v^2}$$

$$k = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \text{--- (9)}$$

So Lorentz transformation eq are -

$$x' = k(x - vt) = \frac{(x - vt)}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = k \left[t - \frac{x}{v} \left(\frac{1 - 1}{k^2} \right) \right] = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left[t - \frac{z}{v} \frac{1 - (c^2 - v^2)}{c^2} \right]$$

$$= \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}}$$

Similarly, inverse Lorentz transformation eqs are

$$x = x' + vt'$$

$$\frac{x}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + x' v/c^2}{\sqrt{1 - v^2/c^2}}$$

When $v \ll c$
 $\frac{v^2}{c^2} \ll 1$

Lorentz eq. reduce to Galilean transformation eq.

Lorentz Fitzgerald contraction (length contraction)

Let us consider S and S' are two inertial frames such that S' is moving with constant velocity v along x axis with respect to S .

Let a rod having end coordinates x_1' and x_2' is placed in moving frame S' .

Length of rod for observer in moving frame S'

Proper length of rod $l_0 = x_2' - x_1'$ — (1)

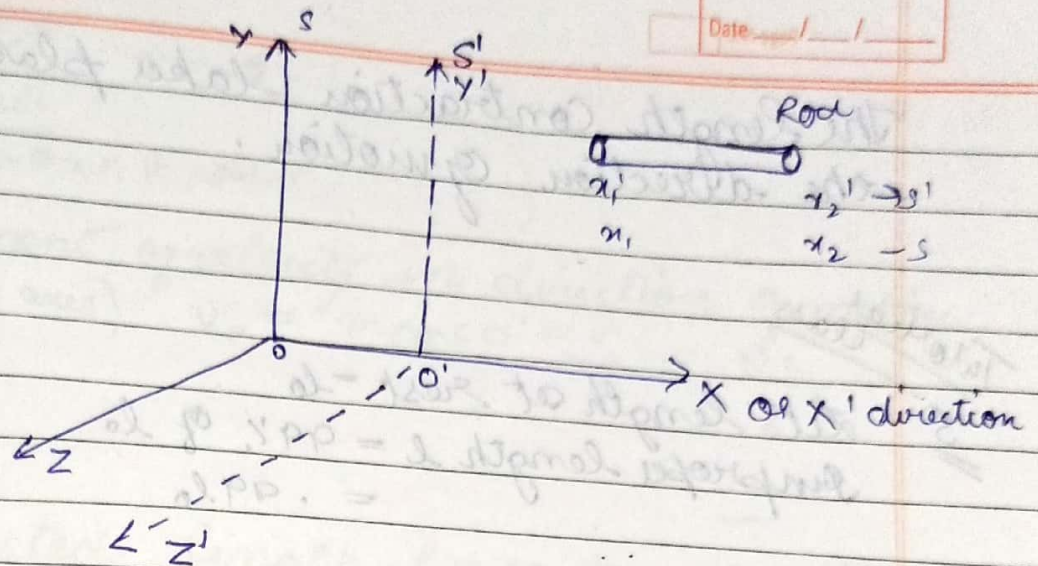
For observer in frame S end coordinates of the rod at the same time are x_1 and x_2 so,

Length of rod for observer in frame S
 Improper length of rod $l = x_2 - x_1$ — (2)

from Lorentz transformation eq-

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}}$$



$$l_0 (x'_2 - x'_1) = x_2 - x_1$$

$$\sqrt{1 - \frac{v^2}{c^2}}$$

$$l_0 = \frac{l}{\sqrt{1 - v^2/c^2}}$$

$$l = l_0 \sqrt{1 - v^2/c^2}$$

improper or contracted length $\rightarrow l < l_0$

Proper length

The length of the object measured in frame w.r.t. to which it is at rest

Improper length

The length of the object measured in frame w.r.t. to which it is moving,

Improper $\rightarrow$ contracted length $\rightarrow$ apparent length

The length contraction takes place only in the direction of motion.

Tutorial class

5 Let length at rest = l_0
Unproper length $l = 99\%$ of l_0
 $= .99 l_0$

from length contraction formula

$$l = l_0 \sqrt{1 - v^2/c^2}$$
$$.99 l_0 = l_0 \sqrt{1 - v^2/c^2}$$

$$.99 = \sqrt{1 - v^2/c^2}$$

$$.9801 = \frac{1 - v^2}{9 \times 10^{16}}$$

$$.9801 \times 9 \times 10^{16} = 1 - v^2$$

$$v^2 = 1 - .9801 \times 9 \times 10^{16}$$

$$\frac{v^2}{9 \times 10^{16}} = 1 - .9801$$

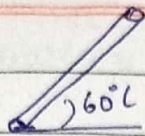
$$v^2 = .0199 \times 9 \times 10^{16}$$

$$v^2 = .1791 \times 10^{16}$$

$$v = .4232 \times 10^8$$

$$v = 4.23 \times 10^7 \text{ m/s}$$

6.



$$v = 0.8c$$

$$v_x = v \cos 60^\circ$$

Component of velocity in direction of motion
(x axis) $v_x = v \cos 60^\circ = \frac{v}{2} = \frac{.8c}{2}$

$$= .4c$$

Contracted length $l = \cancel{l_0} = l_0 \sqrt{1 - \frac{v_x^2}{c^2}}$

% Contraction in length $= \left(\frac{l_0 - l}{l_0} \right) \times 100\%$

$$= \left(\frac{l_0 - l_0 \sqrt{1 - \frac{v_x^2}{c^2}}}{l_0} \right)$$

$$= \left(1 - \sqrt{1 - \left(\frac{.4c}{c} \right)^2} \right) \times 100$$

$$= \left(1 - \sqrt{1 - .16} \right) \times 100$$

$$= 1 - \sqrt{.84} \times 100$$

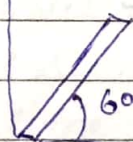
$$= 1 - .916 \times 100$$

$$= .084 \times 100$$

$$= 8.4\%$$

or

$$l_y = l \sin 60$$



$$l_x = l \cos 60$$

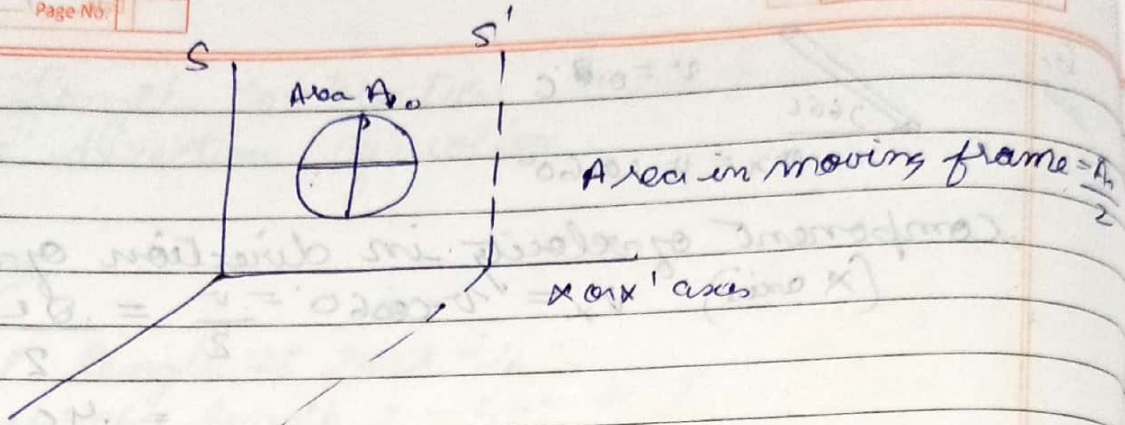
frame is moving in x direction with $v = .8c$
Contraction length in x direction $l'_x = l_x \sqrt{1 - \frac{v^2}{c^2}}$

$$l'_y = l_y = l \sin 60$$

$$\text{Resultant } l = \sqrt{l'^2_x + l'^2_y}$$

$$\% \text{ contraction in length} = \left(\frac{l_0 - l}{l_0} \right) \times 100$$

7.



