

# NETWORK ANALYSIS AND SYNTHESIS.

3/8/18)

Date \_ / \_ / \_

## \* Basic Component of electrical network:-

- Resistor
  - Inductor
  - Capacitor
- } linear, bilateral, passive.

Linear:- proportionality ratio b/w two quantity is constant.

$$x(t) \propto y(t)$$

$$\uparrow x(t) = k y(t) \uparrow$$

$$\phi(t) = LI(t)$$

$$V(t) = \frac{\partial \phi(t)}{\partial t} = L \frac{d\phi(t)}{\partial t}$$

$$q(t) \propto V(t)$$

$$[q(t) = CV(t)]$$

$$i(t) = \frac{\partial q(t)}{\partial t} = C \frac{\partial V(t)}{\partial t}$$

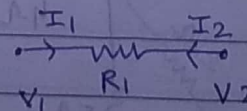
\* Bilateral:- Bilateral means if we change the polarity of Voltage Source than characteristics of the element remain same.

\* passive:- It means response or output across the element can't be greater than the excitation or input.

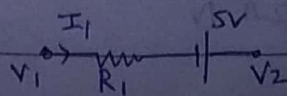
## 7/8/18 Nodal and mesh Analysis:-

### \* Nodal:-

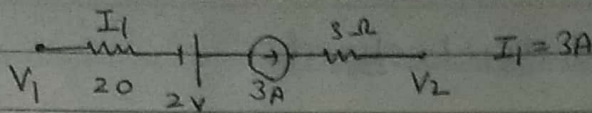
- 1) Selection of Nodes.
- 2) Assumption of Nodes
- 3) Apply the KCL



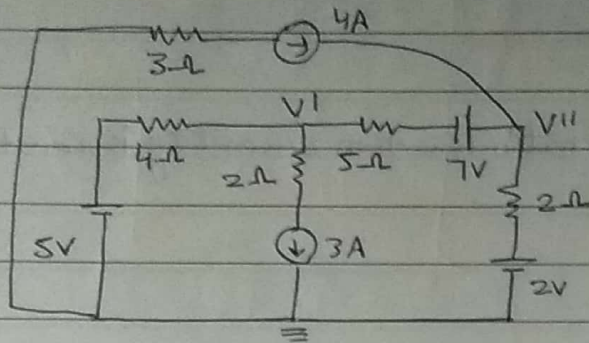
$$I_1 = \frac{V_1 - V_2}{R_1}, I_2 = \frac{V_2 - V_1}{R_1}, I_1 = -I_2$$



$$[I_1 = \frac{V_1 + 5 - V_2}{R}]$$



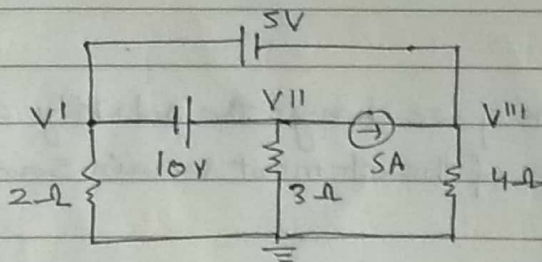
Q2)



$$\frac{V_1 - 5}{4} + 3 + \frac{V_1 - V_2 + 7}{5} = 0$$

$$\frac{V_2 - V_1 - 7}{5} + \frac{V_2 - 2}{2} = 0$$

Super nodal analysis:- find  $I$  in 3 ohm using nodal:-

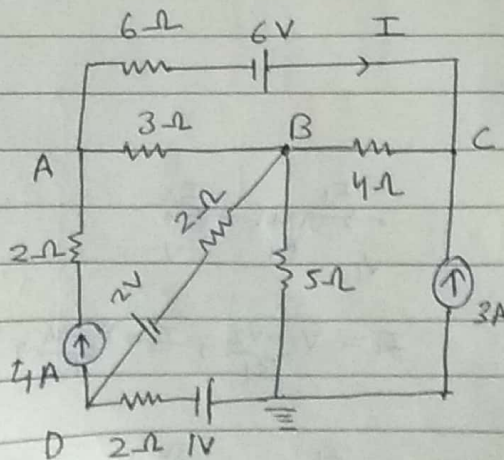


$$-\frac{V_1}{2} - \frac{V_2}{3} - \frac{V_3}{4} = 0$$

$$\begin{cases} V_1 - V_2 = -10 \\ V_1 - V_3 = 5 \end{cases}$$

9/8/18 Tut

①



at A

$$-4 + \frac{V_A - V_B}{3} + \frac{V_A - V_C + 6}{6} = 0$$

at B

$$\frac{V_B - V_A}{3} + \frac{V_B - V_C}{4} + \frac{V_B}{5} + \frac{V_B - V_D + 2}{2} = 0$$

at C

$$\frac{V_C - V_A - 6}{6} + \frac{V_C - V_B}{4} - 3 = 0$$

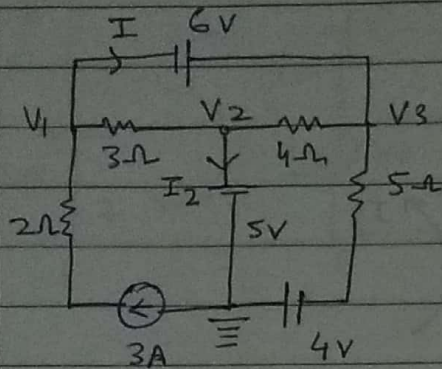


at D

$$\frac{V_D + 1}{2} + \frac{V_D - V_B - 2}{2} + 4 = 0$$

Date \_ / \_ / \_

2)



$$\frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{5} = 0$$

assume  $I_1 \neq I_2$

$$V_1 - V_3 = -6$$

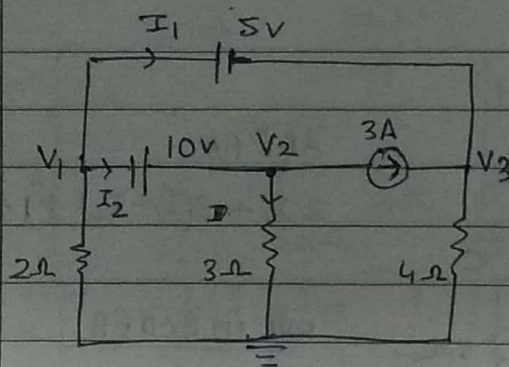
$$V_2 = 5$$

$$\frac{V_3 + 4}{5} + \frac{V_3 - V_2}{4} - I_1 = 0$$

$$-3 + \frac{V_1 - V_2}{3} + I_1 = 0$$

$$\frac{V_2 - V_1}{3} + I_2 + \frac{V_2 - V_3}{4} = 0$$

3)



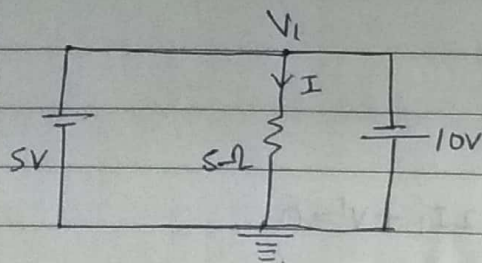
assume  $I_1 \neq I_2$

$$V_1 - V_3 = 5$$

$$V_1 - V_2 = -10$$

$$\frac{V_1}{2} + I_2 + I_1 = 0$$

4)



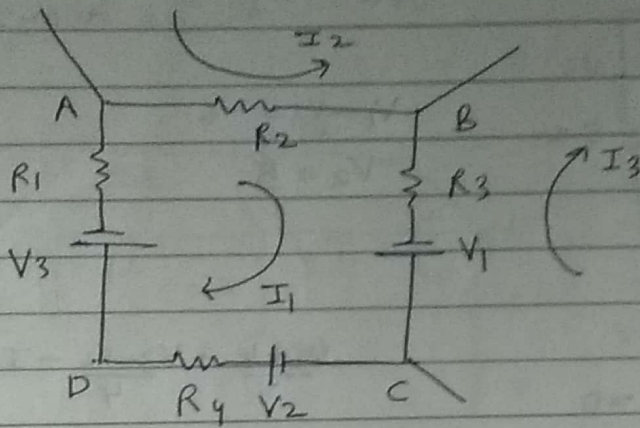
Not possible.

we cannot apply || two battery.

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# Mesh Analysis (KVL)

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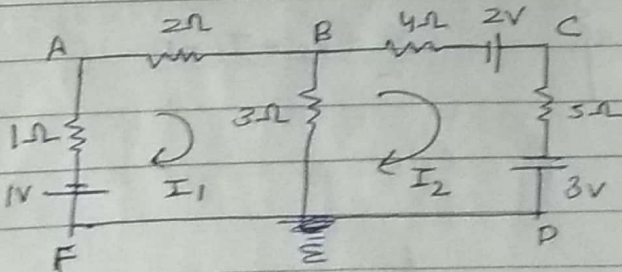
KVL in ABCDA :

$$(I_1 + I_2)R_2 + (I_1 - I_3)R_3 - V_1 + I_1R_4 + V_2 + V_3 + I_1R_1 = 0$$

- 1) Selection of Mesh.
- 2) Assumption of mesh Current.
- 3) Apply KVL.

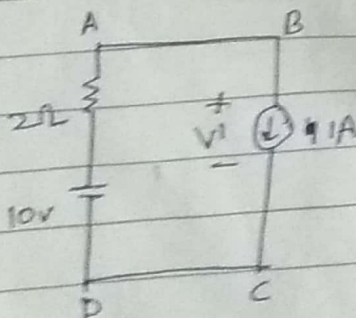
ABEFA

$$2I_1 + 3(I_1 - I_2) + 1 + I_1 = 0$$



KVL in BCDEB

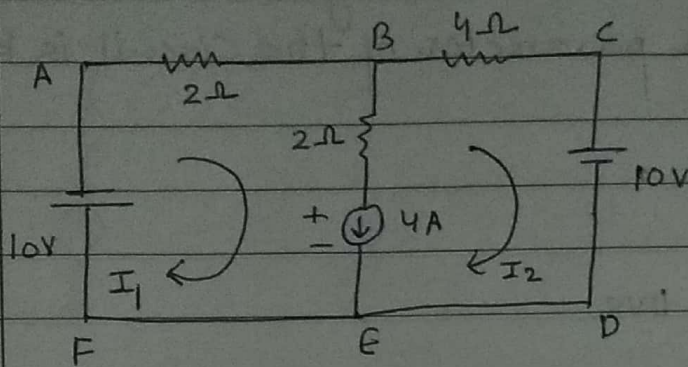
$$4I_2 - 2 + 5I_2 - 3 + (I_2 - I_1)3 = 0$$



$$-10 + 2I_1 + V_1 = 0$$



Q=)



find  $I_1$  &  $I_2$  and Voltage drop across 4A as shown in fig.

$$I_1 - I_2 = 4.$$

KVL in ABEFA.

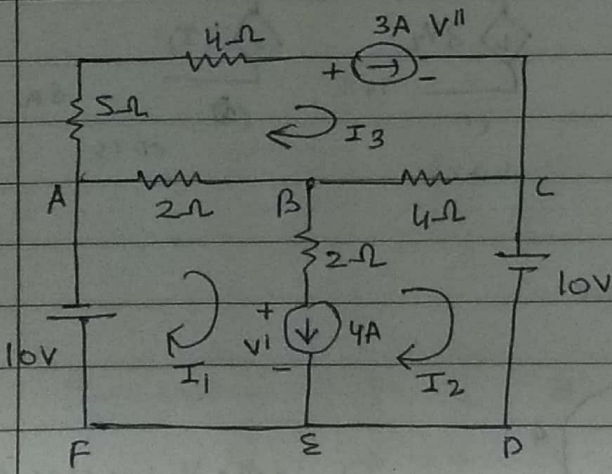
$$2I_1 + 2(I_1 - I_2) + V' + 10 = 0$$

$$2I_1 + V' + 18 = 0 \quad \text{--- (1)}$$

KVL in BCDEB

$$4I_2 + 10 - V' + (I_2 - I_1)2 = 0$$

$$4I_2 - V' + 2 = 0 \quad \text{--- (2)}$$



$$I_3 = 3A$$

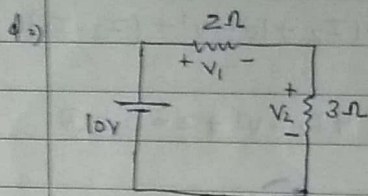
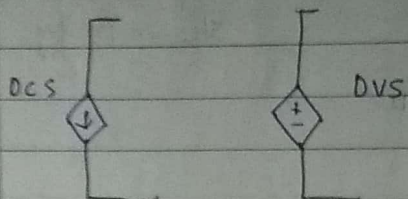


## \* Concept of Dependent Source:-

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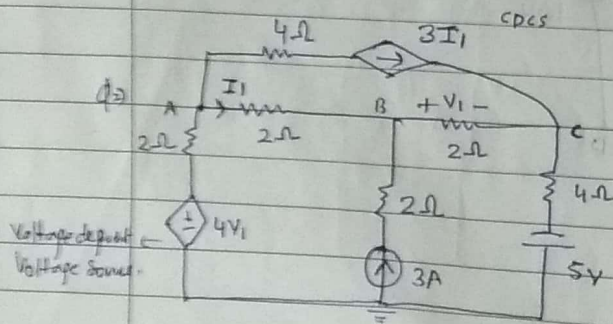
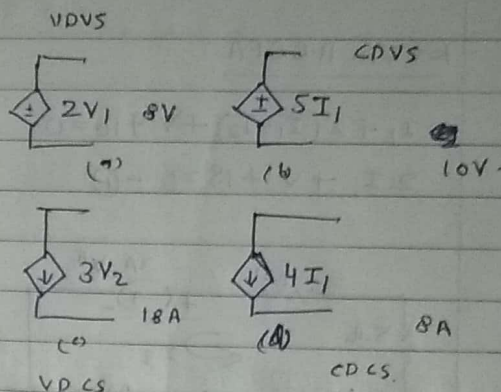
A Voltage or Current Source which magnitudes depends on another parameters of the circuit is known as dependent Source.



$$I_1 = \frac{10}{2+3} = \frac{10}{5} = 2A$$

$$V_1 = 2I_1 = 4V$$

$$V_2 = 3I_1 = 6V$$



find the Value of  $V_1$  &  $I_1$  as shown in fig using nodal analysis.

at A

$$\frac{V_A - V_1}{2} + \frac{V_A - V_B}{2} + 3I_1 = 0$$

at B

$$\frac{V_B - V_A}{2} - 3 + \frac{V_B - V_C}{2} = 0$$

at C

$$\frac{V_C + 5}{4} - 3I_1 + \frac{V_C - V_B}{2} = 0$$

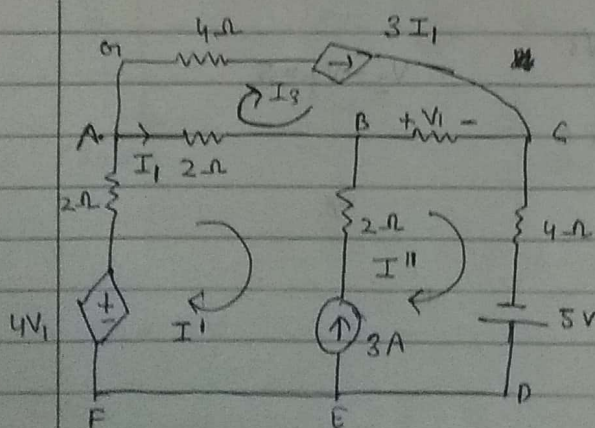
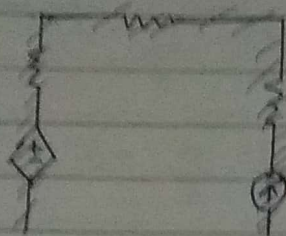
$$I_1 = \frac{V_A - V_B}{2}$$

$$V_1 = 2 \times \left( \frac{V_B - V_C}{2} \right)$$



using mesh analysis:- find value of  $I_1$  &  $V_1$ .

Date - / - /



at ABFEFA (KVL not apply due to Current source)

$$2I_1' + 2(I_1' - I_1'') - 3 = 4V_1 + 2I_1' = 0$$

$$I_1' - I_1'' = -3$$

passion



at abcdefg.

~~$I_1 = I'' = I'''$~~

$$2(I^I I^{II}) + 2(I^{II} I^{III}) + 4I^{II} - 5 - 4V_1 + 2I^I = 0 \quad \text{Date } - / - / -$$

$$I^{III} = 3I^I$$

$$I_1 = I^{II} - I^{III}$$

$$V_1 = 2(I^{II} - I^{III})$$

$$\Rightarrow 2I^I - 2I^{III} + 2I^{II} - 2I^{III} + 4I^{II} - 5 - 4V_1 + 2I^I = 0$$

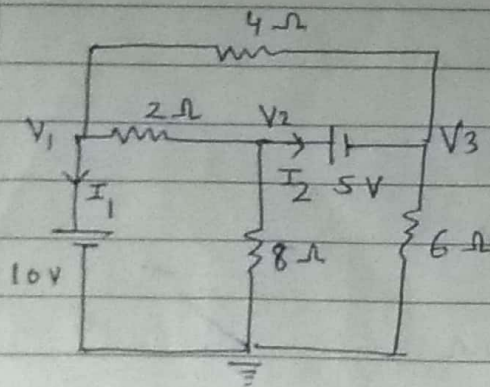
$$-4I^{III} + 6I^{II} + 4I^I - 4V_1 - 5 = 0$$

$$\Rightarrow -4(3I^I) + 6I^{II} + 4I^I - 4(2I^{II} - 2I^{III}) - 5 = 0$$

$$-12I^I + 6I^{II} + 4I^I - 8I^{II} + 8I^{III} - 5 = 0$$

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Q. find  $V_1, V_2, V_3$  using nodal analysis:-



$$V_2 - V_3 = 5$$

$$V_1 = 10V$$

$$V_1 - V_2 + \frac{V_1 - V_3}{4} = 0$$

$$\frac{V_2 + V_2 - V_1}{8} = 0 \quad \Rightarrow \quad \frac{V_2 + V_2 - 10}{8} = 0$$

$$\frac{V_3 - V_1}{4} + \frac{V_3}{6} = 0$$



$I_1$  &  $I_2$  assume:-

$$I_1 + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0 \quad \text{--- (1)}$$

$$\begin{array}{r|l} 2 & 8 \\ 2 & 4 \\ 2 & 2 \end{array}$$

3 date - / - /

$$-I_1 - \frac{V_2}{8} - \frac{V_3}{6} = 0$$

$$I_1 = -\frac{V_2}{8} - \frac{V_3}{6} = \frac{-3V_2 - 4V_3}{24} \quad \text{put in (1)}$$

$$\begin{array}{r|l} 2 & 2, 4, 24 \\ 2 & 1, 2, 12 \\ 2 & 1, 1, 6 \\ \hline 3 & 3, 1 \end{array}$$

$$\frac{-3V_2 - 4V_3}{24} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0$$

$$\frac{-3V_2 - 4V_3 + 12V_1 - 12V_2 + 6V_1 - 6V_3}{24} = 0$$

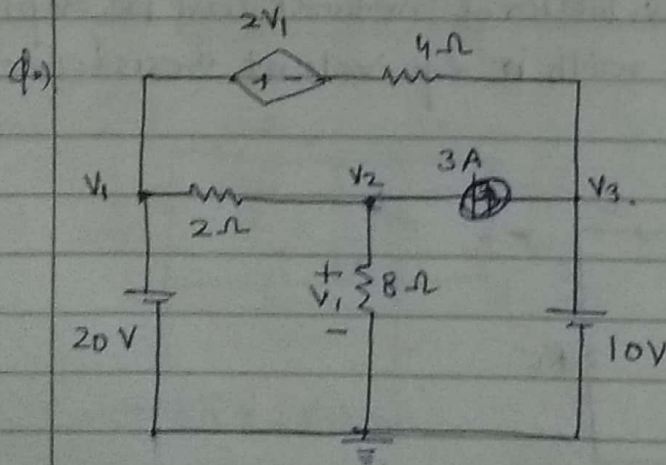
$$18V_1 - 15V_2 - 10V_3 = 0$$

$$18(10) - 15V_2 - 10V_3 = 0$$

$$-15V_2 - 10V_3 = -180$$

$$V_2 - V_3 = 5$$

$$\begin{bmatrix} V_2 = 9.2V \\ V_3 = 4.2V \end{bmatrix}$$



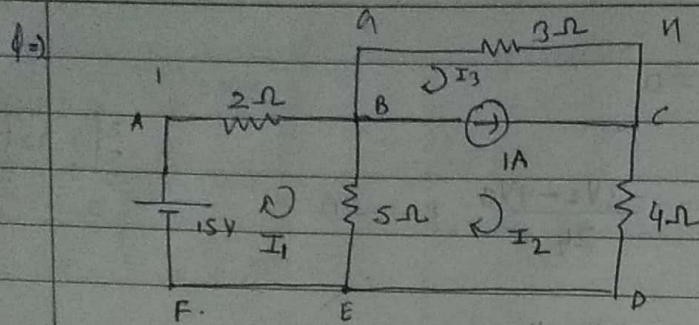
find value of  $V_1$  and using nodal and mesh analysis.

$$V_1 = 20V$$



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find the current in all the branch.

$$I_2 - I_3 = 1A$$

2. ABEFA

$$2I_1 + 5(I_1 - I_2) - 15 = 0$$

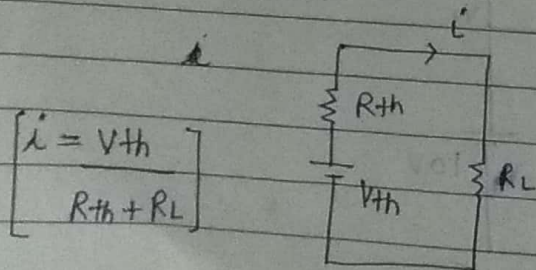
KVL in GHCD EBGH

$$3I_3 + 4I_2 + 5(I_2 - I_1) = 0$$

★ THEVENIN'S THEOREM:-

(linear, bilateral, passive).

Any two terminal linear, bilateral network can be replaced by a DC Voltage Source Series with an equivalent resistance. (Equivalent impedance).



$$I = \frac{V_{th}}{R_{th} + R_L}$$

★ procedure to find the  $V_{th}$  &  $R_{th}$ :-

→ have only independent source

$V_{oc}$

→ have both independent + dependent

$V_{oc}$

→ only dependent source.

$$V_{th} = V_{oc} = 0$$



R<sub>th</sub> :-

1. Deactivate all V's & C's (Independent) & find resistance b/w the terminal  
Date - / -

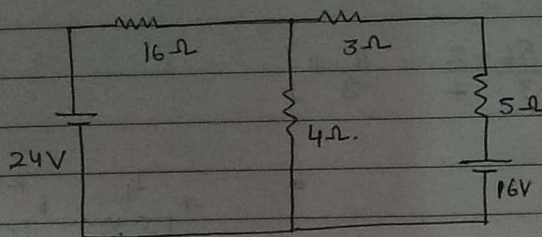
2. Find Sc Current b/w the terminal &

$$R_{th} = \frac{V_{th}}{I_{sc}}$$

3. Deactivate all Independent V's, C's & assume 1A current b/w the terminal & find Voltage b/w the terminal & the Voltage is equal to R<sub>th</sub> b/w the terminal.

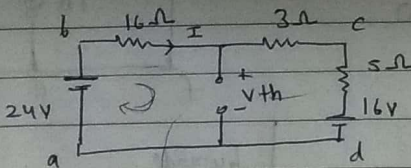
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Q:-



find the Current in 4Ω resistance using thevenin theorem.

Soln:-



$$-24 + 16I + 3I + 5I + 16 = 0$$

$$24I = 8$$

$$I = \frac{8}{24} = \frac{1}{3} A$$

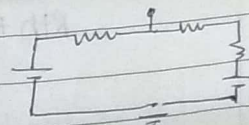
$$-24 + 16I + V_{th} = 0$$

$$-24 + 16 \times \frac{1}{3} + V_{th} = 0$$

$$[V_{th} = -18.66V]$$

by nodal

$$\frac{V_{th} - 24}{16} + \frac{V_{th} - 16}{8} = 0$$



$$\frac{24 - 16}{3}$$

$$= \frac{8}{3}$$

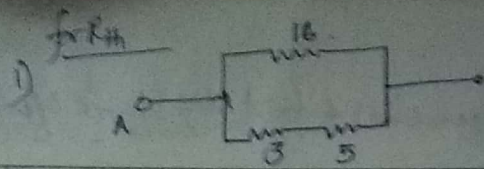
$$= \frac{8}{3}$$

$$= \frac{8}{3}$$

$$= \frac{8}{3}$$

passion





Date - / - /

$$R_{th} = 16 \parallel 8$$

$$\left[ \frac{16 \times 8}{24} = \frac{16}{3} \Omega \right]$$

(2) 2n abefg

$$16I_1 - 24 = 0$$

$$I_1 = 3/2$$

2n bcded

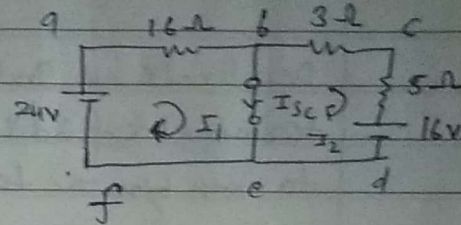
$$3I_2 + 5I_2 + 16 = 0$$

$$I_2 = -2$$

$$I_{sc} = I_1 - I_2$$

$$\frac{3}{2} + 2 = 7/2 A$$

$$\left[ R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{56 \times 2}{3 \times 7} = \frac{16}{3} \Omega \right]$$



(3)

KVL in abcdefg

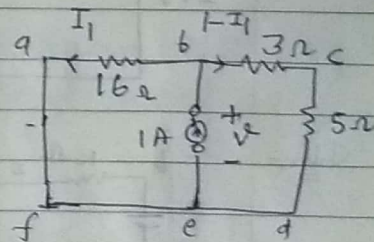
$$-16I_1 + 3(1 - I_1) + 5(1 - I_1) = 0$$

$$24I_1 = 8$$

$$I_1 = 8/24$$

$$-16I_1 + V = 0$$

$$\left[ R_{th} = V = 16I_1 = 16 \times \frac{8}{24} = \frac{16}{3} \Omega \right]$$



by node

$$\frac{V}{16} + \frac{V}{8} - 1 = 0$$

$$\begin{aligned} R &= \frac{V}{I} \\ I &= 1 \\ R &= V \end{aligned}$$

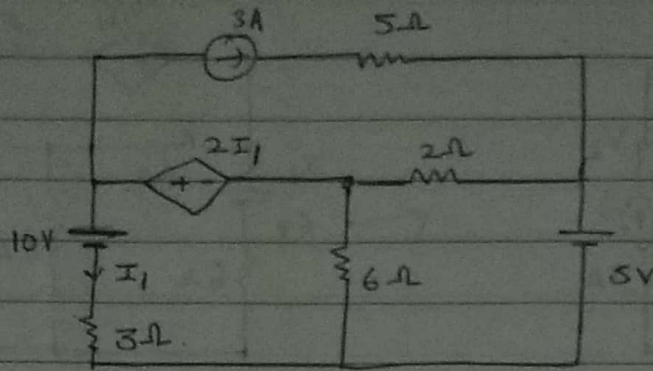
$$I' = \frac{V_{th}}{R_{th} + R_L}$$



\* Circuit with Independent and dependent Source.

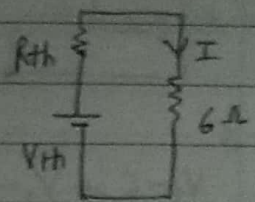
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Q $\Rightarrow$

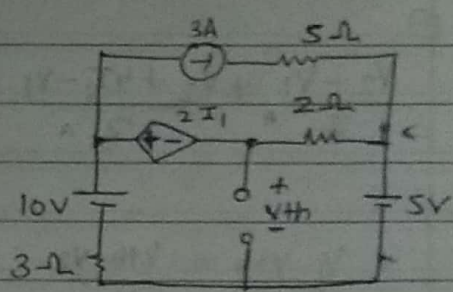


find current in  $6\Omega$  resistance.

Soln:-



$$I = \frac{V_{th}}{R_{th} + 6}$$



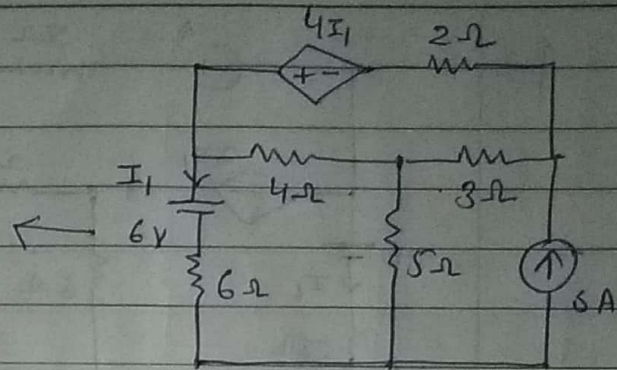
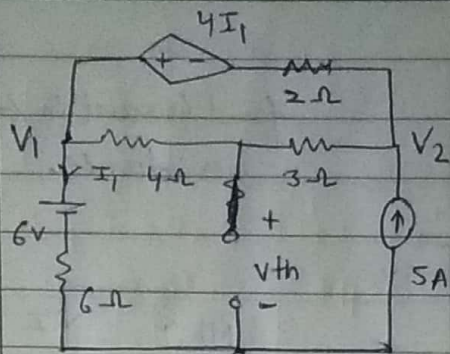


24/8/18

Find the current in  $5\Omega$  resistance using the theorem.

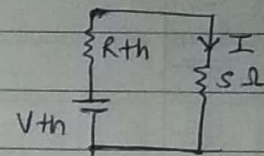
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Ans:  $\Rightarrow$



①

$$\frac{V_1 - 6}{6} + \frac{V_1 - V_2}{3 + 4} + \frac{V_1 - V_2 - 4I_1}{2} = 0$$



②

$$\frac{V_2 - V_1}{7} + \frac{V_2 + 4I_1 - V_1}{2} - 5 = 0$$

$$I_1 = \frac{V_1 - 6}{6}$$

$$\frac{V_1 - V_{th}}{4} = \frac{V_{th} - V_2}{3}$$



By mesh

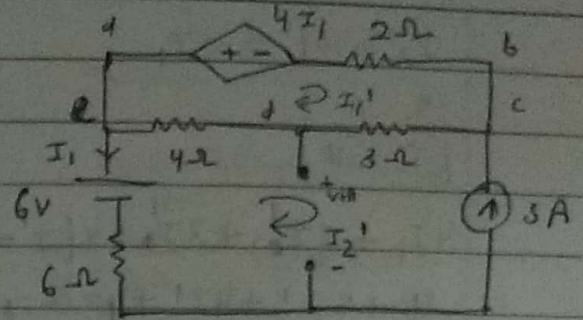
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$$I_2' = -5A$$

$$I_1 = -I_2'$$

$$[I_1 = 5A]$$

In abcdea



$$4I_1 + 2I_1' + 3(I_1' - I_2') + 4(I_1' - I_2') = 0$$

$$4(I_2' - I_1') + V_{th} + 6I_2' - 6 = 0$$

for Rth

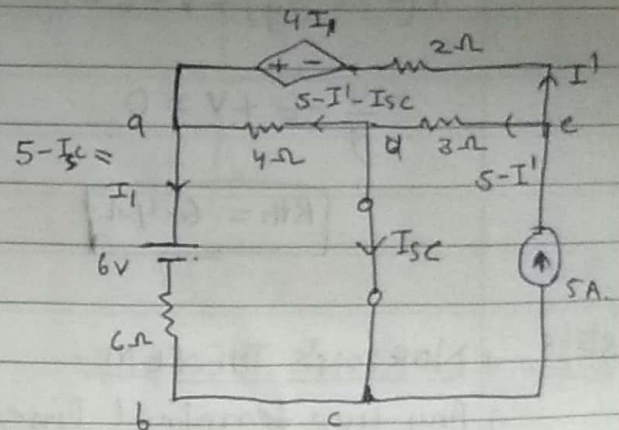
$$I_1 + I_{sc} = 5A$$

KVL in abcd

$$6 + 6I_1 + 4(5 - I_1' - I_{sc}) = 0$$

KVL in adea

$$-4(5 - I_1' - I_{sc}) - 3(5 - I_1') + 2I_1' - 4I_1 = 0$$



$$R_{th} = \frac{V_{oc}}{I_{sc}}$$



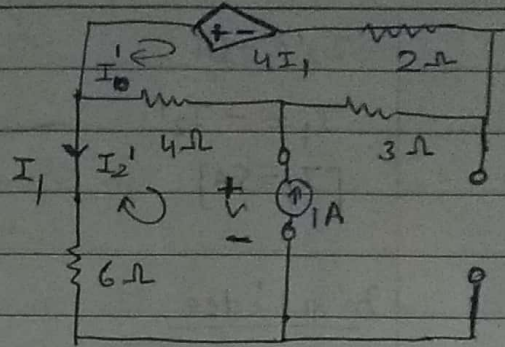
$$I_2' = -1 = -I_1$$

$$\begin{cases} I_1 = 1A \\ I_2' = -1A \end{cases}$$

$$4I_1 + 2I_1' + 3I_1' + 4(I_1' - I_2') = 0$$

$$4 + 5I_1' + 4I_1' + 4 = 0 \quad | \quad I_1' = -8/9$$

$$9I_1' = -8$$



$$4(I_2' - I_1') + V + 6(I_2') = 0$$

$$V = R_{th}$$

$$4(-1 - 1) + V - 6 = 0$$

$$V = 14$$

$$R_{th} = 14\Omega$$

$$4(-1 + 8/9) + V - 6 = 0$$

$$-\frac{4}{9} - 6 + V = 0$$

$$V = 7.7 = R_{th} = 7.7\Omega$$

$$V = 6.1$$

$$R_{th} = 6.1\Omega$$

$$4(-1 + 8/9) + V - 6 = 0$$

$$-\frac{4}{9} - 6 + V = 0$$

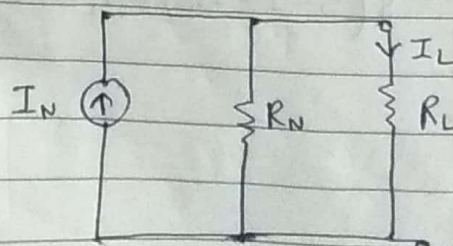
$$I = \frac{V_{th}}{R_{th} + R_L}$$

$$R_{th} = 6.4\Omega$$

### 28/8/18 \* NORTON'S THEOREM:-

Any two terminal linear, bilateral network can be converted into a single voltage source current source parallel with  $R_{eq}$  impedance.

$$I_L = \frac{R_N}{R_N + R_L} \times I_N$$

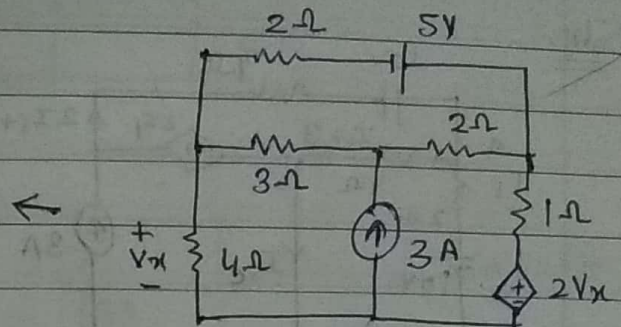
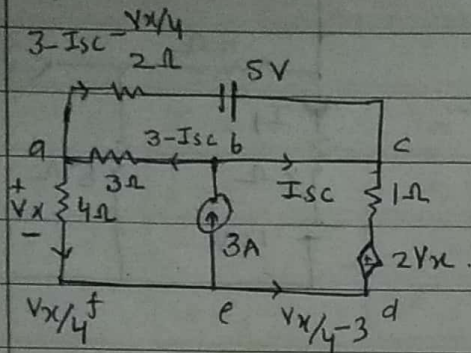




$I_N$ : Short Circuit current b/w the terminal.  
 $[R_N = R_{th}]$

Date \_ / \_ / \_

Q2) find current in 2A resistor.