Let D_0 be original diameter of circular lamina in rest frame, original area $A_0 = \frac{\pi D_0^2}{4}$ — (1)

if frame S' is moving w.r.t S along x axis

$$D'_x = D_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$D'_y = D_0$$

Area of circular lamina from moving S'

$$A = \pi \frac{D'_x}{2} \frac{D'_y}{2}$$

$$= \pi \frac{D_0 \sqrt{1 - \frac{v^2}{c^2}}}{2} \frac{D_0}{2}$$

$$= \frac{\pi D_0^2}{4} \sqrt{1 - \frac{v^2}{c^2}}$$

given $A = \frac{A_0}{2}$

$$\frac{\pi D_0^2}{4} \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2} \left(\frac{\pi D_0^2}{4} \right)$$

$$\frac{1}{2} \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = 1$$

squaring,

$$\frac{1}{4} = \frac{1 - \frac{v^2}{c^2}}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{4}$$

$$v^2 = \frac{3}{4} \times 9 \times 10^{16}$$

$$= \frac{27 \times 10^{16}}{4}$$

$$v = \sqrt{\frac{27 \times 10^{16}}{4}} = 2.598 \times 10^8 \text{ m/s}$$

Time dilation

Let us consider two inertial frames S and S' such that S' is moving w.r.t x with constant velocity v along x -axis

Let a clock is placed position x in frame S at rest, acc. to this clock an event starts at time t_1 and ends at time t_2 so, time interval measured by clock in frame S

$$\text{Proper time } t_0 = t_2 - t_1 \quad \text{--- (1)}$$

for observer in frame S' the same event starts at t'_1 time and ends at time t'_2 , so

$$\text{Apparent or improper time } t = t'_2 - t'_1 \quad \text{--- (2)}$$

Using Lorentz transformation eq.

$$t'_1 = \frac{t_1 - vx/c^2}{\sqrt{1 - v^2/c^2}} \quad ; \quad t'_2 = \frac{t_2 - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

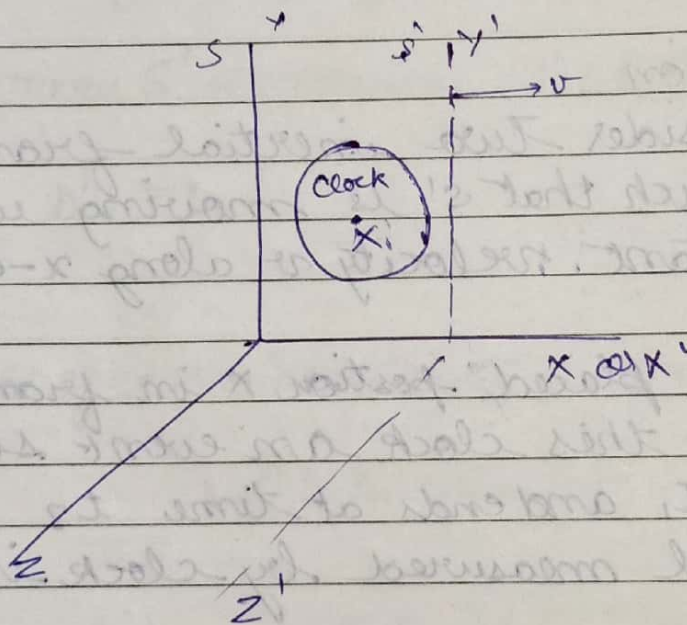
$$t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{1 - v^2/c^2}}$$

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

Apparent time
or dilated time
interval

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$t > t_0$$



So moving clock appears to go slow to the observer at rest and this effect is called dilation.

Proper time

The time interval measured by clock in rest inertial frame in which it is at rest.

Example

To prove that time dilation is a real effect
 as Mesons are unstable fundamental particles
 created at high altitude from earth by fast
 cosmic rays photons and have the velocity of
 $2.994 \times 10^8 \text{ m/s}$ and lifetime (t_0) is $2 \times 10^{-6} \text{ sec}$
 after this time they decay into e^- or positron e^+

$$v = 2.994 \times 10^8 \text{ m/s}$$

$$t_0 = 2 \times 10^{-6} \text{ sec}$$

Actual distance travelled $D = v \times t_0$

$$= 2.994 \times 10^8 \times 2 \times 10^{-6}$$

$$\approx 600 \text{ m}$$

for observer on earth

$$\text{Dilated time } t = \frac{t_0}{\sqrt{1 - v^2/c^2}} = \frac{2 \times 10^{-6}}{\sqrt{1 - \left(\frac{2.994}{3}\right)^2}}$$

$$= 3.17 \times 10^{-5} \text{ sec}$$

Apparent distance travelled

$$D = v t = 2.994 \times 10^8 \times 3.17 \times 10^{-5}$$

$$\approx 9500 \text{ m} = 9.5 \text{ km}$$

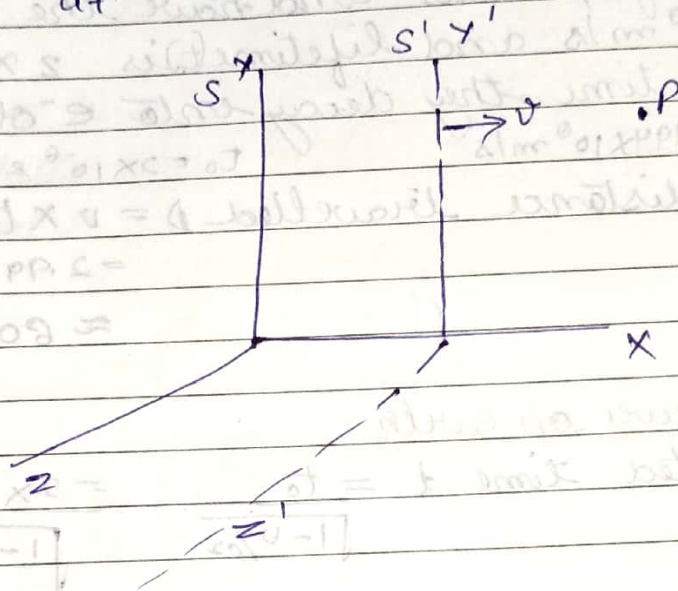
Relativistic velocity addition theorem

Let us consider two inertial frames S and S'
 such that S' is moving with constant
 velocity v wrt to S along x axis

Components of velocity of moving particle
 for observer in rest frame S are u_x , u_y and u_z

for observer in S' , component of velocity of moving particle are-

$$u_x' = \frac{dx'}{dt'} \quad u_y' = \frac{dy'}{dt'} \quad u_z' = \frac{dz'}{dt'}$$



from inverse Lorentz transformation eq's

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} \quad y = y' \quad z = z' \quad t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

differentiating above eq.

$$dx = \frac{dx' + v dt'}{\sqrt{1 - v^2/c^2}} \quad dy = dy' \quad dz = dz'$$

$$dt = \frac{dt' + \frac{v}{c^2} dx}{\sqrt{1 - v^2/c^2}}$$

~~for~~

$$\sqrt{1 - v^2/c^2}$$

velocity of

$$= \frac{dz'}{dt'}$$

p

velocity of

$$u_x = \frac{dx}{dt} = \frac{dx' + v dt'}{dt' + \frac{v dx'}{c^2}} \quad \text{--- (1)}$$

$$u_y = \frac{dy}{dt} = \frac{dy' \sqrt{1 - \frac{v^2}{c^2}}}{dt' + \frac{v dx'}{c^2}} \quad \text{--- (2)}$$

$$u_z = \frac{dz}{dt} = \frac{dz' \sqrt{1 - \frac{v^2}{c^2}}}{dt' + \frac{v dx'}{c^2}} \quad \text{--- (3)}$$

Dividing num and den. of eq (1) (3) by dt' we get

$$u_x = \frac{\frac{dx'}{dt'} + \frac{v dt'}{dt'}}{\frac{dt'}{dt'} + \frac{v}{c^2} \frac{dx'}{dt'}} = \frac{u_x' + v}{1 + \frac{v}{c^2} u_x'}$$

relation eq's

$$t = \frac{t' + \frac{v x'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{dz'}{dt'}$$

$$u_y = \frac{\frac{dy'}{dt'} \sqrt{1 - \frac{v^2}{c^2}}}{\frac{dt'}{dt'} + \frac{v}{c^2} \frac{dx'}{dt'}}$$

$$= \frac{u_y' \sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2} u_x'}$$

$$u_z = \frac{\frac{dz'}{dt'} \sqrt{1 - \frac{v^2}{c^2}}}{\frac{dt'}{dt'} + \frac{v}{c^2} \frac{dx'}{dt'}}$$

$$= \frac{u_z' \sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2} u_x'}$$

Special cases

Case 1 $\Rightarrow$ To prove that Einstein II postulate is consistent with relativistic velocity addition theorem

Let particle moving in S' frame is photon having $u_x' = c$ and frame S' is moving with velocity v along x axis w.r.t S .