KVL in abcdefa

$$-3(3 - I_{sc}) - \left(\frac{V_x}{4} - 3\right) + 2V_x - V_x = 0.$$

KVL in acba

$$(3 - I_{sc} - \frac{V_x}{4})2 - 5 + 3(3 - I_{sc}) = 0$$

for Rth

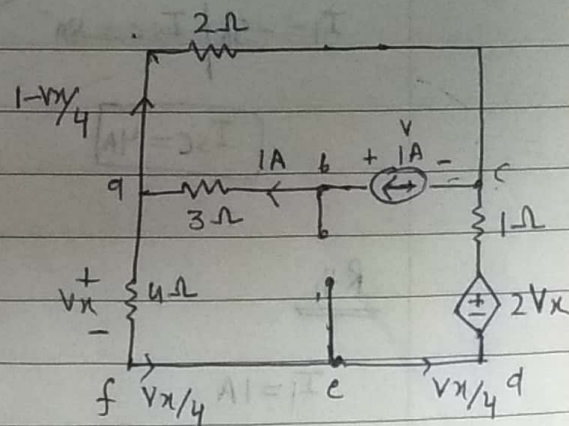
KVL in adefa.

$$2(1 - \frac{V_x}{4}) - \frac{V_x}{4} + 2V_x - V_x = 0.$$

$$2 - \frac{V_x}{2} - \frac{V_x}{4} + V_x = 0.$$

KVL in. abca.

$$2(1 - \frac{V_x}{4}) - V + 3 = 0.$$



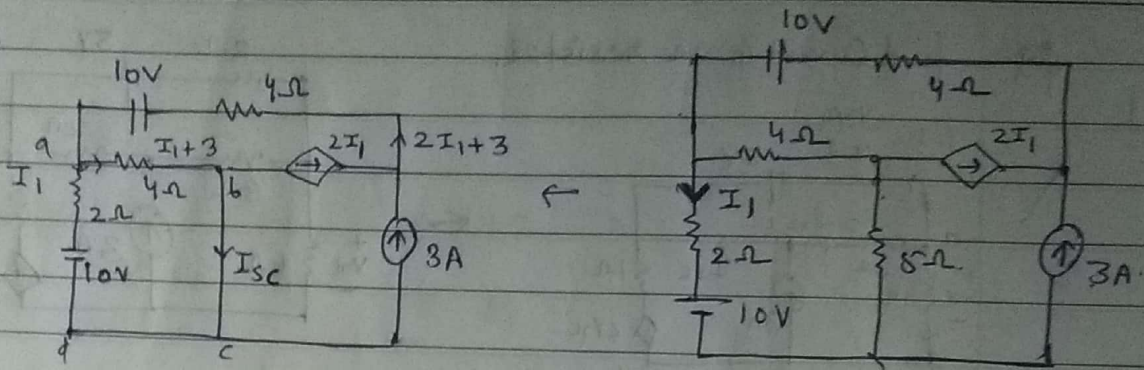
$$[V = R_{th}] = R_N.$$



Q<sub>2</sub> find the current in 5Ω using Norton Theorem:-

Date \_/\_/\_

Soln:-



$$I_1 + 3 = 2I_1 + I_{sc}$$

$$I_1 + I_{sc} = 3 \quad \text{--- (1)}$$

KVL in abcdq.

$$4(I_1 + 3) + I_{sc} - 10 - 2I_1 = 0$$

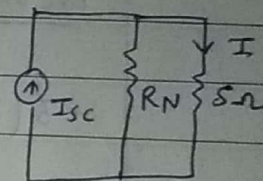
$$4I_1 + 12 + I_{sc} - 10 - 2I_1 = 0$$

$$2I_1 + I_{sc} = -2 \quad \text{--- (2)}$$

$$[I_1 = -1]$$

$$I_1 = -1 \text{ A} \quad I_{sc} = 4 \text{ A}$$

$$I_{sc} = 4 \text{ A}$$



$$I = \frac{R_N}{R_N + 5} \times I_{sc}$$

R<sub>th</sub>

$$I_1 = 1 \text{ A}$$

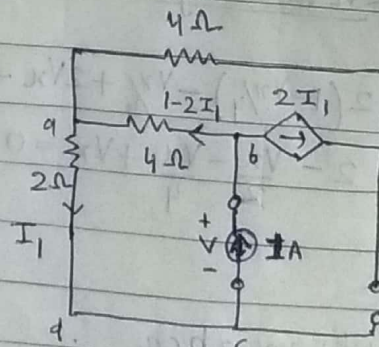
KVL in abcdq.

$$-4(1 - 2I_1) + V - 2I_1 = 0$$

$$-4 + 8I_1 + V - 2I_1 = 0$$

$$6 - 4 + V = 0$$

$$[V = -2] = R_{th}$$

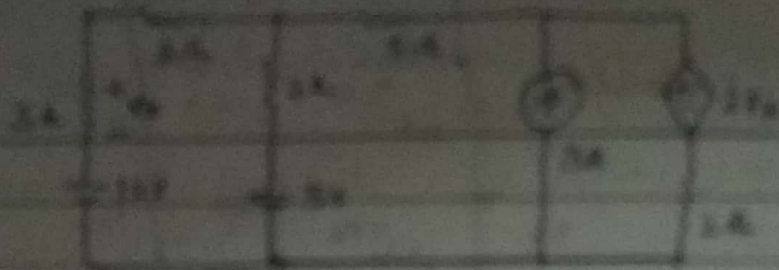


$$I = \frac{-2}{-2 + 5} \times 4$$

$$I = \frac{-2}{3} \times 4 = -8/3 = -2.6 \text{ A}$$

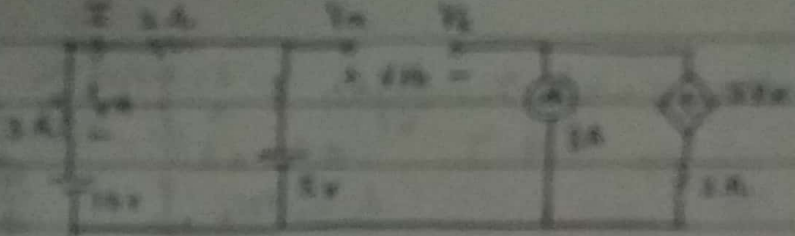


4.



Find  $V_x$

5.



Find  $V_x$



# Tutorial

Q) Find the Current in 5- $\Omega$  Resistance using norton's theorem.

Soln:

In abef a KVL

$$3(I_{sc} + V_x/6) - 2V_x - 5 - 4(I_1 - I_{sc} - V_x/6) = 0$$

$$7I_{sc} - \frac{5}{6}V_x - 4I_1 = -5$$

In abca

$$3(I_{sc} + V_x/6) + V_x + 10 + 2I_1 = 0$$

$$3I_{sc} + \frac{3}{2}V_x + 2I_1 = -10$$

In bcdeb

$$V_x - 2(I_1 - V_x/6) + 10 = 0$$

$$\frac{4}{3}V_x - 2I_1 = -10$$

for  $R_{th}$

$$I_{sc} = 0.027 \quad V_x = -7.08$$

In abef

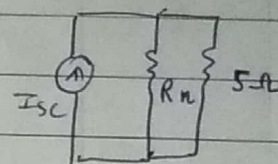
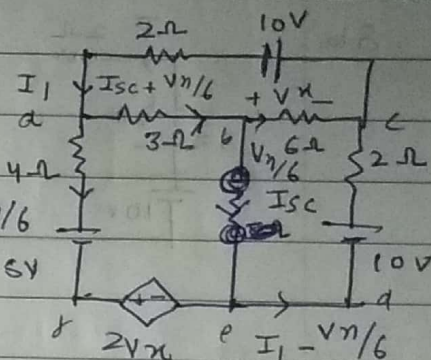
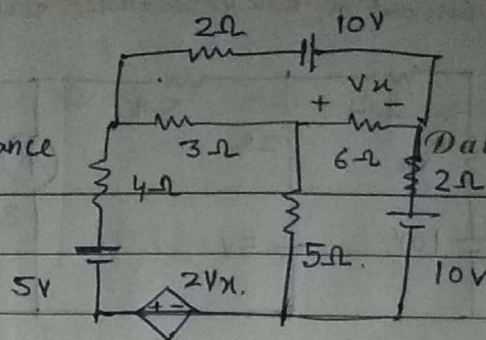
$$-3(1 - V_x/6) + V_x - 2I_1 = 0$$

In abcdef

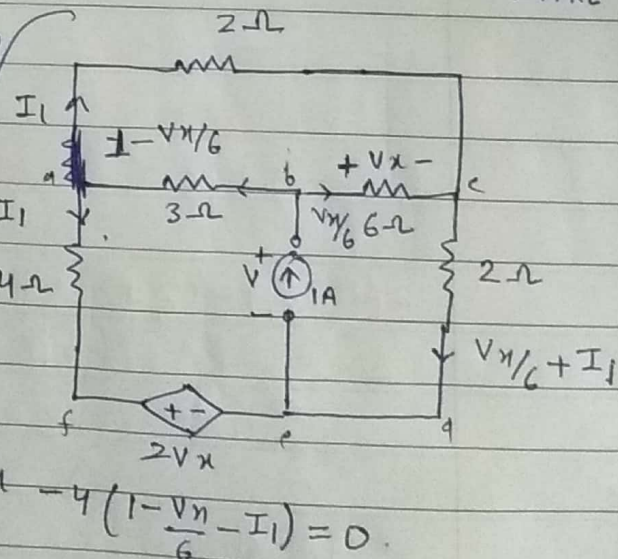
$$-3(1 - \frac{V_x}{6}) + V_x + 2(\frac{V_x}{6} + I_1) - 2V_x - 4(1 - \frac{V_x}{6} - I_1) = 0$$

$$-3(1 - V_x/6) + V - 2V_x - 4(1 - V_x/6 - I_1) = 0$$

$$[V = 6.09]$$



$$I = \frac{R_n \times I_{sc}}{R_n + R_L}$$



$$V = -3.8$$



### 3/8/18 \* Maximum power Transfer Theorem:-

1 In a resistive load maximum power is transferred across the load when the value of load resistance become equal to the total input resistance.

I case

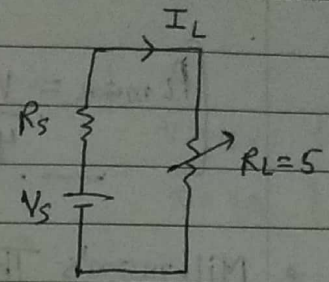
$$I_L = \frac{V_S}{R_S + R_L}$$

$$P_L = I_L^2 \times R_L$$

$$P_L = \frac{V_S^2 \times R_L}{(R_S + R_L)^2}$$

$$\frac{\partial^2 P_L}{\partial R_L^2} = -ve \quad \frac{\partial P_L}{\partial R_L} = 0$$

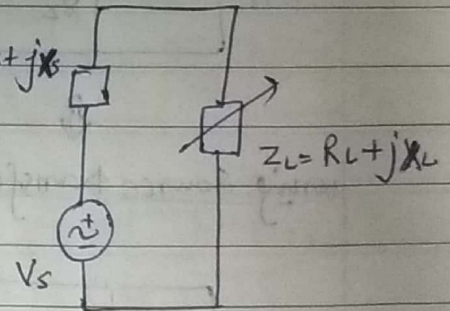
$$[R_L = R_S]$$



II case

AC load: or Complex load:-

for the complex load maximum power is transferred across the load when the value of load impedance become equal to complex conjugate of the input impedance or source impedance.



$$I = \frac{V_S}{(R_S + R_L) + j(X_S + X_L)}$$

$$Z_L = Z_S^*$$

$$R_S + jX_S = R_L' - jX_L$$

$$|I| = \frac{V_S}{\sqrt{(R_S + R_L)^2 + (X_S + X_L)^2}}$$

$$\begin{cases} R_L = R_S \\ X_L = -X_S \end{cases}$$

$$P_L = \frac{V_S^2}{(R_S + R_L)^2 + (X_S + X_L)^2} \times R_L$$



$$\frac{\partial P_L}{\partial R_L} = 0$$

$$R_L = R_S$$

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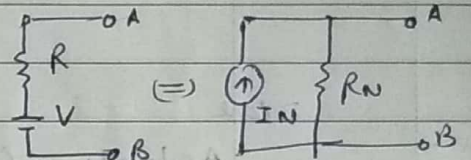
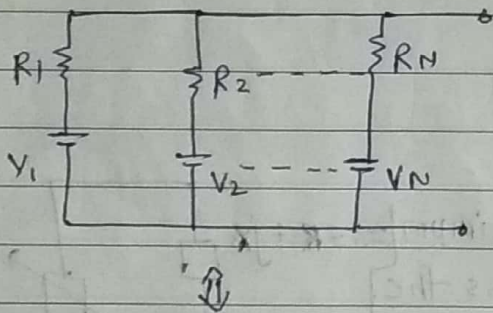
$$\frac{\partial P_L}{\partial X_L} = 0$$

$$X_L = -X_S$$

$$P_{L \max} = \frac{V_S^2}{4R_S}$$

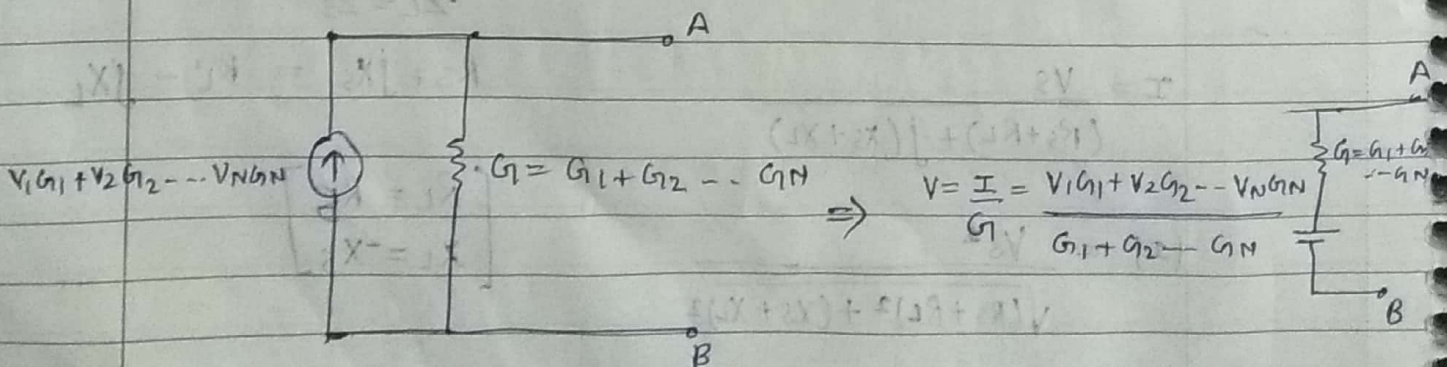
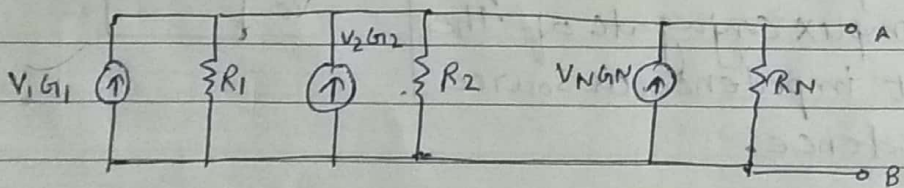
### \* Millman's Theorem:-

If number of Voltage Source.  $V_1, V_2, \dots, V_N$  Series with Req resistance  $R_1, R_2, \dots, R_N$  are connected in parallel can be converted into a single Voltage Source. Series with Equivalent resistance.



$$\begin{aligned} I_N &= I_{SC} = \frac{V_{th}}{R_{th}} \\ R_{th} &= R_N \end{aligned}$$

using Source transformation



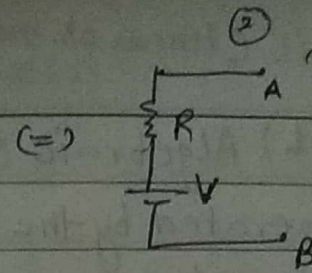
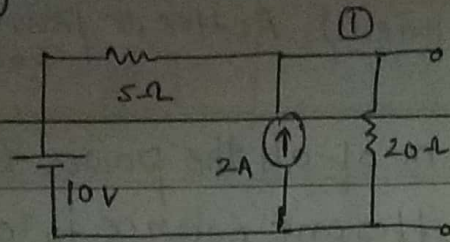
$$\frac{1}{R_{th}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

$$G = G_1 + G_2 + \dots + G_N$$

$$R = \frac{1}{G}$$

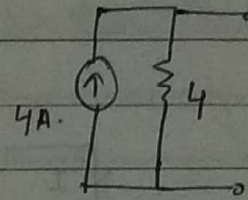
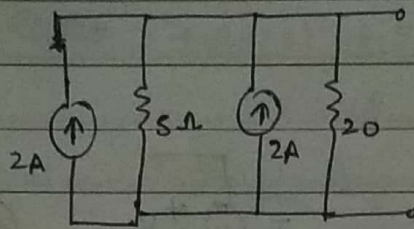


Q2) Using Source transformation find Value of  $V$  and  $R$ .



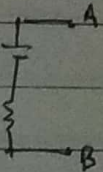
Date - / - / - <sup>①=② expanded</sup>

Soln:-



$$V = 16V$$

$$R = 4\Omega$$



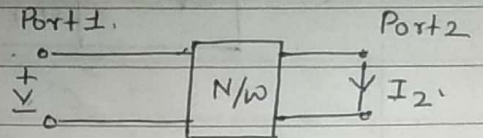
4/9/18

### \* Reciprocity Theorem:-

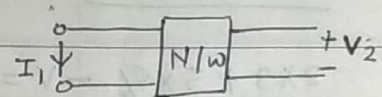
In a two port network if we apply a Voltage  $V_1$  at port 1 produce a short current in port 2.

If we apply a Voltage  $V_2$  at port 2 produce a short circuit  $I_1$  current in port 1. then network is reciprocal and condition of reciprocity is

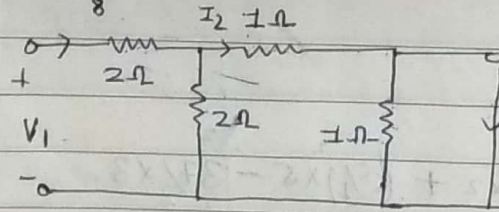
$$\frac{V_1}{I_2} (V_2=0) = \frac{V_2}{I_1} (V_1=0)$$



$$V_1 = V_2, I_1 = I_2$$



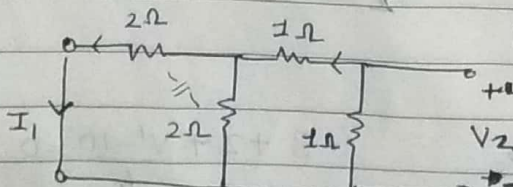
$$I = \frac{V_1 \times 3}{8}$$



$$I_2 = \frac{2}{2+1} \times \frac{3V_1}{8}$$

$$I_2 = \frac{V_1}{4} \Rightarrow \left[ \frac{V_1}{I_2} = 4 \right]$$

$$\frac{V_2}{4} \quad \frac{1}{1+2} \times \frac{3V_2}{2}$$



$$I = \frac{V_2 \times 3}{2}$$

$$I_1 = \frac{V_2}{4}$$

$$\left[ \frac{V_2}{I_1} = 4 \right]$$



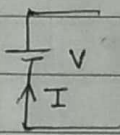
(based on conservation of power)

Active, pass, loss of  
max. loss?