Velocity for observer in rest frame S

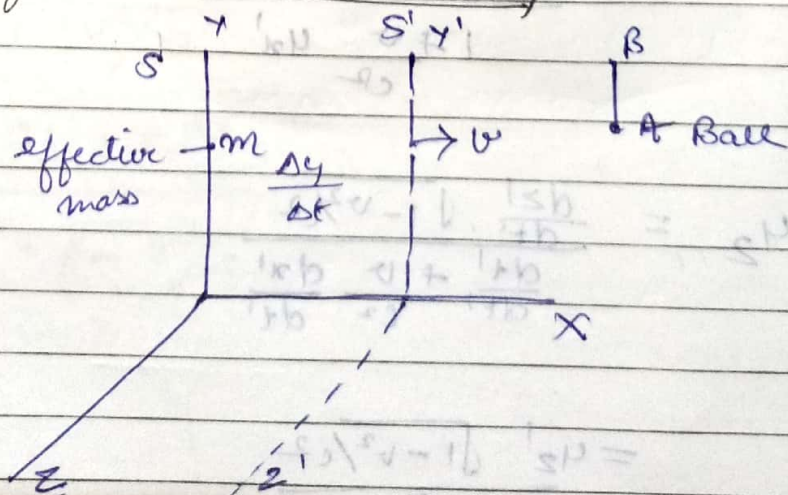
$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} = \frac{c + v}{1 + \frac{vc}{c^2}} = \frac{c(c+v)}{c+v} = c$$

Case 2: To prove that no signal can travel faster than light.

Considering moving particle velocity $u_x' = c$ when frame S' is moving with velocity $v = c$.

$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} = \frac{c + c}{1 + \frac{c^2}{c^2}} = \frac{2c}{2} = c$$

Variation of mass with velocity



Variations of mass with velocity

Let S and S' be the two inertial frames such that S' is moving with constant velocity v w.r.t to S along x direction.

Let a ball of rest mass m_0 is thrown in y direction so that it get displaced by $\Delta y'$ distance in time $\Delta t'$.

So, momentum of ball in y direction in S' frame

$$p_{y'} = m_0 v_{y'} = m_0 \frac{\Delta y'}{\Delta t'} \quad (1)$$

For observer in rest frame S if the ball get displaced by Δy distance in time Δt and has effective mass m then momentum of ball in y direction for observer in S

$$p_y = m v_y = m \frac{\Delta y}{\Delta t} \quad (2)$$

As momentum is same for all ~~at~~ inertial frames and also there is no relative velocity in y direction,

$$p_y = p_{y'}$$

$$\frac{m \Delta y}{\Delta t} = m \frac{\Delta y'}{\Delta t'} \quad (3)$$

As there is no relative velocity in x direction $\{$ ④

$$\Delta y = \Delta y'$$

$\Delta t'$ = proper time

Δt = improper time

from time dilation formula $\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$

$$\text{So, } m \frac{dy}{dt'} \sqrt{1 - v^2/c^2} = \frac{m_0 \Delta y'}{\Delta t'}$$

effective
mass or

~~relativistic~~
mass

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Einstein mass energy relation

Let us consider a particle of mass m moving with velocity v

Then force F acting on the particle is given by

$$F \frac{dp}{dt} = \frac{d}{dt} (-mv) = m \frac{dv}{dt} + v \frac{dm}{dt} \quad \text{--- (1)}$$

According to work energy theorem, work done on the particle if it is displaced by ds when force F acts on it, is equal to increase in kinetic energy dk ,

$$\begin{aligned} dw &= dk = F \cdot ds \\ &= \left(m \frac{dv}{dt} + v \frac{dm}{dt} \right) \cdot ds \end{aligned}$$

We take velocity of particles, which are moving in space as $m_0 c^2$

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As $\frac{ds}{dt} = v$, $dk = m_0 v dv + v^2 dm$ - (2)

Mass varies with velocity according to relation

$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ (3) where m = effective or relativistic mass

m_0 = rest mass

squaring both sides,

$$m^2 = \frac{m_0^2}{(1 - v^2/c^2)}$$

$$m^2 (1 - v^2/c^2) = m_0^2$$

$$m^2 c^2 - m^2 v^2 = m_0^2 c^2 \quad - (4)$$

diff eq (4)

$$c^2 2m dm - [m^2 \cdot 2v dv + v^2 2m dm] = 0$$

$$c^2 2m dm = 2m (m v dv + v^2 dm)$$

$$c^2 dm = m v dv + v^2 dm \quad - (5)$$

Comparing eq (2) & (5)

$$dk = c^2 dm$$

integrating above eq -

$$\int_0^k dk = c^2 \int_{m_0}^m dm$$

$$K = c^2 (m - m_0)$$

Relativistic
KE.

$$K = (m - m_0) c^2 \quad - (6)$$

Rest mass energy is $m_0 c^2$

As total relativistic energy of particle is sum of relativistic KE K and rest mass energy $m_0 c^2$,

$$\bar{E} = K + m_0 c^2$$

$$= (m - m_0) c^2 + m_0 c^2$$

$$\boxed{\bar{E} = m c^2}$$

Einstein mass energy relation

Ex m

i.e. Energy corresponds to mass and mass corresponds to energy at high velocity ($v \rightarrow c$)

from relativistic mechanics

$$K = (m - m_0) c^2$$

$$K = \left[\frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 \right] c^2 = m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right]$$

$$K = m_0 c^2 \left(\left(1 - \frac{v^2}{c^2} \right)^{-1/2} - 1 \right)$$

When $v \ll c$, expanding binomially upto 1 term,

$$\boxed{K = m_0 c^2 \left[1 + \frac{1}{2} \frac{v^2}{c^2} + \dots - 1 \right] = \frac{1}{2} m_0 v^2}$$

Expression of KE in classical mechanics when $v \ll c$

Derive relation b/w E, p and rest mass energy

Mass varies with velocity acc. to relation -
 $m = m_0 / \sqrt{1 - v^2/c^2}$

where m = effective or relativistic mass
 m_0 = rest mass
 $m_0 c^2$

Squaring both sides,

$$m^2 = \frac{m_0^2}{(1 - v^2/c^2)}$$

$$m^2 (c^2 - v^2) = m_0^2 c^2$$

$$m^2 c^2 - m^2 v^2 = m_0^2 c^2$$

Multiplying by c^2

$$m^2 c^4 - m^2 v^2 c^2 = m_0^2 c^4$$

$$E^2 - p^2 c^2 = (m_0 c^2)^2$$

$$\boxed{E^2 = p^2 c^2 + m_0^2 c^4}$$

Derive relation b/w p and rest mass energy $m_0 c^2$

Total relativistic energy,

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

$$E = K + m_0 c^2$$

$$K + m_0 c^2 = \sqrt{p^2 c^2 + m_0^2 c^4}$$

Squaring both sides

$$K^2 + 2K m_0 c^2 + m_0^2 c^4 = p^2 c^2 + m_0^2 c^4$$

$$p^2 c^2 = K^2 + 2K m_0 c^2$$

$$\boxed{p = \sqrt{\frac{K^2}{c^2} + 2K m_0}}$$

Mass less particles

Particles which do not exist at rest i.e. they have zero rest mass and when they came into existence they travel with the velocity of light are called mass less particles.

At rest $m_0 = 0$

$$E = pc$$

$$E = mc^2, v = c$$

examples are $\rightarrow$ photons, gravitons, Neutrinos.

Relativistic form of Newton II law

$$F = \frac{d(p)}{dt} = \frac{d(mv)}{dt}$$

Relativistic mass $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

then $F = \frac{d}{dt} \left(\frac{m_0 v}{\sqrt{1 - v^2/c^2}} \right)$

$$= \frac{m_0 \left[\frac{d}{dt} \left(\frac{v}{\sqrt{1 - v^2/c^2}} \right) \right]}{(1 - v^2/c^2)^{3/2}}$$

$$F = m_0 \left(\frac{1 - v^2/c^2}{} \right)^{-3/2} \frac{dv}{dt}$$

Wave mechanics

Duality of matter and radiation.

1) Dual nature of light

- | | |
|-------------------------------------|---|
| <p>— Wave —</p> <p>— Particle —</p> | <p>① Interference ② Diffraction</p> <p>③ Polarisation</p> <p>① Photoelectric effect</p> <p>② Compton effect - X rays</p> <p>③ Black body radiation</p> |
|-------------------------------------|---|

Matter waves

De Broglie hypothesis

Extended the idea of dual nature to moving material particles.