# ★ Tellegen's Theorem:-

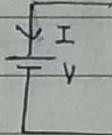
In a given Any circuit (linear or non linear), Active or passive, Unilateral or Bilateral) Algebraic Sum of all the power is Zero. or power generated by the different source is equal to the power dissipate.

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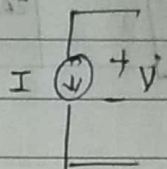
$$P = -VI$$

generate

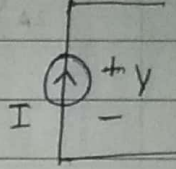


$$P = +VI$$

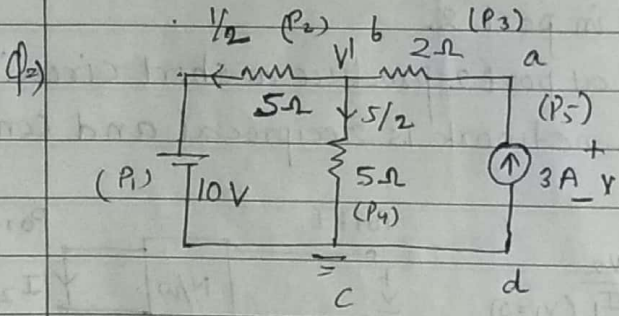
dissipate.



$$P = VI$$



$$P = -VI$$



$$\frac{V1-10}{5} + \frac{V1}{5} - 3 = 0$$

$$\frac{V1-10+V1-15}{5} = 0$$

$$2V1 = 25$$

$$[V1 = 12.5V]$$

KVL in abcd

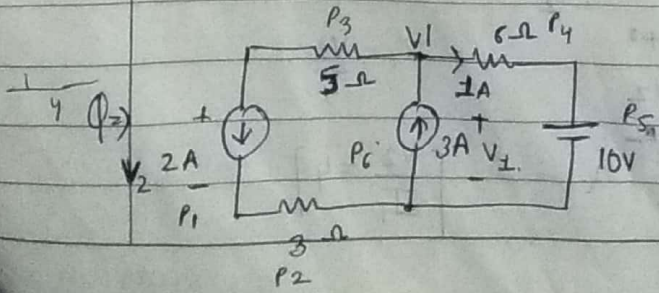
$$2 \times 3 + 5 \times \frac{5}{2} - V = 0$$

$$[V = 37.5]$$

$$P1 + P2 + P3 + P4 + P5$$

$$= 10 \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 \times 5 + (3)^2 \times 2 + \left(\frac{5}{2}\right)^2 \times 5 - 37.5 \times 3$$

$$= 5 + \frac{5}{4} + 18 + \frac{25 \times 5}{4} - \frac{111}{2} = 55.5 - 55.5 = 0$$



$$-3 + 2 + \frac{V1-10}{6} = 0$$

$$\frac{V1-10}{6} = 1 \quad [V1 = 16]$$



$$\begin{aligned} 6 + 10 - V_1 &= 0 & -10 + 6 + 10 - V_2 &= 0 \\ [V_1 = 16V] & & [V_2 = 20V] & \end{aligned}$$

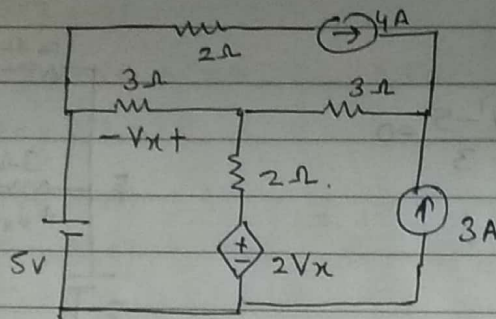
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$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6$$

$$\begin{aligned} \Rightarrow & 0 + 3 \times (2)^2 + 5 \times (2)^2 + 6 + 10 - 16 \times 3 \\ &= 48 - 48 \\ &= \underline{\underline{0 \text{ Am}}} \end{aligned}$$

6/9/18 Tutorial :-

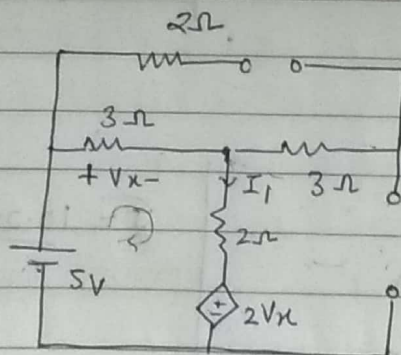
find current in  $2\Omega$  using Superposition.



$$I = I_1 + I_2 + I_3$$

3V   3A   4A

Soln.:-



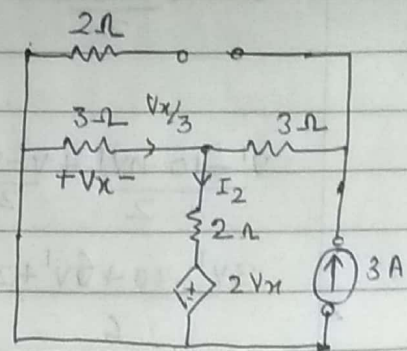
$$-5 + V_x + 2I_1 + 2V_x = 0$$

$$-5 + 3 \times 3I_1 + 2I_1 = 0$$

$$11I_1 = 5$$

$$I_1 = 5/11$$

$$V_x = 3I_1$$



$$\frac{V_x}{3} + 3 = I_2$$

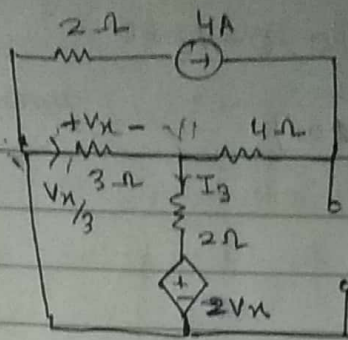
$$V_x + 2I_2 + 2V_x = 0$$

$$3V_x + 2I_2 = 0$$

$$9(I_2 - 3) + 2I_2 = 0$$

$$I_2 = 27/11$$





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$$I = 5/11 + 27/11 + 36/11$$

$$= 68/11 \text{ A}$$

$$Vx/3 + 4 = I_3$$

$$Vx + 2I_3 + 2Vx = 0$$

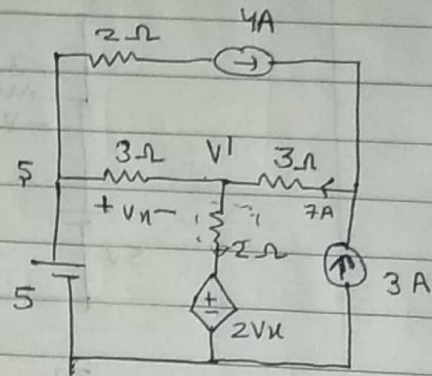
$$I = 36/11$$

by nodal

$$\frac{V' - 2Vx}{2} - 7 + \frac{V' - 5}{3} = 0$$

$$Vx = 5 - V'$$

$$I = \frac{V' - 2Vx}{2}$$



$$\frac{V' - 10 + 2V'}{2} + \frac{V' - 5}{3} - 7 = 0$$

$$\frac{3V' - 30 + 6V' + 2V' - 10 - 42}{6} = 0$$

$$11V' = 82$$

$$[V' = 10.25]$$

$$7.45$$

$$Vx = 5 - 10.25 = -5.25$$

$$I = \frac{10.25 + 10.5}{2}$$

$$I = 6.175 \text{ A}$$

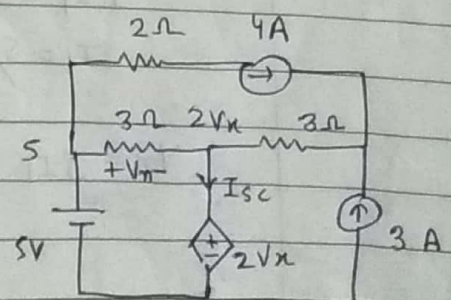
By nodal

$$5 - 2Vx = Vx$$

$$Vx = 5/3$$

$$\frac{Vx}{3} + 7 = I_{sc}$$

$$I_{sc} = 68/9$$



passion



$$V_x = -3 \times 1$$

$$V_n = -3$$

$$V = 3 \times 3 = 9$$

$$[R_{th} = 9]$$

$$[I = \frac{9}{9+2} \times \frac{68}{9} = \frac{68}{11} A]$$

by thevenin's

$$\frac{V_{th} - 5 - 3}{3} = 0$$

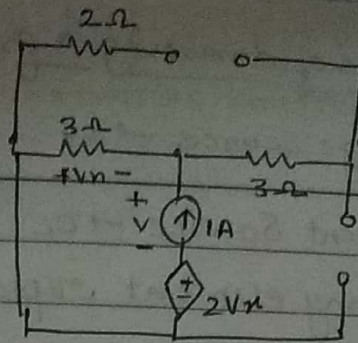
$$V_x = -7 \times 3 = -21$$

$$-V_n - 2V_n - V_{th} + 5 = 0$$

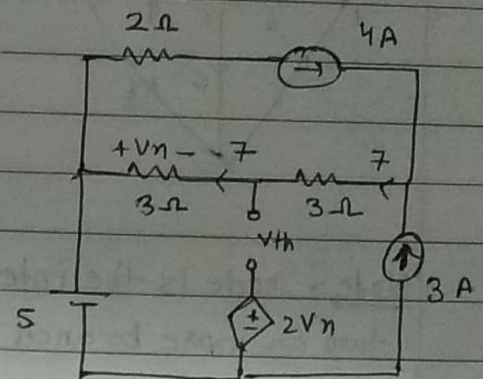
$$-3V_n - V_{th} = -5$$

$$63 + 5 = V_{th}$$

$$[V_{th} = 68]$$



Date - / - / -



$$[I = \frac{68}{9+2} = \frac{68}{11}]$$

7/9/18

\* Graph theorem:-

CKT Analysis:-

- ↳ Nodal theorem
- ↳ Graph theory
  - procedure of drawing a graph
  - Some definition related to GT
  - Matrix design
  - using these matrix we analyse circuit.



## \* Procedure of drawing a graph:-

All Voltage Source  $\rightarrow$  SC

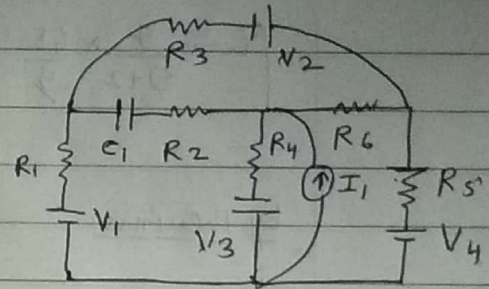
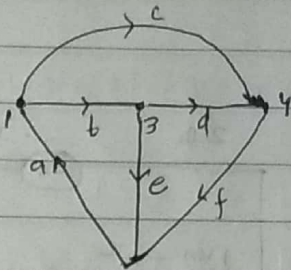
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All Current Source  $\rightarrow$  OC

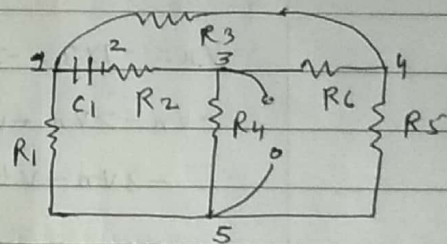
Each & Every element replaced by a line.

Q. draw graph of circuit as shown in fig:-

Sch



1. Node:- node is the interconnection of two or more branch.



2. Mesh:- mesh is a continuous drawing start from a point and finish at the same point.

3. Directional and non-directional graph:- If direction of flow is represented in each and every branch than this is known as directional graph otherwise non directional graph.

4. Planar and Non-planar graph:-

If a graph can be drawn on a paper without intersecting the branch is known as planar graph. otherwise non-planar.

5. Tree - co-Tree:-

Connection of all the node without having any closed loop is known as the tree.

Co-Tree is the remaining part of the tree graph.

Total graph = Tree + Co-tree.

6) Twigs & links:- Branch of tree is known as twigs (represented by continuous line).



Number of twigs  $\rightarrow$  [Twigs =  $N-1$ ]

Link - Branch of co-tree is known as co-tree Link. (by dotted line)  
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Total no Branch (B)

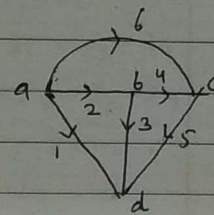
$$B = \text{Link} + \text{twigs}$$

$$B = \text{Link} + N - 1$$

$$\boxed{\text{Link} = B - N + 1}$$

(i) Incident matrix:- (KCL).

$\rightarrow$  Draw No of node & no of Branch.



| Node | 1  | 2  | 3  | 4  | 5  | 6  |
|------|----|----|----|----|----|----|
| a    | +1 | +1 | 0  | 0  | 0  | +1 |
| b    | 0  | +1 | +1 | +1 | 0  | 0  |
| c    | 0  | 0  | 0  | -1 | +1 | -1 |
| d    | -1 | 0  | -1 | 0  | -1 | 0  |

If B-Direction leave the node = +1

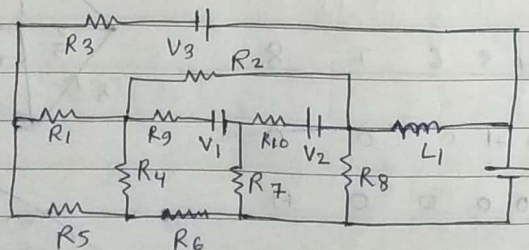
toward the node = -1

otherwise = 0

$\rightarrow$  Reduce-Incident matrix  $\rightarrow$  It has Reduce one row from the incident matrix is called Reduce-Incident matrix.

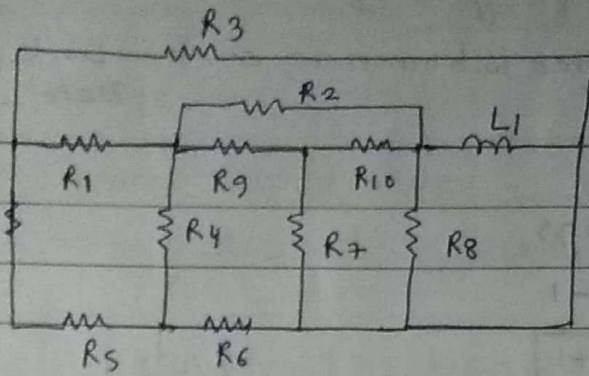
Q $\Rightarrow$  Draw the graph of network as shown in fig. and draw incident matrix.

Soln:-





Soln:-



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\* f-Tie Set matrix :-

→ It is based on KVL

→ matrix draw b/w No of loop and No of branch.

→ No. of Loop = No of link.

→ Every loop have only one link and remaining are twigs.

→ Direction of loop current in the direction of link current.

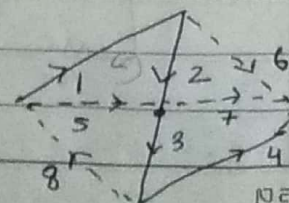
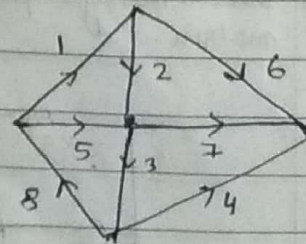
→ If BC in the direction of loop current = +1

opposite = -1

otherwise = 0

Ex:- Draw the fundamental tie Set matrix, taking 1, 2, 3, 4 as a tree.

| loop              | → branch |    |    |    |    |    |    |    |
|-------------------|----------|----|----|----|----|----|----|----|
|                   | 1        | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
| $L_1(1, 2, 5)$    | -1       | -1 | 0  | 0  | +1 | 0  | 0  | 0  |
| $L_2(1, 2, 3, 8)$ | +1       | +1 | +1 | 0  | 0  | 0  | 0  | +1 |
| $L_3(3, 4, 7)$    | 0        | 0  | -1 | +1 | 0  | 0  | +1 | 0  |
| $L_4(4, 3, 2, 9)$ | 0        | -1 | -1 | -1 | 0  | +1 | 0  | 0  |



Scanned by CamScanner



$$I_L = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

$$[L_b] = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8 \end{bmatrix}$$

Date \_ / \_ / \_

$$[L_b] = [B^T] [I_L]$$

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8 \end{bmatrix} = \begin{bmatrix} -1 & +1 & 0 & 0 \\ -1 & +1 & 0 & -1 \\ 0 & +1 & -1 & -1 \\ 0 & 0 & -1 & -1 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

$$L_1 = -I_1 + I_2$$

$$L_2 = -I_1 + I_2 - I_4$$

$$L_3 = I_2 - I_3 - I_4$$

$$L_4 = -I_3 - I_4$$

$$L_5 = I_1$$

$$L_6 = I_4$$

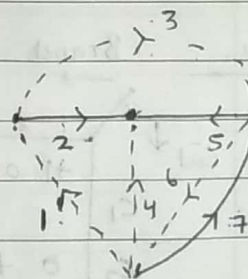
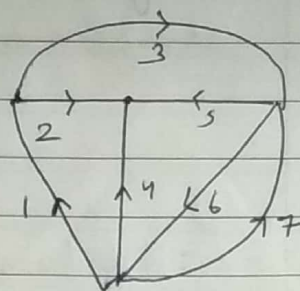
$$L_7 = I_3$$

$$L_8 = I_2$$

Q2) taking 2, 5, 7 as the tree draw a tie set matrix.

Soln:

| loop       | branch | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|------------|--------|----|----|----|----|----|----|----|
| L(2,3,5)   |        | 0  | -1 | +1 | 0  | +1 | 0  | 0  |
| L(4,5,7)   |        | 0  | 0  | 0  | +1 | -1 | 0  | -1 |
| L(6,7)     |        | 0  | 0  | 0  | 0  | 0  | +1 | +1 |
| L(1,2,5,7) |        | +1 | +1 | 0  | 0  | -1 | 0  | -1 |

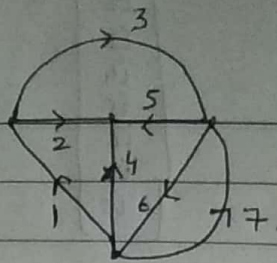


$$[L_b] = [B^T] [I_L]$$

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & +1 \\ -1 & 0 & 0 & +1 \\ +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ +1 & -1 & 0 & -1 \\ 0 & 0 & +1 & 0 \\ 0 & -1 & +1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$



Date \_ / \_ / \_



12/9/18

\* f-Cut Set matrix:-

Drawn between no of cut and No. of Branch.

No of cut = No. of twigs.

In a cut only one twig and max remaining are link.

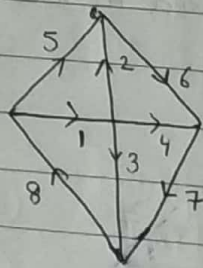
Direction of cut = Direction of twig.

if branch direction (BD) in the direction of cut = +1

opposite = -1

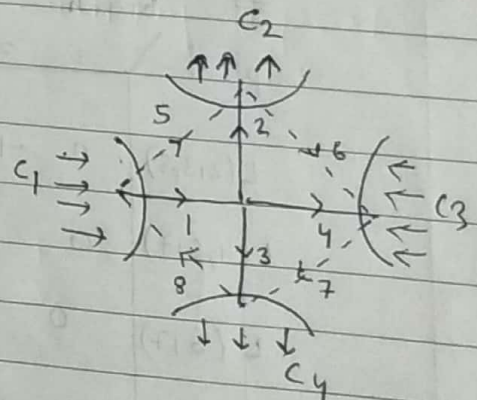
otherwise = 0

→ Taking 1, 2, 3, 4 as a tree draw the f-cut set matrix.



Soln:-

| Cut ↓          | Branch → |    |    |    |    |    |    |    |
|----------------|----------|----|----|----|----|----|----|----|
|                | 1        | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
| C <sub>1</sub> | +1       | 0  | 0  | 0  | +1 | 0  | 0  | -1 |
| C <sub>2</sub> | 0        | +1 | 0  | 0  | +1 | -1 | 0  | 0  |
| C <sub>3</sub> | 0        | 0  | +1 | 0  | 0  | 0  | +1 | -1 |
| C <sub>4</sub> | 0        | 0  | 0  | +1 | 0  | -1 | +1 | 0  |





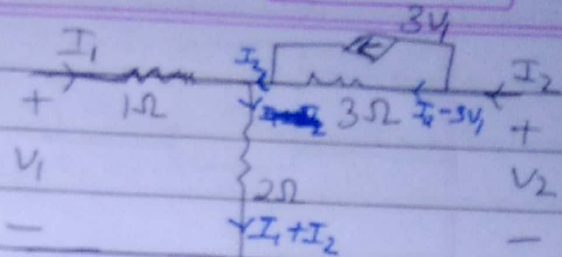
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$$V_1 = I_1 + 2(I_1 + I_2)$$

$$V_1 = 3I_1 + 2I_2$$



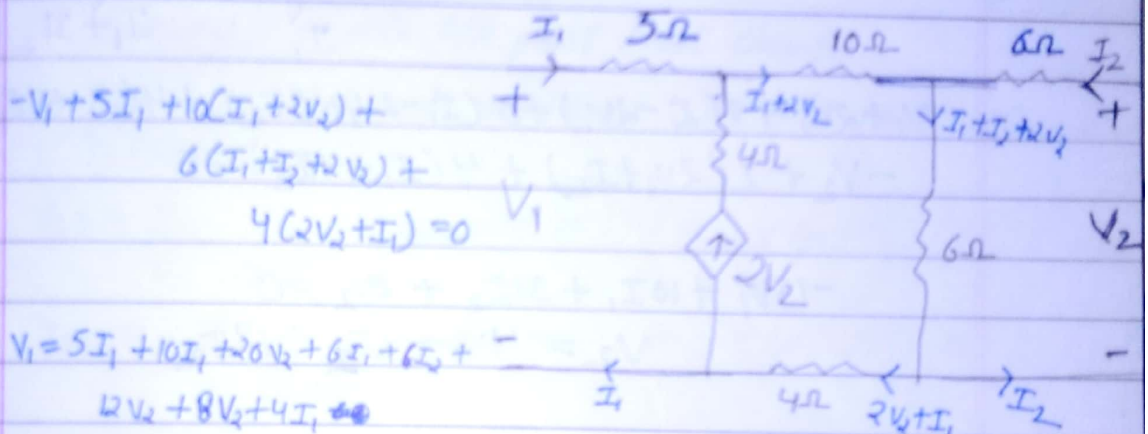
$$V_2 = 3(I_2 - 3V_1) + 2(I_1 + I_2)$$

$$V_2 = 3(I_2 - 3(3I_1 + 2I_2)) + 2I_1 + 2I_2$$

$$= 3(I_2 - 9I_1 - 6I_2) + 2I_1 + 2I_2$$

$$= 3I_2 - 27I_1 - 18I_2 + 2I_1 + 2I_2$$

$$V_2 = -25I_1 - 13I_2$$



$$-V_1 + 5I_1 + 10(I_1 + 2V_2) +$$

$$6(I_1 + I_2 + 2V_2) +$$

$$4(2V_2 + I_1) = 0$$

$$V_1 = 5I_1 + 10I_1 + 20V_2 + 6I_1 + 6I_2 +$$

$$12V_2 + 8V_2 + 4I_1$$

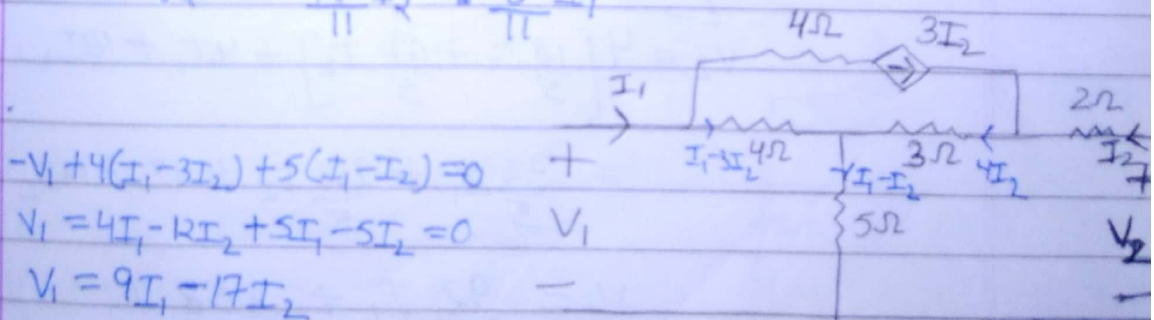
$$= 25I_1 + 32V_2 + 8V_2 + 6I_2$$

$$V_1 = 25I_1 + 40V_2 + 6I_2$$

$$-V_2 + 6I_2 + 6I_1 + 6I_2 + 12V_2 = 0$$

$$V_2 = 12I_2 + 6I_1 + 12V_2$$

$$V_2 = -\frac{12}{11}I_2 + \frac{6}{11}I_1$$



$$-V_1 + 4(I_1 - 3I_2) + 5(I_1 - I_2) = 0$$

$$V_1 = 4I_1 - 12I_2 + 5I_1 - 5I_2 = 0$$

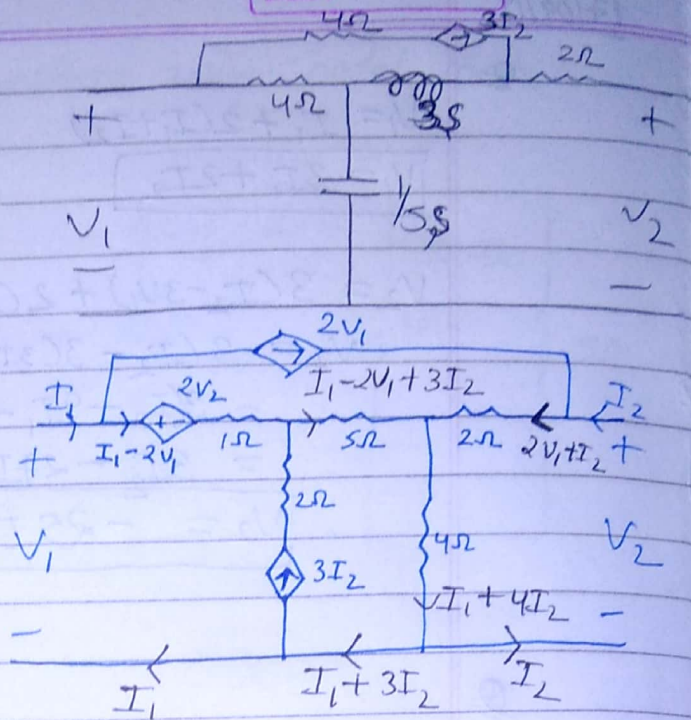
$$V_1 = 9I_1 - 17I_2$$

$$V_2 + 2I_2 + 12I_2 + 5I_1 - 5I_2 = 0$$

$$V_2 = 5I_1 + 9I_2$$



19/09/19 Find the Z-parameter



$$-V_1 + 2V_2 + 1(I_1 - 2V_1) + 5(I_1 - 2V_1 + 3I_2) + 4(I_1 + 4I_2) = 0$$

$$-V_2 + 2(2V_1 + I_2) + 4(I_1 + 4I_2) = 0$$

$$-13V_1 + 10I_1 + 31I_2 + 2V_2 = 0$$

$$V_2 = 4V_1 + 4I_1 + 18I_2$$

$$-13V_1 + 10I_1 + 31I_2 + 2(8V_1 + 8I_1 + 36I_2) = 0$$

$$5V_1 = 18I_1 + 67I_2$$

$$V_1 = \frac{18I_1 + 67I_2}{5}$$

$$Z_{11} = \frac{18}{5}, \quad Z_{12} = \frac{67}{5}$$

$$V_2 = 4\left[\frac{18I_1 + 67I_2}{5}\right] + 4I_1 + 18I_2$$

$$= \frac{72}{5}I_1 + \frac{268}{5}I_2 + 4I_1 + 18I_2$$

$$V_2 = \frac{92}{5}I_1 + \frac{358}{5}I_2$$

$$Z_{21} = \frac{92}{5}, \quad Z_{22} = \frac{358}{5}$$



$\frac{V}{I} = \text{Complex impedance}$

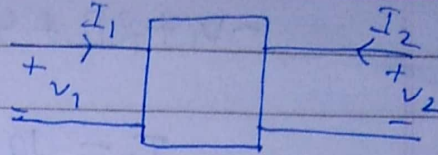
$\frac{I}{V} = \text{admittance parameter}$

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## Y-Parameter

$$(I_1, I_2) = f(V_1, V_2)$$



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$Y_{11} = \frac{I_1}{V_1} (V_2 = 0)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{21} = \frac{I_2}{V_1} (V_2 = 0)$$

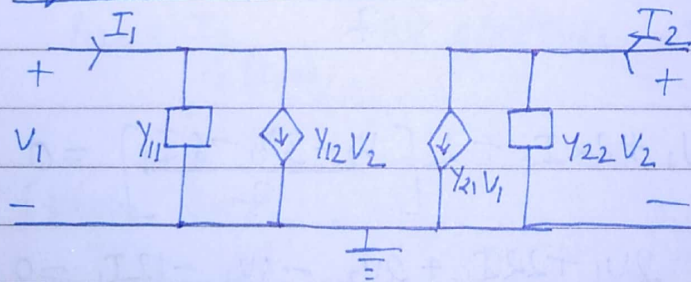
$$Y_{11} = \frac{I_1}{V_1} (V_2 = 0) \quad \text{Input admittance with o/p port short circuit.}$$

$$Y_{21} = \frac{I_2}{V_1} (V_2 = 0) \quad \text{Forward admittance with o/p port S.C.}$$

$$Y_{12} = \frac{I_1}{V_2} (V_1 = 0) \quad \text{Reverse " " I/p " "}$$

$$Y_{22} = \frac{I_2}{V_2} (V_1 = 0) \quad \text{o/p " " I/p " "}$$

## Equivalent circuit



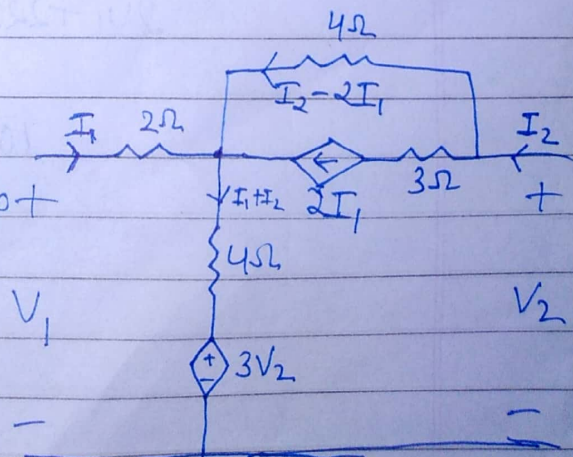
$$\text{KVL: } -V_1 + 2I_1 + 4(I_1 + I_2) + 3V_2 = 0$$

$$-V_2 + 4(I_2 - 2I_1) + 4(I_1 + I_2) + 3V_2 = 0$$

$$-V_2 + 4I_2 - 8I_1 + 4I_1 + 4I_2 + 3V_2 = 0$$

$$8I_2 - 4I_1 + 2V_2 = 0$$

$$I_2 = \frac{4I_1 - 2V_2}{8}$$



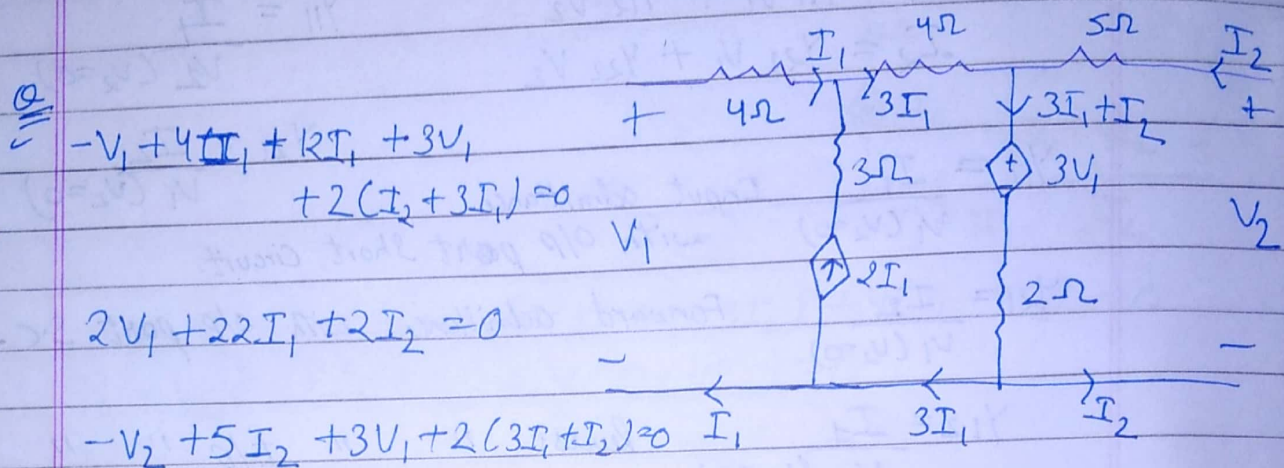


$$-V_1 + 2I_1 + 4I_1 + 4\left(\frac{I_1}{2} - \frac{1}{4}V_2\right) + 3V_2 = 0$$

$$I_1 = 12V_1 - 8V_2$$

$$I_2 = \frac{12V_1 - 8V_2 - \frac{1}{4}V_2}{2}$$

$$I_2 =$$



$$-V_2 + 5I_2 + 3V_1 + 6I_1 + 2I_2 = 0$$

$$7I_2 + 6I_1 + 3V_1 - V_2 = 0$$

$$7I_2 = V_2 - 3V_1 - 6I_1$$

$$I_2 = \frac{V_2 - 3V_1 - 6I_1}{7}$$

$$2V_1 + 22I_1 + 2\left[\frac{V_2 - 3V_1 - 6I_1}{7}\right] = 0$$

$$2V_1 + 22I_1 + \frac{2V_2}{7} - \frac{6V_1}{7} - \frac{12I_1}{7} = 0$$

$$10I_1 + 2V_1 - \frac{6V_1}{7} + \frac{2V_2}{7} = 0$$

$$\frac{70I_1 + 14V_1 - 6V_1 + 2V_2}{7} = 0$$

$$70I_1 + 8V_1 + 2V_2 = 0$$

$$I_1 = -\frac{8V_1}{70} - \frac{2V_2}{70}$$



$$7I_2 + 6 \left[ \frac{-8V_1}{70} - \frac{2V_2}{70} \right] + 3V_1 - V_2 = 0$$

$$I_2 = \underline{\hspace{2cm}}$$

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### h-parameter

$$(V_1, I_2) = f(I_1, V_2)$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

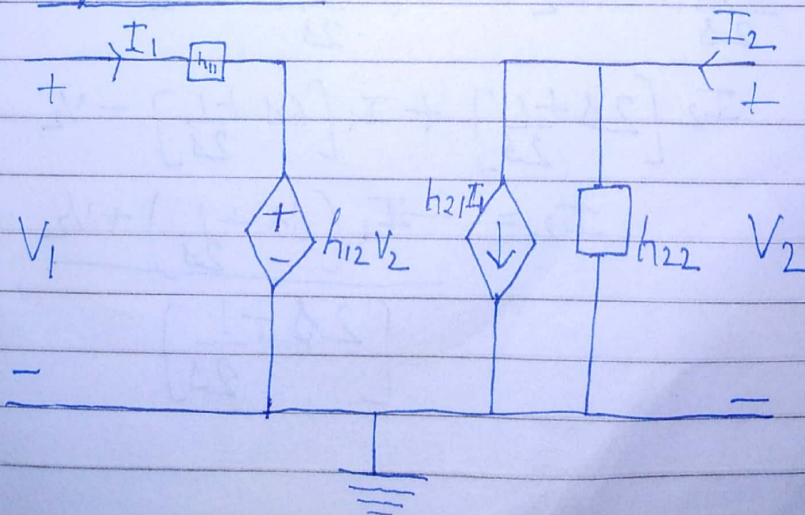
$$h_{11} = \frac{V_1}{I_1 (V_2=0)} \Rightarrow \text{Input impedance with o/p port S.C.}$$

$$h_{21} = \frac{I_2}{I_1 (V_2=0)} \Rightarrow \text{Current gain with o/p port S.C.}$$