Particle of m moves with velocity v

De Broglie wave $\lambda = \frac{h}{mv} = \frac{h}{p}$

Waves associated with moving material particles $\rightarrow$ Matter waves

Davison and Germer expt to prove the dual nature of matter particles

Other relations of λ

① Non relativistic cases ($v \ll c$)

i) for a particle having mass m and velocity v .

$$\lambda = \frac{h}{mv}$$

ii) for a particle having kinetic energy $K = \frac{1}{2}mv^2$

$$K = \frac{1}{2}mv^2$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

$$K = \frac{1}{2} \frac{(mv)^2}{m}$$

$$mv = \sqrt{2mK}$$

iii) for charged particle having charge q and accelerated through a potential of V volts

$$K = qV$$

$$q \rightarrow \text{coulombs}$$
$$V \rightarrow \text{volts}$$

$$\text{As } K = qV$$

$$\text{De Broglie } \lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

Special case

i) If charged particle is electron, and accelerated through a pot. diff of V volt,

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\lambda_e = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}$$

$$\lambda_e = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

for e^- , $h = \text{Planck's constant}$

$$= 6.63 \times 10^{-34} \text{ Js}$$

$$m = \text{mass of } e^- = 9.1 \times 10^{-31} \text{ kg}$$

$$q = e = 1.6 \times 10^{-19} \text{ C}$$

2) for alpha particle

charge on α particle $q = 2e = 2 \times 1.6 \times 10^{-19} \text{ C}$

mass of α particle $= 4 m_{\text{proton}}$

$$= 4 \times 1.6 \times 10^{-27} \text{ kg}$$

$$\lambda_{\alpha} = \frac{h}{\sqrt{2 m_{\alpha} q_{\alpha} V}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 42.75 \times 10^6}}$$

Relativistic cases

$v \rightarrow c$ $v \approx 10^6 - 10^8 \text{ m/s}$ (high velocity)

i) for a particle having mass m and velocity v

$$\lambda = \frac{h}{mv}$$

where relativistic ~~kinetic~~ mass $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

$$\lambda = \frac{h \sqrt{1 - v^2/c^2}}{m_0 v}$$

where m_0 = rest mass of particle

ii) Derive expression of λ if relativistic K.E., K and rest mass energy of particle $m_0 c^2$ is given.

If particle is moving with high velocity v

$$E = mc^2$$

$$E = K + m_0 c^2 \quad \text{--- (1)}$$

$$E = \sqrt{p^2 c^2 + m_0^2 c^4} \quad \text{--- (2)}$$

equating (1) & (2), momentum

$$\sqrt{p^2 c^2 + m_0^2 c^4} = K + m_0 c^2$$

Both charge move ^{or de Broglie's for} matter wave & for

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Squaring both sides, we get

$$\frac{h^2 c^2}{\lambda^2} + m_0^2 c^4 = (K + m_0 c^2)^2$$

$$\frac{h^2 c^2}{\lambda^2} + m_0^2 c^4 = K^2 + m_0^2 c^4 + 2K m_0 c^2$$

$$\frac{h^2 c^2}{\lambda^2} = K(K + 2m_0 c^2)$$

$$\lambda^2 = \frac{h^2 c^2}{K(K + 2m_0 c^2)}$$

$$\lambda = \frac{hc}{K(K + 2m_0 c^2)}$$

valid for
high velocity
for low
velocity

$$K > m_0 c^2$$

When $K \ll m_0 c^2$

$$\lambda = \frac{hc}{\sqrt{K(K + 2m_0 c^2)}}$$

$$(K + 2m_0 c^2)^2 \approx 2 m_0 c^2$$

Then,

$$\lambda = \frac{hc}{\sqrt{K \cdot 2m_0 c^2}}$$

$$\lambda = \frac{h}{\sqrt{2m_0 K}}$$

$$\lambda = \frac{h}{\sqrt{2m_0 K}}$$

non relativistic
(for low velocity)

ii) λ if $KE = 1 \text{ MeV}$
 $(m_0 c^2)_{\text{electron}} = 9.1 \times 10^{-31} \times 9 \times 10^{16} \text{ J}$
 $= \frac{9.1 \times 10^{-31} \times 9 \times 10^{16}}{1.6 \times 10^{-13}} \text{ MeV}$
 $\approx 0.51 \text{ MeV}$

$1 \text{ MeV} = 1.6 \times 10^{-19} \times 10^6 \text{ J}$
 $1 \text{ J} = \frac{1}{1.6 \times 10^{-13}} \text{ MeV}$

$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\sqrt{(1.6 \times 10^{-13}) (1.6 \times 10^{-13} + 2 \times 0.51 \times 1.6 \times 10^{-13})}}$
 $= 15.56 \times 10^{-13}$

either $v \rightarrow 0$ or $\lambda \rightarrow \infty$
 iii) for charged $KE = qV$

$\lambda = \frac{hc}{\sqrt{qV (qV + 2m_0 c^2)}}$

Matter waves

$\lambda = \frac{h}{mv}$

$\lambda \propto \frac{1}{m} ; \lambda \propto \frac{1}{v}$

Detection of matter waves

i) Stone $m = .1 \text{ Kg}$ $v = 100 \text{ m/s}$
 $\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{.1 \times 100}$

$\lambda = 6.63 \times 10^{-36} \times 10$
 $= 6.63 \times 10^{-35} \text{ m}$

Subatomic particle

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$v = 100 \text{ m/s}$$

$$\lambda = 6.63 \times 10^{-34}$$

$$9.1 \times 10^{-31} \times 100$$

Wave velocity or Phase velocity v_p

Let us consider a particle of mass m moving with velocity v then wavelength of the wave associated with moving particle

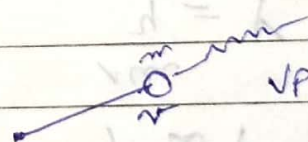
$$\lambda = \frac{h}{mv} \quad \text{--- (i) (from de-Broglie concept)}$$

* If ν is frequency of wave, then $\nu = \frac{E}{h}$

$$\text{So, wave velocity } v_p = \nu \lambda = \frac{E}{h} \cdot \frac{h}{mv}$$

from Einstein mass energy relation $E = mc^2$

$$\text{So } v_p = \frac{mc^2}{mv}$$



Phase
Wave velocity
or velocity of
single
wave

$$v_p = \frac{c^2}{v}$$

Particle
velocity

As $v \ll c$ then
 $v_p > c$ which is
not possible.

Schrodinger had resolved this discrepancy by considering that a moving particle is equivalent to a group of waves or wave packet instead of a single wave

Wave packet

A group of waves of slightly different velocities and wavelength get superposed and the probability of finding the particle is maximum in the region where constructive interference takes place and at points of destructive interference particle cannot be located.

Group velocity

The velocity with which wave packet moves forward in the medium is called group velocity and its formula is -

$$v_g = \frac{d\omega}{dk}$$

It is also defined as the ^{ratio of change in} angular frequency to the change in propagation constant

$$\omega = \text{angular frequency} \\ = 2\pi\nu$$

$$k = \text{propagation constant} = \frac{2\pi}{\lambda}$$

Phase velocity v_p

$$v_p = \nu \lambda = \frac{2\pi\nu \cdot \lambda}{2\pi}$$

$$\text{Phase velocity } v_p = \frac{\omega}{k}$$

$$y = a \sin(\omega t - kx)$$

$$\omega t - kx = \text{constant}$$

$$\omega dt - k dx = 0$$

$$v_p = \frac{dx}{dt} = \frac{\omega}{k}$$

When a single phase wave of definite frequency and wave length travels in the medium its velocity of propagation is called phase velocity.

To prove.