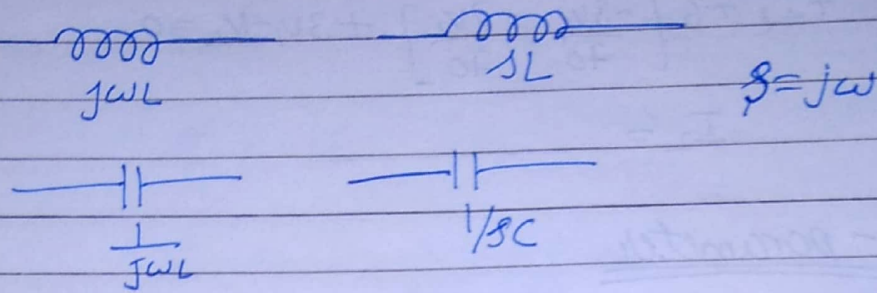
$$h_{12} = \frac{V_1}{V_2 (I_1=0)} \Rightarrow \text{Reverse voltage gain with I/p port open circuit}$$

$$h_{22} = \frac{I_2}{V_2 (I_1=0)} \Rightarrow \text{O/p admittance with I/p port open ckt.}$$

### Equivalent Circuit

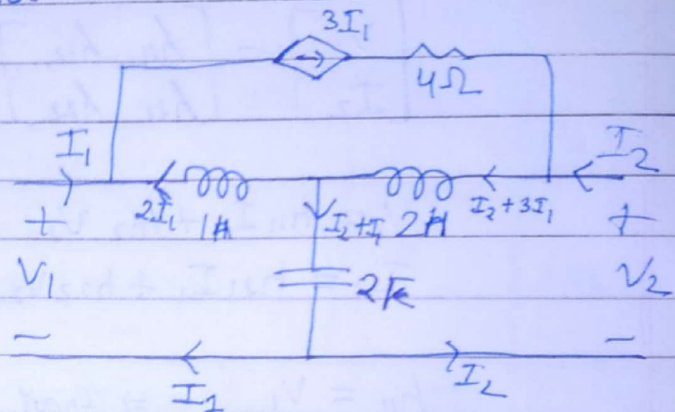






Q Find the h-parameter

Sol



$$-V_1 - 2I_1 \times s + \frac{1}{2s}(I_1 + I_2) = 0$$

$$-V_2 + 2s(I_2 + 3I_1) + \frac{1}{2s}(I_1 + I_2) = 0$$

$$V_1 = -2I_1 s + \frac{I_1}{2s} + \frac{I_2}{2s}$$

$$V_2 = 2sI_2 + 6sI_1 + \frac{I_1}{2s} + \frac{I_2}{2s}$$

$$\frac{I_2}{2s} + 2sI_2 + 6sI_1 + \frac{I_1}{2s} = V_2$$

$$I_2 \left[ 2s + \frac{1}{2s} \right] + I_1 \left[ 6s + \frac{1}{2s} \right] = V_2$$

$$I_2 = \frac{-I_1 \left( 6s + \frac{1}{2s} \right) + V_2}{\left[ 2s + \frac{1}{2s} \right]}$$



$$I_2 = \frac{-I_1(12s^2+1)}{(4s^2+1)} + \frac{V_2 \times 2s}{4s^2+1}$$

$$\begin{aligned} V_1 &= -2I_1s + \frac{I_1}{2s} + \frac{1}{2s} \left[ \frac{-I_1(12s^2+1)}{(4s^2+1)} + \frac{V_2 \times 2s}{4s^2+1} \right] \\ &= \frac{-4s^2I_1 + I_1}{2s} + \frac{1}{2s} \left[ \frac{-I_1(12s^2+1)}{(4s^2+1)} + \frac{V_2 \times 2s}{4s^2+1} \right] \\ &= I_1 \left[ \frac{1-4s^2}{2s} \right] + \frac{-I_1(12s^2+1)}{2s(4s^2+1)} + \frac{V_2}{(4s^2+1)} \end{aligned}$$

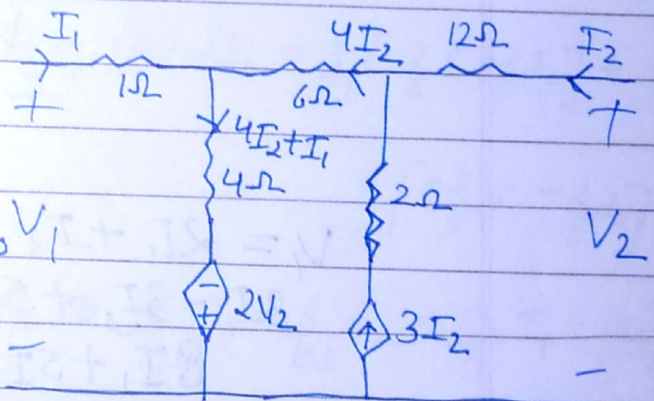
$$V_1 = I_1 \left[ \frac{(1-4s^2) - (12s^2+1)}{2s(4s^2+1)} \right] + \frac{V_2}{(4s^2+1)}$$

$$\begin{aligned} \textcircled{Q} \quad -V_1 + I_1 + 4(4I_2 + I_1) - 2V_2 &= 0 \\ -V_1 + 5I_1 + 16I_2 - 2V_2 &= 0 \end{aligned}$$

$$-V_2 + 12I_2 + 24I_2 + 16I_2 + 4I_1 - 2V_2 = V_1$$

$$-3V_2 + 52I_2 + 4I_1 = 0$$

$$I_2 = \frac{3V_2 - 4I_1}{52} \Rightarrow \frac{-4I_1}{52} + \frac{3V_2}{52}$$





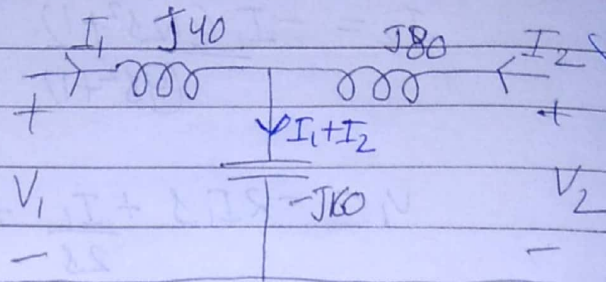
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## Tutorial

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Q Find the Y-parameter

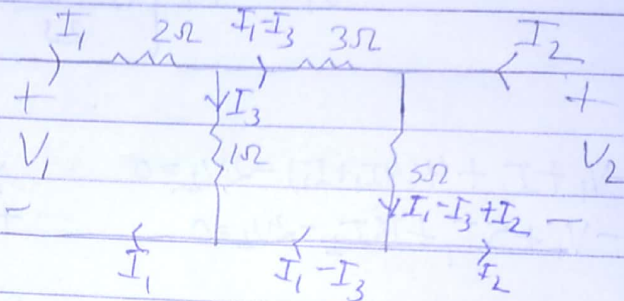


$$\text{Sol) } -V_1 + j40 \times I_1 - j160(I_1 + I_2) = 0$$

$$V_1 = -j40I_1 - j160I_2$$

$$V_2 = -j160I_1 - j80I_2$$

Q



$$V_1 = 2I_1 + I_3$$

$$3I_1 - 3I_3 + 5I_1 - 5I_3 + 5I_2 - I_3 = 0$$

$$8I_1 + 5I_2 - 9I_3 = 0$$

$$V_2 = 5I_1 - 5I_3 + 5I_2$$

$$V_2 = 5I_1 - 5(V_1 - 2I_1) + 5I_2$$

$$V_2 = 5I_1 - 5V_1 + 10I_1 + 5I_2$$

$$V_2 + 5V_1 = 15I_1 + 5I_2$$

$$8I_1 + 5I_2 - 9(V_1 - 2I_1) = 0$$

$$8I_1 + 5I_2 - 9V_1 + 18I_1 = 0$$

$$V_1 = \frac{26I_1 + 5I_2}{9}$$

$$V_2 = 15I_1 + 5I_2 - 5\left[\frac{26I_1 + 5I_2}{9}\right]$$

$$= 15I_1 + \frac{130I_1}{9} - 5I_2 - \frac{25I_2}{9}$$

$$V_2 = \frac{265I_1}{9} - \frac{70I_2}{9}$$



$$-V_1 + I_1 + \frac{1}{2S} (I_1 + 3V_2)$$

$$+ 2(I_1 + 3V_2 + I_2) = 0$$

$$V_2 = 2I_1 + 6V_2 + 2I_2$$

$$V_2 = -\frac{2I_1}{5} - \frac{2I_2}{5}$$

$$V_1 = I_1 + \frac{I_1}{2S} + \frac{3V_2}{2S} + 2I_1 + 6V_2 + 2I_2$$

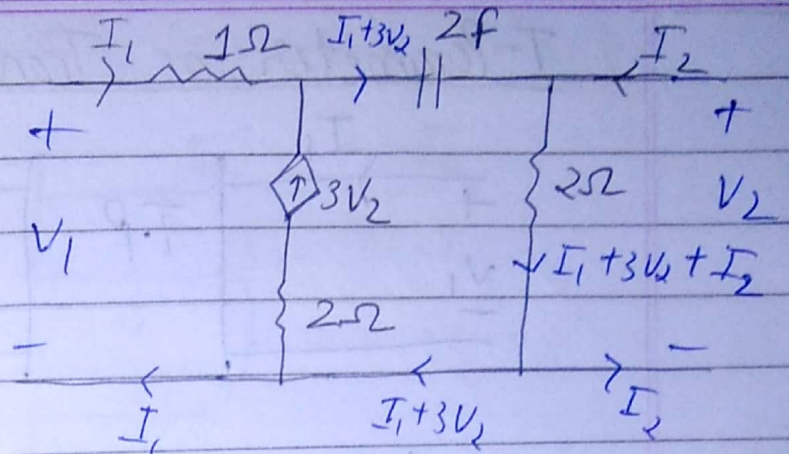
$$V_1 = 3I_1 + \frac{I_1}{2S} + 2I_2 + \frac{3V_2}{2S} + 6V_2$$

$$= \frac{I_1(3+2S)}{2S} = \frac{I_1(1+3 \times 2S)}{2S} + 2I_2 + \frac{3}{2S} \left[ -\frac{2I_1}{5} - \frac{2I_2}{5} \right] +$$

$$6 \left[ -\frac{2I_1}{5} - \frac{2I_2}{5} \right]$$

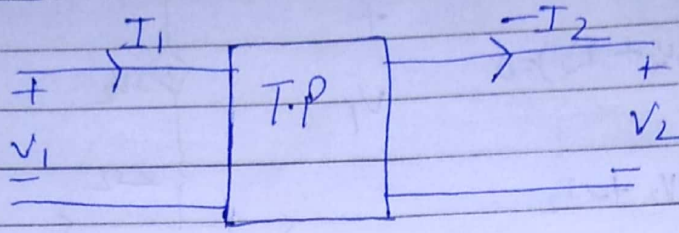
$$V_1 = \left( \frac{1+6S}{2S} \right) I_1 + 2I_2 - \frac{6}{10S} I_1 - \frac{6}{10S} I_2 - \frac{12}{5} I_1 - \frac{12}{5} I_2$$

$$V_1 = \frac{I_1}{2S} + 3I_1 - \frac{6}{10S} I_1 - \frac{12}{5} I_1 + 2I_2 - \frac{6}{10S} I_2 - \frac{12}{5} I_2$$





25/09/19

T-Parameter or Transmission Parameter

$$(V_1, I_1) = f(V_2, -I_2)$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0}$$

reverse voltage gain with  
o/p port o.c

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$$

reverse admittance with  
o/p port o.c

$$B = \left. \frac{V_1}{-I_2} \right|_{V_2=0}$$

Inverse Current gain with  
o/p short circuit.

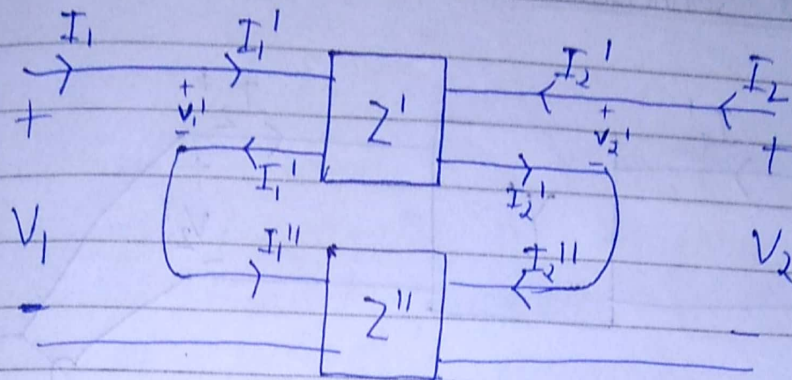
$$D = \left. \frac{I_1}{-I_2} \right|_{V_2=0}$$

We never draw the equivalent circuit because  
eq. related to same.

Connection of 2-port network

- ① Series Connection  $[Z]$
- ② Parallel "  $[Y]$
- ③ Series-parallel  $[h]$
- ④ Cascade connection  $[T]$



Series Connection

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{bmatrix} \begin{bmatrix} I_1' \\ I_2' \end{bmatrix}$$

$$\begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = \begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix} \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix}$$

$$I_1 = I_1' = I_2'' \quad , \quad V_1 = V_1' + V_1''$$

$$I_2 = I_2' = I_2'' \quad , \quad V_2 = V_2' + V_2''$$

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11}' I_1 + Z_{12}' I_2 + Z_{11}'' I_1 + Z_{12}'' I_2$$

$$V_1 = [Z_{11}' + Z_{11}'' ] I_1 + [Z_{12}' + Z_{12}'' ] I_2$$

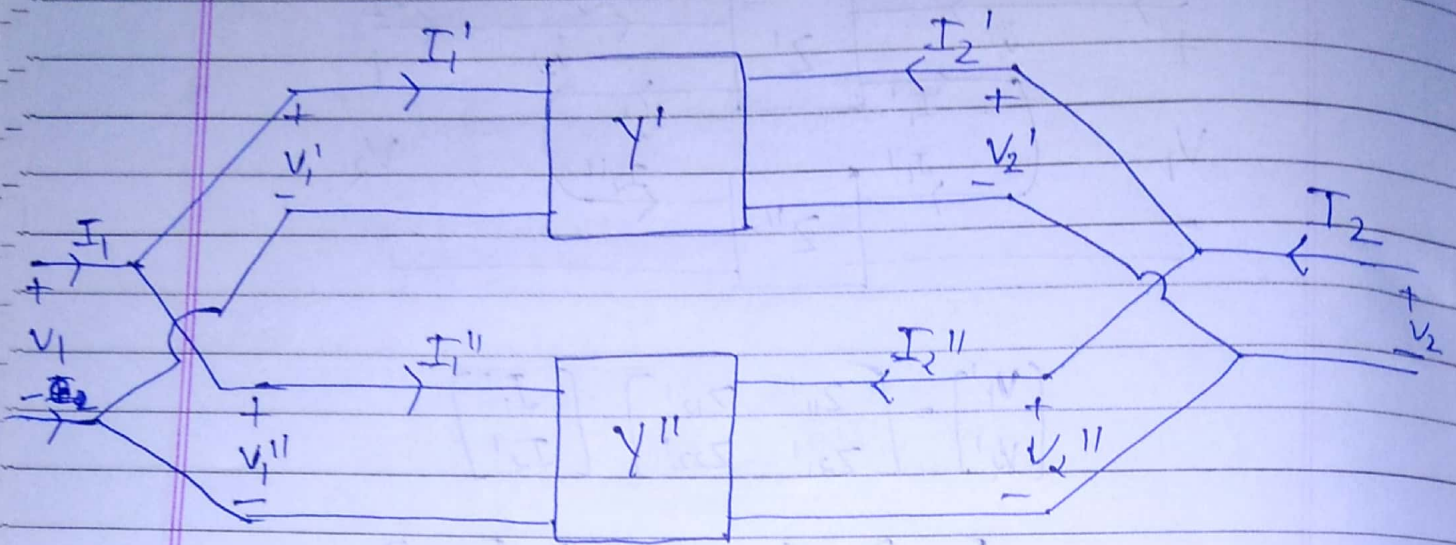
$$V_2 = [Z_{21}' + Z_{21}'' ] I_1 + [Z_{22}' + Z_{22}'' ] I_2$$

$$\begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = \begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[Z] = [Z'] + [Z'']$$

If two  $z$ -parameter are connected in series then the resultant parameter is the matrix sum of individual parameter.



Parallel Connection [Y-parameter]

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \end{bmatrix}$$

$$\begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix}$$

from I/P port  $\Rightarrow V_1 = V_1' = V_1''$  ,  $I_1 = I_1' + I_1''$

from O/P port  $\Rightarrow V_2 = V_2' = V_2''$  ,  $I_2 = I_2' + I_2''$

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11}' V_1 + Y_{12}' V_2 + Y_{11}'' V_1 + Y_{12}'' V_2$$

$$I_1 = [Y_{11}' + Y_{11}''] V_1 + [Y_{12}' + Y_{12}''] V_2$$

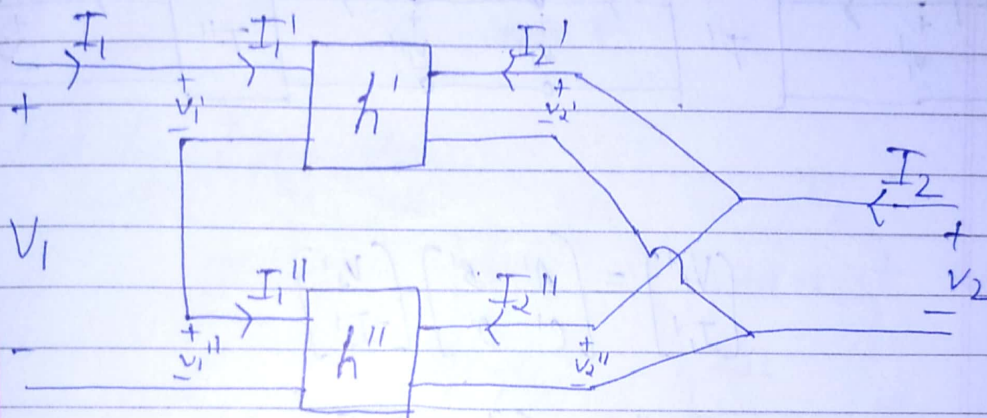


$$I_2 = [Y_{21}' + Y_{21}']V_1 + [Y_{22}' + Y_{22}']V_2$$

$$[Y] = [Y'] + [Y'']$$

If two Y-parameter are connected in parallel then the resultant parameter is the matrix sum of individual parameter.