Good!
Relativistic
case
for 2 marks

Group velocity v_g is equal to particle velocity v .
Let us consider a particle of mass m moving with velocity v ($v \rightarrow c$). The energy and momentum of particle are given by -

$$\text{Energy } E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \quad \text{where } m_0 = \text{rest mass} \quad \text{--- (1)}$$

$$\text{momentum } p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \quad \text{--- (2)}$$

The velocity of wave packet formed when particle behave like group of waves is given by

$$v_g = \frac{d\omega}{dk} \quad \text{--- (3)}$$

$$\text{As } \omega = 2\pi \nu = \frac{2\pi E}{h}$$

$$\text{from (1)} \quad \omega = \frac{2\pi}{h} \frac{m_0 c^2}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

$$\begin{aligned} \text{diff w.r.t } v, \\ \frac{d\omega}{dv} &= \frac{2\pi m_0 c^2}{h} \frac{d}{dv} \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \\ &= \frac{2\pi m_0 c^2}{h} \left(\frac{1}{2}\right) \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \left(\frac{-2v}{c^2}\right) \\ \frac{d\omega}{dv} &= \frac{2\pi m_0 v}{h} \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \quad \text{--- (4)} \end{aligned}$$

As propagation constant $k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$

(from de Broglie concept)

using $\odot$

$$k = \frac{2\pi}{h} \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

diff k wrt v

$$\frac{dk}{dv} = \frac{2\pi m_0}{h} \frac{d}{dv} \left\{ v \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \right\}$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h} \left\{ v \left(\frac{-1}{2} \right) \left(1 - \frac{v^2}{c^2} \right)^{-3/2} \left(\frac{-2v}{c^2} \right) + \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \right\}$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h} \left\{ v \left(\frac{-1}{2} \right) \left(1 - \frac{v^2}{c^2} \right)^{-3/2} \left(\frac{-2v}{c^2} \right) + \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \right\}$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2} \right)^{-3/2} \left\{ \frac{v^2}{c^2} + \left(1 - \frac{v^2}{c^2} \right) \right\}$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2} \right)^{-3/2} \quad - \odot$$

$$\text{sg } v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} = \frac{\frac{2\pi m_0 v}{h} \left(1 - \frac{v^2}{c^2} \right)^{-3/2}}{\frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2} \right)^{-3/2}} = v$$

$$\boxed{v_g = v}$$

particle velocity

for less than 3 marks

Page No.

non relativistic case

Date

II method

The velocity of wave packet formed when particle behave like group of waves is given by, $v_g = \frac{d\omega}{dk}$ (3)

$$\text{As } \omega = 2\pi \nu = \frac{2\pi E}{h}$$

$$\text{If } v \ll c, E = \frac{1}{2} mv^2$$

$$\text{Then } \omega = \frac{2\pi}{h} \cdot \frac{1}{2} mv^2 = \frac{m \times v^2}{h}$$

Diff ω wrt v

$$\frac{d\omega}{dv} = \frac{m \times 2v}{h} \quad (4)$$

As propagation constant $k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$ (from de Broglie concept)

using (4)

$$k = \frac{2\pi m v}{h} \quad \frac{dk}{dv} = \frac{2\pi m}{h} \quad (5)$$

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$$

$$= \frac{m \times 2v/h}{2\pi m/h}$$

$$\boxed{v_g = v}$$

Relation b/w group velocity v_g with and phase velocity v_p

$$v_g = \frac{d\omega}{dk}$$

$$v_p = \frac{\omega}{k} \quad \text{--- (1)}$$

$$\text{or } \omega = v_p k$$

$$\text{so } v_g = \frac{d(v_p k)}{dk} = v_p + k \frac{dv_p}{dk} \quad \text{--- (2)}$$

propagation constants $k = \frac{2\pi}{\lambda}$, $\frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2}$

$$\text{then } v_g = v_p + \frac{2\pi}{\lambda} \left(\frac{dv_p}{d\lambda} \right) \left(-\frac{\lambda^2}{2\pi} \right)$$

$$\boxed{v_g = v_p - \lambda \frac{dv_p}{d\lambda}}$$

for dispersion medium

As v_p is λ dependent

$$\therefore v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

for non-dispersive medium

$$v_p \neq f(\lambda)$$

$$\frac{dv_p}{d\lambda} = 0$$

$$\boxed{v_g = v_p}$$

To prove $v_g v_p = c^2$

*

phase velocity $v_p = \frac{c^2}{v}$ $v \rightarrow$ particle velocity

As $v_g = v$

group velocity = particle velocity

$$v_p = \frac{c^2}{v_g}$$

$$v_p \cdot v_g = c^2$$

Heisenberg uncertainty principle

According to this principle it is impossible to determine simultaneously the exact position and exact momentum of the moving particle with perfect accuracy.

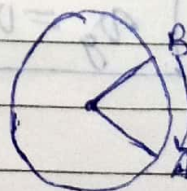
The product of uncertainty in position (Δx) and uncertainty in momentum is greater equal to ($\geq$) $h/2\pi$ where h = planck's constant

Energy time uncertainty relation

$$\Delta E \cdot \Delta t \geq \frac{h}{2\pi}$$

Angular position and angular momentum (ΔJ) uncertainty relation

$$\Delta J \cdot \Delta \theta \geq \frac{h}{2\pi}$$



Canonically conjugate pairs of other physical quantities

Applications

- 1) To prove that e^- cannot exist inside the nucleus of an atom

Let us assume that electron is present inside the nucleus of an atom, as radius of the nucleus is of the order of 10^{-14} m, so uncertainty in position of e^- is $\rightarrow$

$$\Delta x = 2r = 2 \times 10^{-14} \text{ m}$$

from Heisenberg uncertainty principle.

$$\text{Uncertainty in momentum } \Delta p \approx \frac{h}{2\Delta x}$$

$$\approx \frac{6.63 \times 10^{-34} \text{ Kg m}^2/\text{s}}{2 \times 3.17 \times 10^{-14} \text{ m}}$$

$$\Delta p \approx 0.527 \times 10^{-20} \text{ Kg m/s}$$

Let $|\Delta p| \approx |p| = 0.527 \times 10^{-20} \text{ Kg m/sec}$
from relativistic mechanics,

$$\text{Energy } E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

$$= \sqrt{(pc)^2 + (m_0 c^2)^2}$$

$$pc = 0.527 \times 10^{-20} \times 3 \times 10^8 = 1.581 \times 10^{-12} \text{ J}$$
$$= \frac{1.581 \times 10^{-12}}{1.6 \times 10^{-13}} \text{ MeV}$$

$$= 9.88 \text{ MeV}$$

$$(m_0 c^2) = 9.1 \times 10^{-31} \times 9 \times 10^{16} \text{ J}$$
$$= \frac{9.1 \times 10^{-31} \times 9 \times 10^{16}}{1.6 \times 10^{-13}} \text{ MeV} = 0.51 \text{ MeV}$$

$$m_0 c^2 \ll pc \Rightarrow E \approx pc \approx 9.888 \text{ MeV}$$

Energy of e^- present inside the nucleus

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But from experimental data the energy emitted by radioactive nuclei during β decay process is about 3-4 MeV

So our assumption is wrong and e^- cannot exist inside the nucleus of the atom.

2. To find the radius of Bohr's first orbit using Heisenberg principle.

Let Δx is uncertainty in position and Δp is uncertainty in momentum for an e^- present in an orbit of an atom then from Heisenberg uncertainty principle

from Heisenberg uncertainty principle:

$$\Delta x \Delta p \approx \frac{h}{2\pi}$$

$$\text{or } \Delta p \approx \frac{h}{2\pi \Delta x} \quad \text{--- (1)}$$

Uncertainty in kinetic energy (ΔK) of an e^- is given by -

$$\Delta K = \frac{\Delta p^2}{2m} = \frac{h^2}{4\pi^2 \Delta x^2 \cdot 2m} = \frac{h^2}{8\pi^2 m (\Delta x)^2} \quad \text{--- (2)}$$

Uncertainty in potential energy of e^- (ΔV) is given by

$$\Delta V = \frac{-Ze^2}{4\pi \epsilon_0 \Delta x} \quad \text{--- (3)}$$

Total uncertainty in energy of e^-

$$\Delta E = \Delta K + \Delta V$$

$$\Delta E = R^2 = \frac{Z^2 e^2}{4\pi \epsilon_0 \Delta x} \quad \text{--- (4)}$$

Binding energy of e^-

for energy to be minimum as radius of first orbit is to be determined

$$\frac{d(\Delta E)}{d(\Delta x)} = 0 \quad \text{and} \quad \frac{d^2(\Delta E)}{d(\Delta x)^2} = +ve$$

diffⁿ eq (4) w.r.t Δx and putting $\frac{d(\Delta E)}{d(\Delta x)} = 0$, we get

$$\frac{d(\Delta E)}{d(\Delta x)} = \frac{h^2(-2)}{8\pi^2 m (\Delta x)^3} - \frac{Ze^2(-1)}{4\pi\epsilon_0 (\Delta x)^2} = 0$$

$$-\frac{2h^2}{8\pi^2 m (\Delta x)^3} + \frac{Ze^2}{4\pi\epsilon_0 (\Delta x)^2} = 0$$

$$\frac{Ze^2}{4\pi\epsilon_0 (\Delta x)^2} = \frac{2h^2}{8\pi^2 m (\Delta x)^3}$$

$$\Delta x = \frac{\epsilon_0 h^2}{m \pi Ze^2}$$

Wave function and its physical significance
Wave function: is the function of
 The quantity whose variation make up the matter waves is called wave function. It is the function of space coordinates and time

$$\Psi = \Psi_0 \sin(\omega t - kx)$$

$$= \Psi_0 e^{i(\omega t - kx)}$$

↓ Normalization
 Atomic Constant

The square of the magnitude of ψ gives the probability of finding the particle in the given volume at any point at time t

$$\text{Probability } P(r, t) = \int |\psi|^2 d\tau = \int \psi^* \psi d\tau \quad \text{where } d\tau = dx dy dz$$

volume element

Normalization condition

As the particle is certainly located somewhere in space so the probability of finding the particle in the entire space must be unity

$$\int_{-\infty}^{+\infty} |\psi|^2 d\tau = 1$$

Conditions for acceptable wave functions

- 1) ψ must be finite
- 2) ψ must be single valued
- 3) ψ must be continuous
- 4) ψ must satisfy normalization condition

Schrodinger time independent wave equation
Let us consider a group of waves associated with a moving particle.

The differential eq. of wave motion is -

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

here, ψ = wave function = $\psi(r, t)$
 v = velocity

Schrodinger Time independent wave equation

$$\nabla^2 \psi + \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{--- (1) where } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

The solution of eq (1) can be written as-

$$\psi = \psi_0 e^{-i\omega t} \quad \text{--- (2)}$$

diff eq (2) w.r.t t

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t}$$

again diff. above eq $\rightarrow$

$$\frac{\partial^2 \psi}{\partial t^2} = (i\omega)^2 \psi_0 e^{-i\omega t} = -\omega^2 \psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \quad \text{--- (3)}$$

from (1) and (3)

$$\nabla^2 \psi = -\frac{\omega^2 \psi}{v^2} \quad \text{--- (4)}$$

$$\text{or } \nabla^2 \psi + \frac{\omega^2}{v^2} \psi = 0$$

$$\text{As } \omega = 2\pi \nu \quad ; \quad \frac{\omega^2}{v^2} = \frac{4\pi^2 \nu^2}{v^2} = \frac{4\pi^2}{\lambda^2} \quad (\because \nu = \frac{v}{\lambda})$$

from de Broglie concept

$$\lambda = \frac{h}{mv} \quad \text{where } m = \text{mass of particle}$$

$v = \text{velocity}$

$$\text{So } \frac{\omega^2}{v^2} = \frac{4\pi^2 m^2 v^2}{h^2} \quad \text{--- (5)}$$

from (4) & (5)

$$\nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0$$

If E is total energy, K is kinetic and V is the potential energy,

$$E = K + U$$

$$E = \frac{1}{2} m v^2 + U$$

$$E - U = \frac{1}{2} m v^2$$

$$m^2 v^2 = 2m(E - U) \quad \text{--- (7)}$$

using eq (7) in eq (6)

$$\nabla^2 \psi + \frac{2m(E - U)}{h^2} \psi = 0$$

$$\boxed{\nabla^2 \psi + \frac{2m(E - U)}{h^2} \psi = 0} \quad \text{--- (8) in 3-D}$$

In 1D (Particle moving in x direction)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E - U)}{h^2} \psi = 0$$

for free particle RE. ($U=0$)

also,

$$\hbar = \frac{h}{2\pi}$$

then,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E - 0)}{\hbar^2} \psi = 0$$

Schrodinger time dependent eq.

The schrodinger time independent wave eq. is as follows—

1/2 pot energy

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad (1)$$

The solution of eq (1) can be written as—

$$\psi = \psi_0 e^{-i\omega t}$$

then $\frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t} = -i\omega \psi = -i(2\pi\nu) \psi$

As $E = h\nu$

$$\frac{\partial \psi}{\partial t} = -i \left(\frac{2\pi E}{h} \right) \psi$$

As $\hbar = h/2\pi$

$$\frac{\partial \psi}{\partial t} = -\frac{i E \psi}{\hbar}$$

$$E \psi = -\hbar \frac{\partial \psi}{\partial t} = -\frac{i \hbar \partial \psi}{i^2 \partial t} \quad (\because i^2 = -1)$$

$$\boxed{E \psi = i \hbar \frac{\partial \psi}{\partial t}} \quad (2)$$

from eq (1) & (2)

$$\nabla^2 \psi + \frac{2m}{\hbar^2} \left[i \hbar \frac{\partial}{\partial t} - V \right] \psi = 0$$

$$\frac{2m}{\hbar^2} \left[i \hbar \frac{\partial}{\partial t} - V \right] \psi = -\nabla^2 \psi$$

$$\left[i \hbar \frac{\partial}{\partial t} - V \right] \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi$$

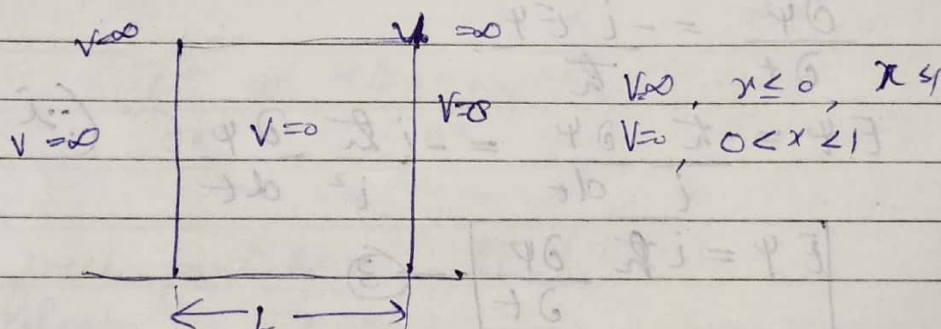
$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi$$

Applications Schrodinger time independent eq.

i) Particle in one dimensional box

Let us consider a particle ~~long~~ trapped in one dimensional potential ^{of length L} ~~box~~ such that particle is moving freely in b/w $0 \leq x \leq L$. $\therefore 0 \leq x \leq L$
~~of length L~~
 energy $V=0$

Potential energy is infinite at the walls of the box as well be outside the box



The schrodinger time ~~independent~~ eq. for particle $\frac{\partial^2 \psi}{\partial x^2}$ moving in x direction -

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial x^2 m}{\hbar^2} (E - V) \psi = 0$$

Inside the box $V=0$,

$$\text{So, } \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial x^2 m E}{\hbar^2} \psi = 0 \quad \text{--- (1)}$$

Let $\frac{\partial^2 \psi}{\partial x^2} = -k^2$ where $k = \text{Propagation constant}$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0 \quad (3)$$

Let the solution of eq (3) is of the form -

$\psi = A \sin kx + B \cos kx$ where A and B are constants

As particle cannot be at $x=0$ and $x=L$, so $\psi=0$ at $x=0$ & $x=L$

Putting $\psi=0$ at $x=0$ in eq (4)

$$0 = A \sin 0 + B \cos 0$$

$$0 = 0 + B \Rightarrow B = 0$$

Putting $\psi=0$ at $x=L$

$$\text{Then } \psi = A \sin kx \quad (5)$$

Putting $\psi=0$ and $x=L$ in eq (5)

$$0 = A \sin kL$$

$$\sin kL = 0$$

$$kL = n\pi$$

$$k = \frac{n\pi}{L} \quad (6)$$

$$(n=1, 2, 3, \dots)$$

$$n \neq 0$$

using (1) & (5)

$$\frac{2 \times 2 m E}{h^2} = \frac{n^2 \pi^2}{L^2} \quad n=1, 2, \dots$$

~~$$E = \frac{h^2 \pi^2 A^2}{2 m L^2}$$~~

~~$$E = \frac{h^2 \pi^2 h^2}{2 m L^2}$$~~

~~$$E = \frac{\pi^2 h^2}{2 m L^2}$$~~

$$E = \frac{h^2 \pi^2 n^2}{2 m L^2}$$

or
energy eqn
values

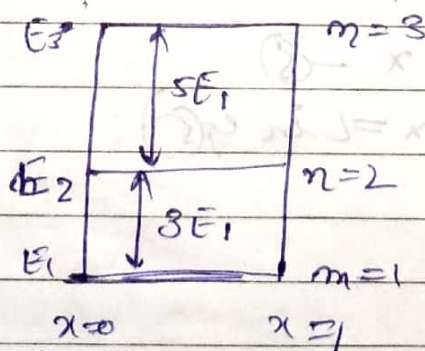
$$E = \frac{n^2 h^2}{2 m L^2}$$

$$E \propto n^2$$

if $n=1$ $E_1 = E_1 = \frac{E_1}{2 m L^2}$ (ground state)