### Series-Parallel Connection [h-parameter]



$$\begin{bmatrix} V_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{bmatrix} \begin{bmatrix} I_1' \\ V_2' \end{bmatrix}$$

$$\begin{bmatrix} V_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} h_{11}'' & h_{12}'' \\ h_{21}'' & h_{22}'' \end{bmatrix} \begin{bmatrix} I_1'' \\ V_2'' \end{bmatrix}$$

I/P port

$$I_1 = I_1' + I_1''$$

$$V_1 = V_1' + V_1''$$

O/P Port

$$I_2 = I_2' + I_2''$$

$$V_2 = V_2' = V_2''$$

$$\begin{bmatrix} V_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = (h_{11}' + h_{11}'')I_1 + (h_{12}' + h_{12}'')V_2$$

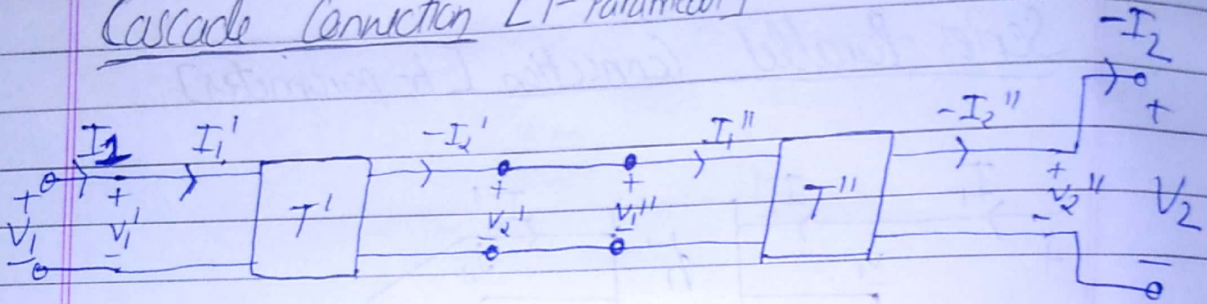


$$\begin{bmatrix} V_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} h_{11}'' & h_{12}'' \\ h_{21}'' & h_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_2 = [h_{21}' + h_{21}''] I_1 + [h_{22}' + h_{22}''] V_2$$

$$[h] = [h'] + [h'']$$

Cascade Connection [T-Parameter]



$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

$$\begin{bmatrix} V_1'' \\ I_1'' \end{bmatrix} = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} V_2'' \\ -I_2'' \end{bmatrix}$$

I/P port

$$V_1 = V_1'$$

$$I_1 = I_1'$$

O/P port

$$V_2 = V_2''$$

$$I_2 = I_2''$$

Cascading Terminal

$$-I_2' = I_1''$$

$$V_2' = V_1''$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1'' \\ I_1'' \end{bmatrix}$$



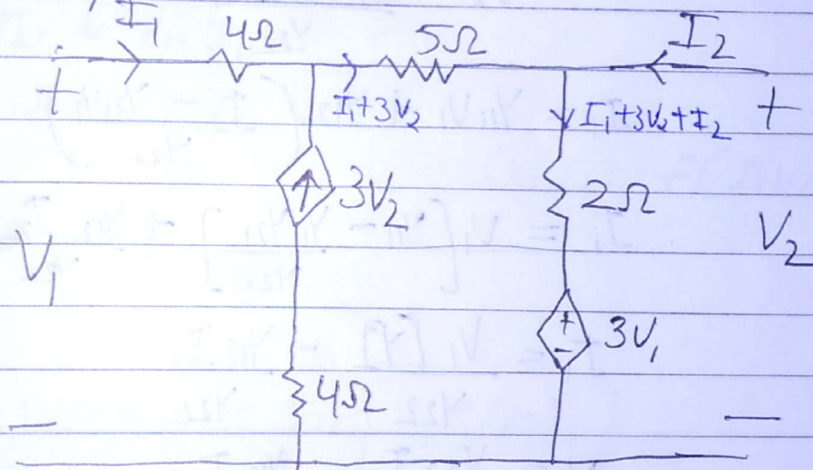
$$\begin{bmatrix} V_1'' \\ I_1'' \end{bmatrix} = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$[T] = [T'] [T'']$$

here the resultant parameter is the matrix product of the two individual T-parameter

Q Find the T-parameter



Sol

$$-V_1 + 4I_1 - 3V_2(4) = 0 \quad -V_1 + 4I_1 + 5(I_1 + 3V_2) + 2(I_1 + 3V_2 + I_2) + 3V_1 = 0$$

$$V_1 = 4I_1 - 12V_2 \quad + 3V_1 = 0$$

$$-V_1 + 4I_1 + 5I_1 + 15V_2 + 2I_1 + 6V_2 + 2I_2 + 3V_1 = 0$$

$$2V_1 + 21V_2 + 11I_1 + 2I_2 = 0$$

$$-V_2 + 2I_1 + 6V_2 + 2I_2 + 3V_1 = 0$$

$$5V_2 + 3V_1 + 2I_1 + 2I_2 = 0$$



30/09/19

Conversion of one parameter in term of another parameter

e Find Z-parameter in term of Y-parameter.

Sol) all Z in term of [Y]

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{--- (1)} \quad (V_1, V_2) \rightarrow f(I_1, I_2)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{--- (2)}$$

find  $V_2$  from eq (2) & put in eq (1)

$$V_2 = \frac{I_2 - Y_{21}V_1}{Y_{22}}$$

$$I_1 = Y_{11}V_1 + Y_{12} \left( \frac{I_2 - Y_{21}V_1}{Y_{22}} \right)$$

$$I_1 = V_1 \left[ Y_{11} - \frac{Y_{11}Y_{21}}{Y_{22}} \right] + \frac{Y_{12}I_2}{Y_{22}}$$

$$I_1 = \frac{V_1 [Y]}{Y_{22}} + \frac{Y_{12}I_2}{Y_{22}}$$

$$V_1 = \frac{Y_{22}I_1}{[Y]} - \frac{Y_{12}I_2}{[Y]}$$

$$Z_{11} = \frac{Y_{22}}{[Y]}$$

$$Z_{12} = \frac{-Y_{12}}{[Y]}$$

find  $V_1$  from eq (1) & put in eq (2)

$$V_1 = \frac{I_1 - Y_{12}V_2}{Y_{11}}$$

$$I_2 = Y_{21} \left[ \frac{I_1 - Y_{12}V_2}{Y_{11}} \right] + Y_{22}V_2$$



$$I_2 = \frac{Y_{21} I_1}{Y_{11}} - \frac{Y_{12} Y_{21} V_2}{Y_{11}} + Y_{22} V_2$$

$$I_2 = \frac{Y_{21} I_1}{Y_{11}} - V_2 \left[ \frac{Y_{12} Y_{21}}{Y_{11}} - Y_{22} \right]$$

$$- \frac{I_1 Y_{21}}{Y_{11}} + \frac{V_2 [Y]}{Y_{11}}$$

$$\frac{V_2 [Y]}{Y_{11}} = I_2 - \frac{I_1 Y_{21}}{Y_{11}}$$

$$V_2 = \frac{-I_1 Y_{21}}{[Y]} + \frac{Y_{11} I_2}{[Y]}$$

Q Find the h-parameter in term of z-parameter

So)  $V_1 = Z_{11} I_1 + Z_{12} I_2$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$(V_1, I_2) = f(I_1, V_2)$$

$$I_2 = \frac{V_2 - Z_{21} I_1}{Z_{22}}$$

$$V_1 = Z_{11} I_1 + Z_{12} \left[ \frac{V_2 - Z_{21} I_1}{Z_{22}} \right]$$

$$V_1 = Z_{11} I_1 + \frac{Z_{12} V_2}{Z_{22}} - \frac{Z_{21} Z_{12}}{Z_{22}} I_1$$

$$V_1 = \left[ \frac{Z_{11} - Z_{21} Z_{12}}{Z_{22}} \right] I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = \frac{[Z]}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$I_2 = \frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1$$



Q Find T-parameter in term of Y-parameter

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$(V_1, I_1) = f(V_2, I_2)$$

$$V_1 = \frac{I_2 - Y_{22} V_2}{Y_{21}}$$

$$I_1 = Y_{11} \left[ \frac{I_2 - Y_{22} V_2}{Y_{21}} \right] + Y_{12} V_2$$

$$I_1 = \frac{Y_{11} I_2}{Y_{21}} - \frac{Y_{11} Y_{22} V_2}{Y_{21}} + Y_{12} V_2$$

~~$$= \left[ \frac{Y_{11}}{Y_{21}} + Y_{12} \right] V_2 + \frac{Y_{11} I_2}{Y_{21}}$$~~

$$I_1 = -\frac{Y_{11}}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

Q Find the Z-parameter in term of T-parameters.



03/10/19

Tutorial

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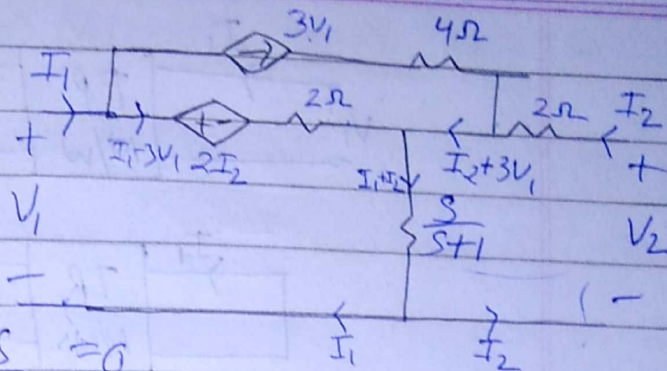
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Z-parameter

$$-V_1 + 2(I_1 - 3V_1) + 2I_2 +$$

$$\frac{s}{s+1}(I_1 + I_2) = 0$$

$$-V_2 + 2(I_2) + (I_1 + I_2) \frac{s}{s+1} = 0$$



$$V_1 = 2I_1 - 6V_1 + 2I_2 + \frac{s}{s+1}I_1 + \frac{s}{s+1}I_2$$

$$7V_1 = \left(2 + \frac{s}{s+1}\right)I_1 + \left(2 + \frac{s}{s+1}\right)I_2$$

$$V_1 = \left(\frac{2+s}{7}\right)I_1 + \left(\frac{2+s}{7}\right)I_2$$

$$V_2 = 2I_2 + \frac{s}{s+1}I_1 + \frac{s}{s+1}I_2$$

$$V_2 = \frac{s}{s+1}I_1 + \left(2 + \frac{s}{s+1}\right)I_2$$

04/10/19

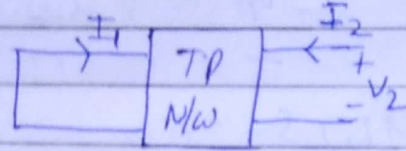
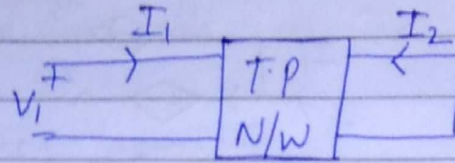
Condition of Reciprocity & Symmetry in two port N/w① Condition of Reciprocity

When we applied voltage  $V_1$  at port ① and find out the S.C current in port ②, now applied the voltage  $V_2$  at port ② and find out the S.C current in port ①

$$\frac{V_1}{I_2 (V_2=0)} = \frac{V_2}{I_1 (V_1=0)} \quad \text{Then, network is said to be}$$

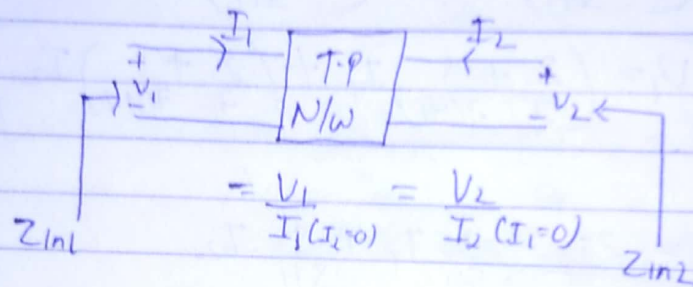
reciprocal network and eq ① is known as reciprocity.





② Condition of ~~reciprocity~~ <sup>Symmetry</sup>

Total input port impedance with output port open ckt should be equal to total O/p port impedance with I/p port impedance.



Q Find the condition of reciprocity & symmetry for Z-parameters

Sol

$$\frac{V_1}{I_2 (V_2=0)} = \frac{V_2}{I_1 (V_1=0)}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{--- ①}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{--- ②}$$

$$0 = Z_{21}I_1 + Z_{22}I_2$$

$$I_1 = -\frac{Z_{22}I_2}{Z_{21}}$$

$$V_1 = Z_{11} \left[ -\frac{Z_{22}I_2}{Z_{21}} \right] + Z_{12}I_2$$

$$\frac{V_1}{I_2} = \frac{-Z_{11}Z_{22} + Z_{21}Z_{12}}{Z_{21}}$$



$$\frac{V_1}{I_2 (V_2=0)} = -\frac{|Z|}{Z_{21}} \Rightarrow$$

$$0 = Z_{11} I_1 + Z_{12} I_2$$

$$I_2 = -\frac{Z_{11}}{Z_{12}} I_1$$

$$V_2 = Z_{21} I_1 - \frac{Z_{11} Z_{22}}{Z_{12}} I_1$$

$$\frac{V_2}{I_1} = \frac{Z_{12} Z_{21} - Z_{11} Z_{22}}{Z_{12}}$$

$$\frac{V_2}{I_1 (V_2=0)} = -\frac{|Z|}{Z_{12}} \Rightarrow$$

$$\boxed{Z_{12} = Z_{21}} \text{ Reciprocity}$$

Condition of symmetry for Z-parameters

$$\frac{V_1}{I_1 (I_2=0)} = \frac{V_2}{I_2 (I_1=0)}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (i)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (ii)}$$

From eq (i)  $I_2=0$

$$\frac{V_1}{I_1} = Z_{11}$$

from eq (ii)  $I_1=0$

$$\frac{V_2}{I_2} = Z_{22}$$

$$\boxed{Z_{11} = Z_{22}} \text{ Symmetry}$$



Q Find the condition of reciprocity & symmetry for h-parameter.

$$\frac{V_1}{I_2 (V_2=0)} = \frac{V_2}{I_1 (V_1=0)}$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

from eq (1)  $V_2 = 0$

$$\frac{V_1}{I_1} = h_{11}$$

from eq (2)  $V_1 = 0$

from eq (2)  $V_2 = 0$

$$I_1 = \frac{I_2}{h_{21}}$$

$$V_1 = \frac{h_{11} I_2}{h_{21}} + h_{12} V_2$$

$$\frac{V_1}{I_2 (V_2=0)} = \frac{h_{11}}{h_{21}}$$

from eq (1)  $V_1 = 0$

$$0 = h_{11} I_1 + h_{12} V_2$$

$$\frac{V_2}{I_1 (V_1=0)} = \frac{-h_{11}}{h_{12}}$$

$$\Rightarrow \boxed{-h_{21} = h_{12}}$$

Symmetry

$$\frac{V_1}{I_1 (I_2=0)} = \frac{V_2}{I_2 (I_1=0)}$$

from eq (2)  $I_1 = 0$

$$\frac{V_2}{I_2 (I_1=0)} = \frac{1}{h_{22}}$$



# Behaviour of inductor, Capacitor in Switching

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$$V_2 = \frac{-h_{21} I_1}{h_{22}}$$

$$V_1 = h_{11} I_1 + \frac{h_{12} (-h_{21}) I_1}{h_{22}}$$

$$V_1 = \left( \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} \right) I_1$$

$$\frac{V_1}{I_1 (I_2=0)} = \frac{|h|}{h_{22}}$$

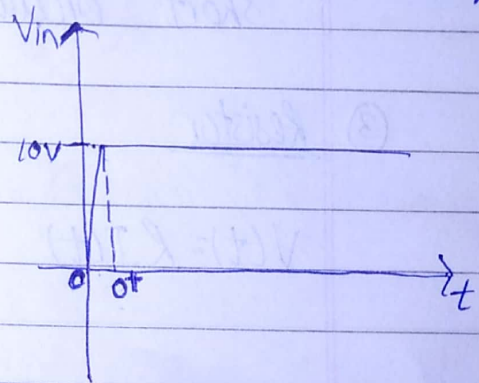
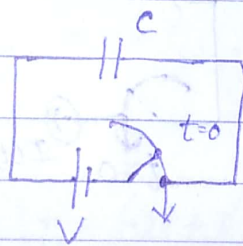
$$\frac{|h|}{h_{22}} = \frac{1}{h_{22}}$$

$$\boxed{|h| = 1}$$

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## Behaviour of resistor, inductor and capacitor in Switching conditions

### ① Capacitor



$$q(t) \propto V(t)$$

$$q(t) = C V(t)$$

$$t = 0 \leftrightarrow 0^+$$

$$t > 0^+$$

$$\boxed{I(t) = \frac{\partial q(t)}{\partial t} = C \frac{\partial V(t)}{\partial t}}$$

In ~~transi~~ transision or switching condition Capacitor work as a short circuit.

In steady state capacitor work as a open ckt.



# \* 1<sup>st</sup> order R-L, R-C circuit

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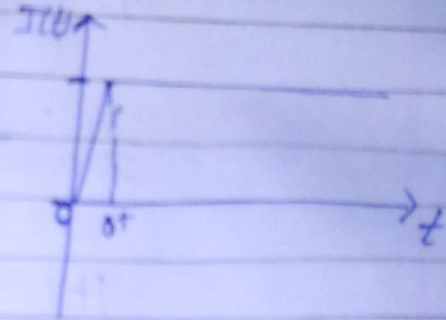
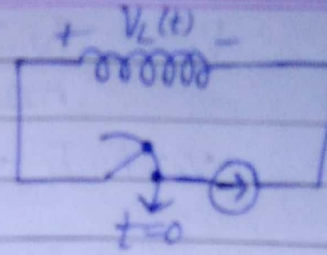
## ② Inductor

$$\phi(t) \propto I(t)$$

$$\phi(t) = LI(t)$$

$$V_L = \frac{\partial \phi(t)}{\partial t} = L \frac{\partial I(t)}{\partial t}$$

$$V_L = L \frac{\partial I(t)}{\partial t}$$

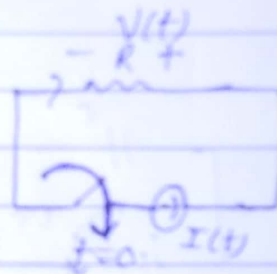


In switching condition inductor work as a open circuit.

In steady state inductor behave as a short circuit.