$n=2$ $E_2 = 4E_1$ (first excited state)

if $n=3$



from (5) & (6)

$$\psi = A \sin \frac{n\pi x}{L}$$

as particle is inside the box must be is b/w
 $0 < x < L$

So probability of finding particle inside the box must be 1. (Normalization condition)

Using normalization conditions,

$$\int_0^L |\psi|^2 dx = 1$$

$$A^2 \int_0^L \left(\frac{\sin^2 n\pi x}{L} \right) dx = 1$$

$$\frac{A^2}{2} \int_0^L \left(1 - \cos \frac{2n\pi x}{L} \right) dx = 1$$

$$\frac{A^2}{2} \left[x - \frac{L \sin \frac{2n\pi x}{L}}{2n\pi} \right]_0^L = 1$$

$$\frac{A^2}{2} \left[L - \frac{L}{2n\pi} \left(\sin \frac{2n\pi L}{L} - \sin \frac{2n\pi \cdot 0}{L} \right) \right] = 1$$

$$\frac{A^2}{2} \left[L - \frac{L(0)}{2n\pi} \right] = 1$$

$$\frac{A^2 L}{2} = 1$$

$$A^2 = \frac{2}{L}$$

$$A = \sqrt{\frac{2}{L}}$$

So normalized wave function or eigen function is

$$\psi = \sqrt{\frac{2}{L}} \frac{\sin n\pi x}{L}$$

Particle in 1D box

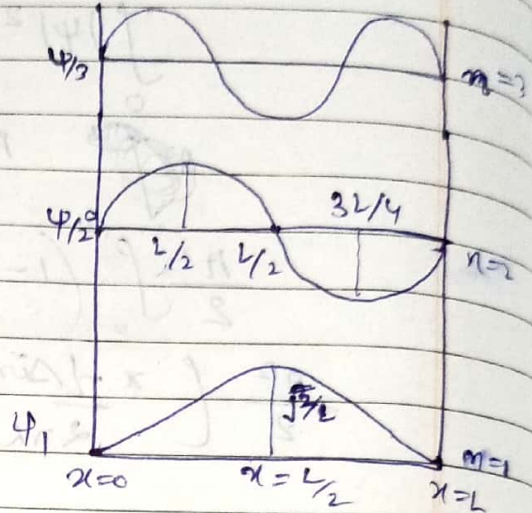
Normalized wave function

$$\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$K = \frac{n\pi}{L}$$

$$\frac{2\pi}{\lambda} = \frac{n\pi}{L}$$

$$L = \frac{n\lambda}{2}$$



if $n=1$, $L = \lambda/2$

if $n=2$, $L = \lambda$

if $n=3$, $L = 3\lambda/2$

for $n=1$, $x = L/2$

$$\psi_1 = \sqrt{\frac{2}{L}} \sin \frac{\pi L}{2L} = \sqrt{\frac{2}{L}}$$

for $n=2$ $x=0$

$$\psi_2 = \sqrt{\frac{2}{L}} \sin \frac{2\pi \cdot 0}{L} = 0$$

$x = L/4$

$$\psi_2 = \sqrt{\frac{2}{L}}$$

$x = L/2$

$$\psi_2 = 0$$

$x = 3L/4$

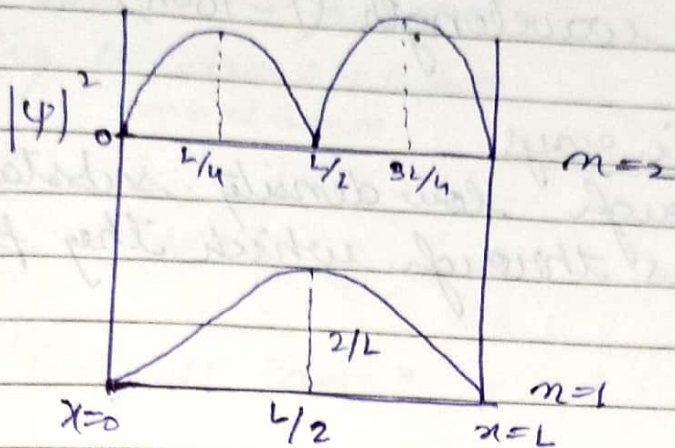
$$\psi_2 = -\sqrt{\frac{2}{L}}$$

$x = L$

$$\psi_2 = 0$$

$$\psi^2 = \frac{2}{L}$$

$$\psi^2 = \frac{2}{L} \sin^2 \frac{n\pi x}{L}$$



$$\psi^2 = \frac{2}{L} \sin^2 \frac{n\pi x}{L}$$

X Ray

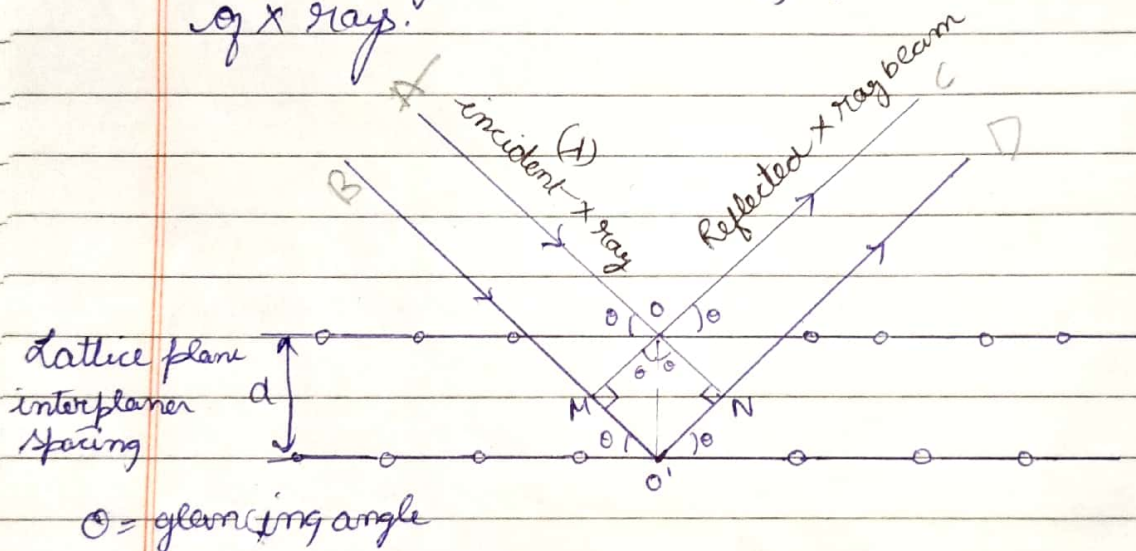
Rays of short wavelength ($1 - 100 \text{ \AA}$)

Electromagnetic rays

Penetrate through low density substance.
Ionise the gas through which they pass
reflect, refract.

Bragg's law

Proposed by William Bragg for constructive diffraction of X rays.



Let us consider a set of n lattice plane of crystal having inter planer spacing d .

Let n X rays of wave length λ falls on lattice planes along AO and BO' at glancing angle θ . As shown in the figure, these rays get reflected along OC and O'C.

To find the path difference b/w the incident and reflected rays draw $OM \perp BO'$ and $ON \perp O'D$

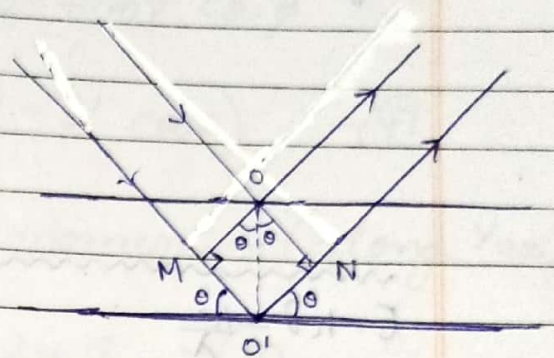
So path difference b/w incident and reflected rays.

$$\Delta OM \Delta = O'M + O'N$$

from fig, $\Delta = d \sin \theta + d \sin \theta$
 $= 2d \sin \theta$

$$\frac{OM}{OO'} = \sin \theta$$

$$O'N = O'M = d \sin \theta$$



for intensity of reflected X ray beam to be maximum

$$\Delta = n\lambda$$

$$n = 0, 1, 2$$

Bragg law of $2d \sin \theta = n\lambda$

$$n = 1, 2, 3, \dots$$

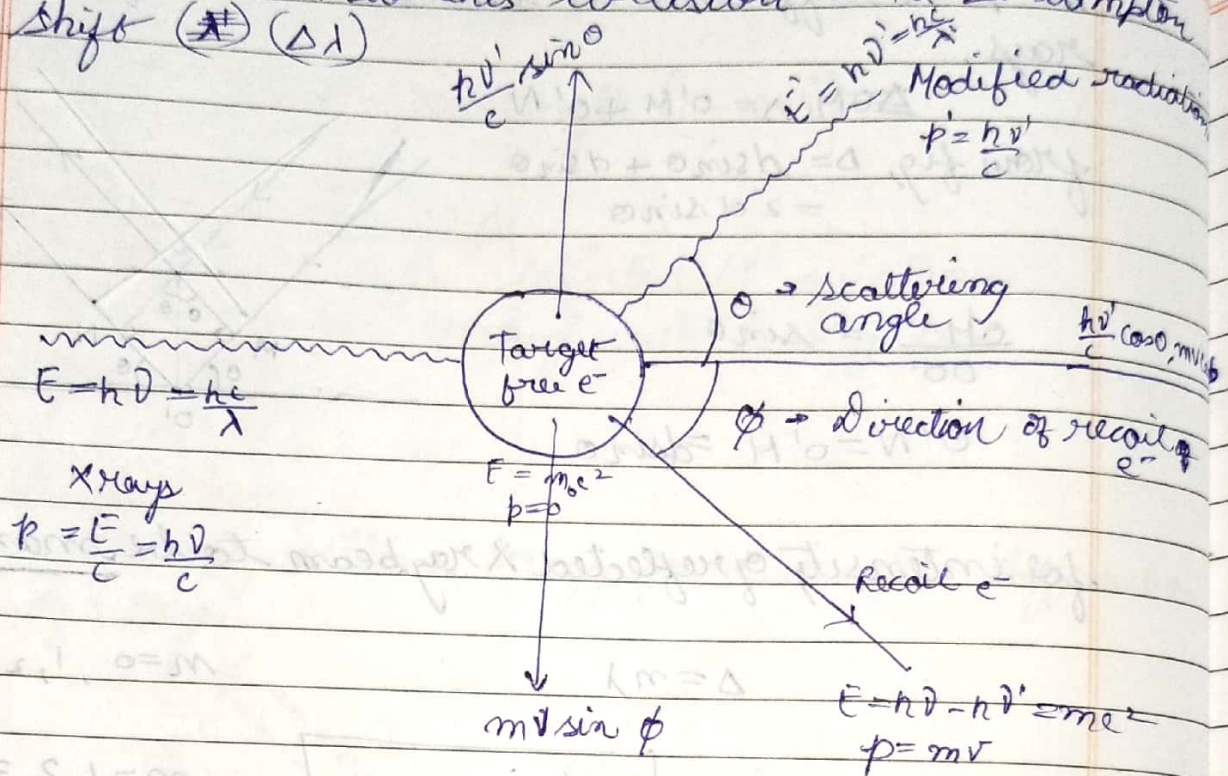
$$n \neq 0$$

Compton Effect

According to Compton effect when radiations of low frequency and ~~low~~ short wavelength and high frequency strike a target, the radiations of lower frequency (modified radiations) than incident beam as well as the radiations of same frequency as incident beam (unmodified radiations) are obtained. This phenomena of scattering with the change in frequency of incident radiations is called Compton effect.

It occurs due to elastic collision b/w incident beam and free e^- of the target initially at rest, the observed change in frequency or wavelength

$\lambda' - \lambda$ due to this collision is called Compton shift ($\Delta\lambda$)



Before collision

Energy of incident radiation $E = h\nu$
 " " free e^- of target $E = mc^2$

After collision

Energy of scattered beam $E' = h\nu'$
 " " recoil e^- $E = mc^2$
 where $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

from law of conservation of energy $\rightarrow$

$$h\nu + m_0 c^2 = h\nu' + mc^2$$

$$\text{or } mc^2 = h(\nu - \nu') + m_0 c^2 \quad (1)$$

Squaring both sides,

$$m^2 c^4 = h^2 (\nu - \nu')^2 + m_0^2 c^4 + 2 h m_0 c^2 (\nu - \nu') \quad (2)$$

As $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$m^2 c^4 = \frac{m_0^2 c^4}{\left(1 - \frac{v^2}{c^2}\right)} = \frac{m_0^2 c^4}{(1 - \frac{v^2}{c^2})} \quad (3)$$

Compton
radiation

from law of conservation of momentum (along X axis) -

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos \theta + m v \cos \phi$$

$$m v \cos \phi = \frac{h}{c} (\nu - \nu' \cos \theta) \quad (4)$$

from law of conservation of momentum (along Y axis) -

$$m v \sin \phi = \frac{h}{c} \nu' \sin \theta \quad (5)$$

Squaring and adding eq (4) & (5) -

$$m^2 v^2 (\cos^2 \phi + \sin^2 \phi) = \frac{h^2}{c^2} (\nu - \nu' \cos \theta)^2 + \frac{h^2}{c^2} \nu'^2 \sin^2 \theta$$

$$m^2 v^2 c^2 = h^2 (\nu^2 + \nu'^2 \cos^2 \theta - 2 \nu \nu' \cos \theta) + h^2 \nu'^2 \sin^2 \theta$$

(6)

eq (2) - eq (6)

$$m^2 c^4 - m^2 v^2 c^2 = h^2 (\nu - \nu')^2 + m_0^2 c^4 + 2 h m_0 c^2 (\nu - \nu') - h^2 (\nu^2 + \nu'^2 - 2 \nu \nu' \cos \theta)$$

$$m^2 c^2 (c^2 - v^2) = h^2 (\nu^2 + \nu'^2 - 2 \nu \nu') + m_0^2 c^4 + 2 h m_0 c^2 (\nu - \nu') - h^2 (\nu^2 + \nu'^2 - 2 \nu \nu' \cos \theta)$$

$$\text{for } m^2 = \frac{m_0^2}{1 - \frac{v^2}{c^2}}$$

$$m^2 c^2 = \frac{m_0^2 c^4}{c^2 - v^2}$$

$$\frac{m_0^2 c^4 (c^2 - v^2)}{(c^2 - v^2)} = -2 h^2 \nu \nu' + m_0^2 c^4 + 2 h m_0 c^2 (\nu - \nu') + 2 h^2 \nu \nu' \cos \theta$$

$$-2h m_0 c^2 (\nu - \nu') = -2h^2 \nu \nu' (1 - \cos \theta)$$

$$\frac{\nu - \nu'}{\nu \nu'} = \frac{h}{m_0 c^2} (1 - \cos \theta)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2} (1 - \cos \theta)$$

$$\text{If } \lambda = \frac{c}{\nu}, \quad \lambda' = \frac{c}{\nu'}$$

$$\text{Then } \frac{c}{\nu'} - \frac{c}{\nu} = \frac{hc}{m_0 c^2} (1 - \cos \theta)$$

$$\text{Compton Shift } \boxed{\Delta \lambda = \lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos \theta)}$$

$\Delta \lambda$ depends only on scattering angle (θ)

$$\frac{h}{m_0 c} = \text{Constant} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} \text{ m} = 0.024 \text{ \AA}$$

Case 1

i) At $\theta = 0^\circ$, $\cos 0^\circ = 1$, $\Delta \lambda = 0$
 $\lambda' = \lambda$ (Unmodified radiation are obtained)

ii) At $\theta = 90^\circ$, $\cos 90^\circ = 0$, $\Delta \lambda = \frac{h}{m_0 c} = \lambda_c$ (Compton wavelength)
 $= 0.024 \text{ \AA}$

iii) At $\theta = 180^\circ$, $\cos 180^\circ = -1$, $\Delta \lambda = \frac{h}{m_0 c} (1 + 1) = \frac{2h}{m_0 c} = 2 \times 0.024 \text{ \AA}$
 $= 0.048 \text{ \AA}$

$$\Delta \lambda_{\max} = 0.045 \text{ \AA} \text{ (max Compton shift)}$$