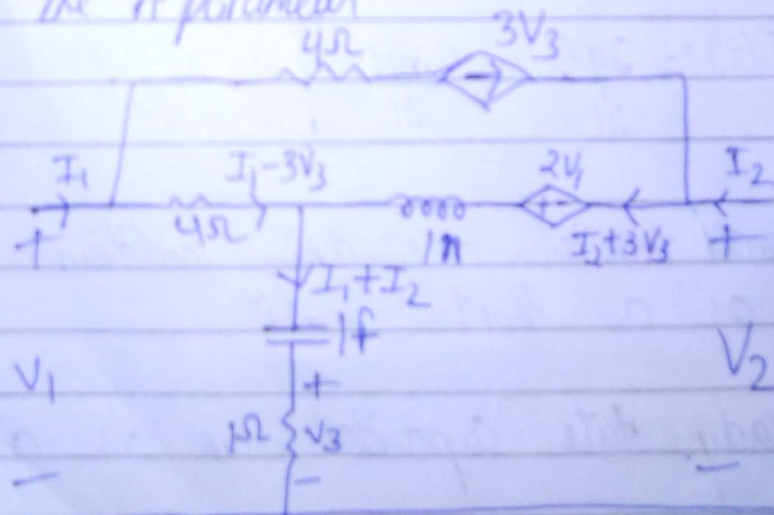
## ③ Resistor

$$V(t) = RI(t)$$



Voltage and current are flow same.

Find the h-parameter





$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$V_3 = I_1 + I_2$$

$$-V_1 + 4(I_1 - 3V_3) + \frac{1}{s}(I_1 + I_2) + I_1 + I_2 = 0$$

$$-V_1 + 4I_1 - 12I_1 + 12I_2 + \frac{I_1}{s} + \frac{I_2}{s} + I_1 + I_2 = 0$$

$$V_1 = \left(-7 + \frac{1}{s}\right)I_1 + \left(-11 + \frac{1}{s}\right)I_2$$

$$V_1 = -6I_1 - 10I_2$$

$$-V_2 - 2V_1 + 5I_2 + 3I_1 + 3I_2 + \frac{I_1}{s} + \frac{I_2}{s} + I_1 + I_2 = 0$$

$$V_2 + 2V_1 = \left(3 + \frac{1}{s} + 1\right)I_1 + \left(4 + \frac{1}{s} + 1\right)I_2$$

$$V_2 + 2\left[\left(-7 + \frac{1}{s}\right)I_1 + \left(-11 + \frac{1}{s}\right)I_2\right] = \left(3 + \frac{1}{s} + 1\right)I_1 + \left(4 + \frac{1}{s} + 1\right)I_2$$

$$V_2 + \left[\left(-14 + \frac{2}{s}\right)I_1 + \left(-22 + \frac{2}{s}\right)I_2\right] = \left(3 + \frac{1}{s} + 1\right)I_1 + \left(4 + \frac{1}{s} + 1\right)I_2$$

$$V_2 = \left[\left(-14 + \frac{2}{s}\right)I_1 + \left(-22 + \frac{2}{s}\right)I_2\right] + \left(3 + \frac{1}{s} + 1\right)I_1 + \left(4 + \frac{1}{s} + 1\right)I_2$$

suppose  
 $s=1$

$$= -\left[(-14+2)I_1 + (-22+2)I_2\right] + (3+1+1)I_1 + (4+1+1)I_2$$

$$= -[-12I_1 - 20I_2] + (5I_1 + 6I_2)$$

$$V_2 = 17I_1 + 26I_2$$

$$I_2 = \frac{1V_2 - 17I_1}{26}$$



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$$V_1 = -10I_2 = -6I_1$$

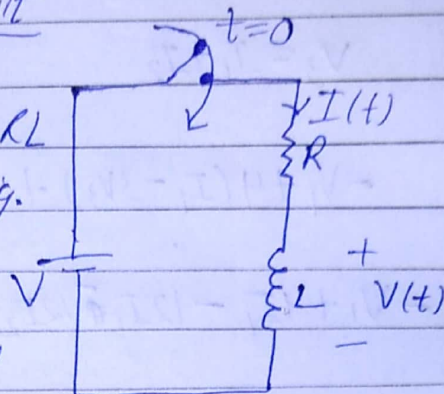
### First order R-L circuit

A combination of series RL element is shown in fig.

The switch is closed at  $t=0$ . Find the

expression for  $I(t)$  and voltage  $V(t)$  as shown in

fig. Assume initial current  $I(0) = 0$

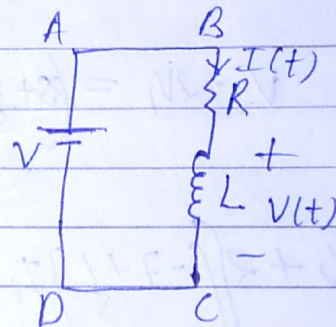


### KVL in ABCDA

$$RI(t) + L \frac{dI(t)}{dt} = V \quad \text{--- (1)}$$

Laplace Transform of eq<sup>n</sup> ①

$$RI(s) + L[sI(s) - I(0)] = \frac{V}{s}$$



$$I(s)[R + Ls] = \frac{V}{s}$$

$$I(s) = \frac{V}{s(R + Ls)}$$

$$I(s) = \frac{V}{Ls(s + \frac{R}{L})} = \frac{V}{L} \left[ \frac{A}{s} + \frac{B}{s + \frac{R}{L}} \right]$$

$$= \frac{V}{R} \left[ \frac{1}{s} - \frac{1}{s + \frac{R}{L}} \right] \quad \text{--- (2)}$$

Taking inverse laplace transform of eq<sup>n</sup> ②

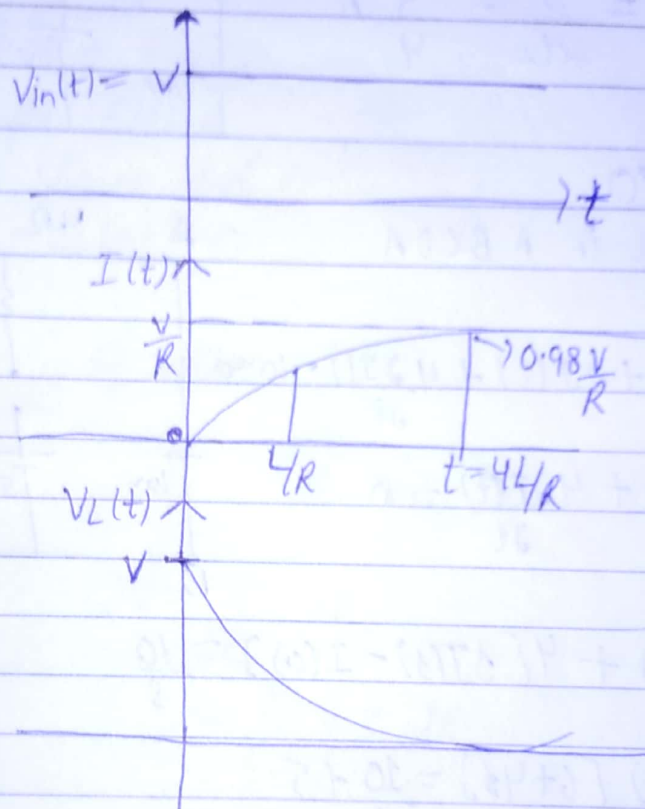
$$I(t) = \frac{V}{R} \left[ 1 - e^{-\frac{Rt}{L}} \right] \quad t > 0$$



$$V_L(t) = L \frac{\partial I(t)}{\partial t} = L \frac{\partial}{\partial t} \left[ \frac{V}{R} (1 - e^{-\frac{Rt}{L}}) \right]$$

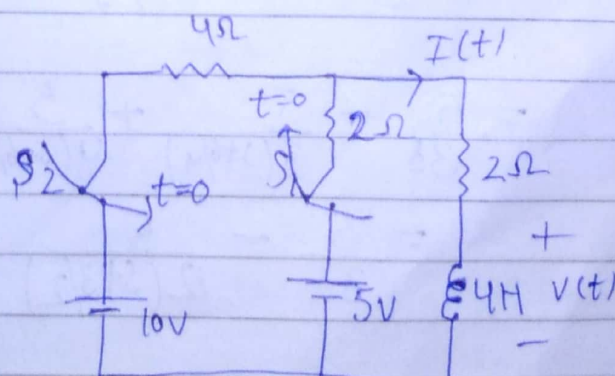
$$= K \times \frac{V}{R} \times -e^{-\frac{Rt}{L}} \times -\frac{R}{L}$$

$$V_L(t) = V e^{-Rt/L}$$



$$\frac{L}{R} = \text{Time Constant}$$

$$t = \frac{4L}{R}$$

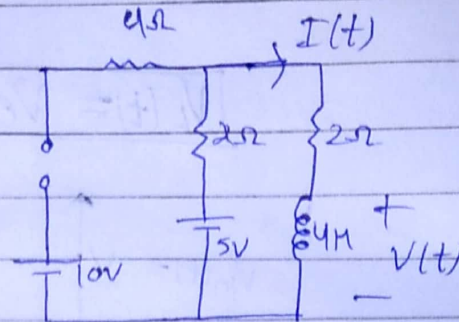


Before  $t=0$  switch  $S_1$  was closed and  $S_2$  was open at  $t=0$  the condition of switch is reversed. Find the expression for  $I(t)$  &  $V(t)$  as shown in fig.

Sol

$t < 0$

$$I_L(0) = \frac{5}{2+2} = \frac{5}{4} \text{ A}$$

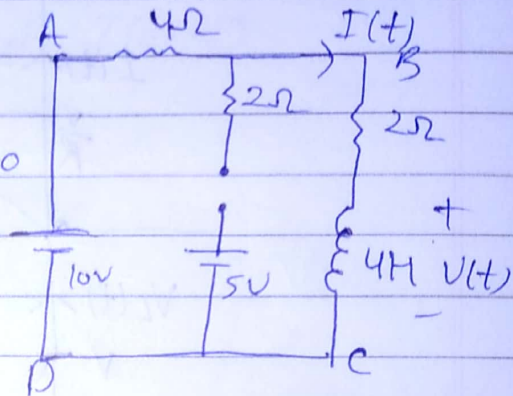


$t \geq 0$

KVL in ABCDA

$$4I(t) + 2I(t) + 4 \frac{dI(t)}{dt} - 10 = 0$$

$$6I(t) + 4 \frac{dI(t)}{dt} = 10$$



$$6I(s) + 4[sI(s) - I(0)] = \frac{10}{s}$$

$$I(s) [6 + 4s] = \frac{10}{s} + 5$$

$$I(s) = \frac{10}{4s(s+6/4)} + \frac{5}{4(s+6/4)}$$

$$= \frac{10}{6} \left[ \frac{1}{s} - \frac{1}{s+6/4} \right] + \frac{5}{4(s+6/4)}$$

$$= \frac{5}{3s} - \frac{5}{3(s+6/4)} + \frac{5}{4(s+6/4)}$$

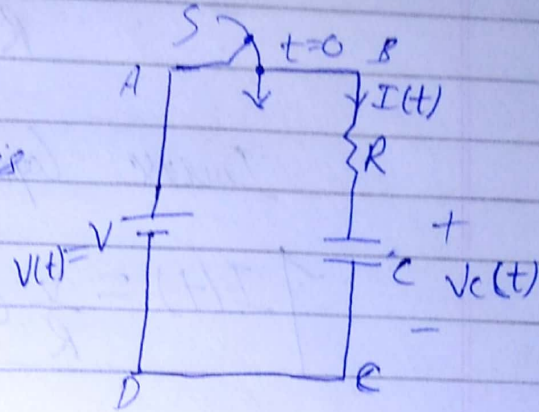
$$= \frac{5}{3s} - \frac{5}{12(s+3/2)}$$



$$I(t) = \frac{5}{3} - \frac{5}{12} e^{-3/2t}$$

### 14/10/19 1st order RC circuit

In series RC Ckt as shown in fig. The switch is closed at  $t=0$ . Find the expression for  $I(t)$  &  $V_c(t)$  as shown and assume  $V_c(0) = 0$



$$q = CV$$

$$I = \frac{\partial q}{\partial t} = C \frac{\partial V}{\partial t}$$

$$\partial V = \frac{I}{C} \partial t$$

$$V = \frac{1}{C} \int I dt$$

Apply KVL in ABCDA

$$RI(t) + \frac{1}{C} \int I dt - V = 0$$

$$RI(t) + \frac{1}{C} \int I dt = V$$

$$RI(s) + \left( \frac{I(s)}{Cs} - \frac{V_c(0)}{s} \right) = \frac{V}{s}$$

$$I(s) \left( R + \frac{1}{Cs} \right) = \frac{V}{s}$$

$$I(s) = \frac{V}{s(RCs + 1)}$$

$$I(s) = \frac{VR}{RC(s + \frac{1}{RC})}$$

$$I(s) = \frac{V}{R(s + \frac{1}{RC})}$$

Inverse Laplace

$$\boxed{I(t) = \frac{V}{R} e^{-t/RC}} \quad t \geq 0$$

$$V_c(t) = \frac{1}{C} \int I(t) dt$$

$$= \frac{1}{C} \int \frac{V}{R} e^{-t/RC} dt$$

$$V_c(t) = \frac{V}{RC} \frac{e^{-t/RC}}{-1/RC} + K$$

$$V_c(t) = -V e^{-t/RC} + K$$

at  $t=0$

$$0 = -V + K$$

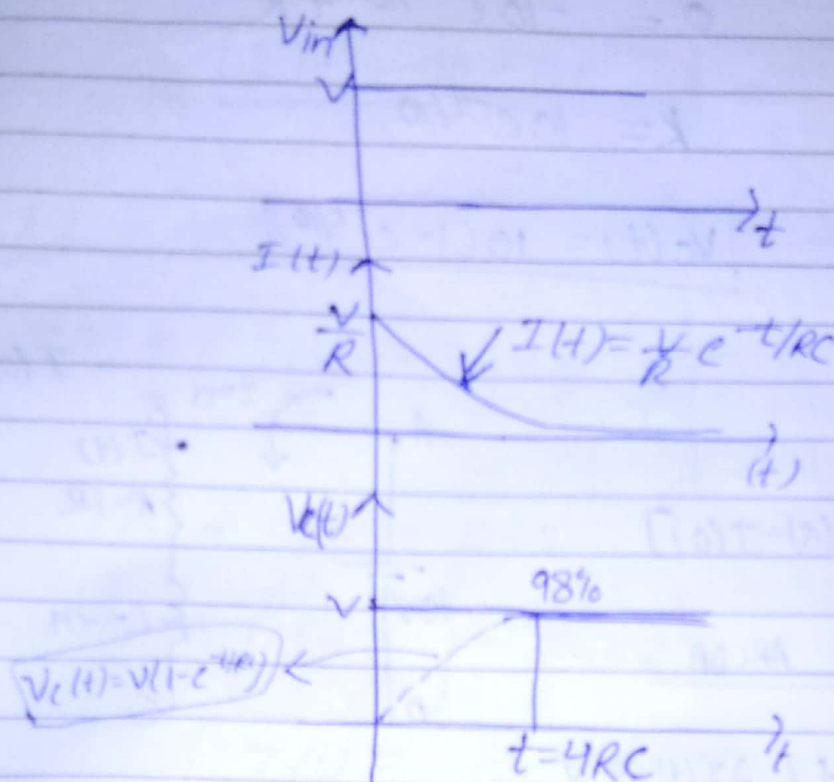
$$\boxed{K = V}$$

$$V_c(t) = V - V e^{-t/RC}$$

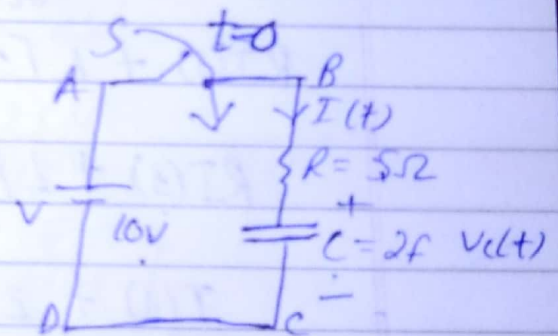
$$\boxed{V_c(t) = V(1 - e^{-t/RC})}$$



$RC = \text{Time Constant}$



Q A series RC ckt as shown in fig. Assume  $V_c(0) = 0$ . Find the expression for  $I(t)$  and  $V_c(t)$ .



Apply KVL ABCDA

Sol  $5I(t) + \frac{1}{2} \int I dt = 10$

$$I(s) \left( 5 + \frac{1}{2s} \right) = \frac{10}{s}$$

$$I(s) = \frac{10 \times 2s}{(5 \times 2s + 1)} \Rightarrow \frac{10 \times 2s}{5 \times 2s \left( s + \frac{1}{10} \right)}$$

$$I(s) = \frac{10}{5 \left( s + \frac{1}{10} \right)}$$

$$\boxed{I(t) = \frac{10}{5} e^{-t/10} \Rightarrow 2e^{-t/10}}$$

$$V_c(t) = \frac{1}{2} \int 2e^{-t/10} dt$$

$$0 = -10e^{-t/10} + k$$

$$k = 10e^{-t/10}$$

$$V_c(t) = 10(1 - e^{-t/10})$$

Q  $L \frac{dI}{dt}$   
 $L[sI(s) - I(0)]$

KVL in ABCDA

$$RI(t) + L \frac{dI(t)}{dt} = V$$

$$RI(s) + L[sI(s) - I_0] = \frac{V}{s}$$

$$RI(s) + L[sI(s) - 2] = \frac{V}{s}$$

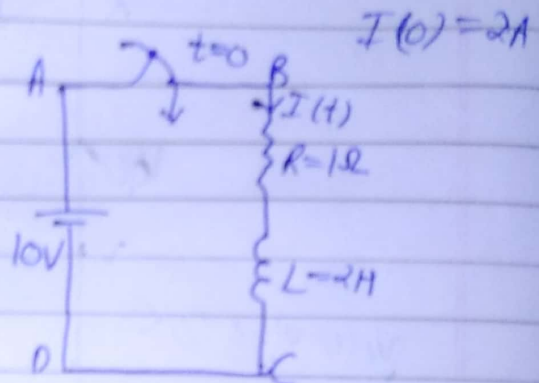
$$I(s) + 2[sI(s) - 2] = \frac{10}{s}$$

$$I(s)[1 + 2s] = \frac{10}{s} + 4$$

$$I(s) = \frac{10}{s(s+1)} + \frac{4}{s+1}$$

$$= \left( \frac{5}{s(s+1/2)} \right) + \frac{2}{(s+1/2)}$$

$$= 10 \left[ \frac{1}{s} - \frac{1}{s+1/2} \right]$$





$$= \frac{10}{s} - \frac{8}{s+1/2}$$

$$I(t) = 10 - 8e^{-t/2}$$

Q KVL in ABCDA

$$I(s) + \left( \frac{I(s)}{2s} - 2 \right) = \frac{10}{s}$$

$$I(s) + \frac{I(s)}{2s} = \frac{10}{s} + 2$$

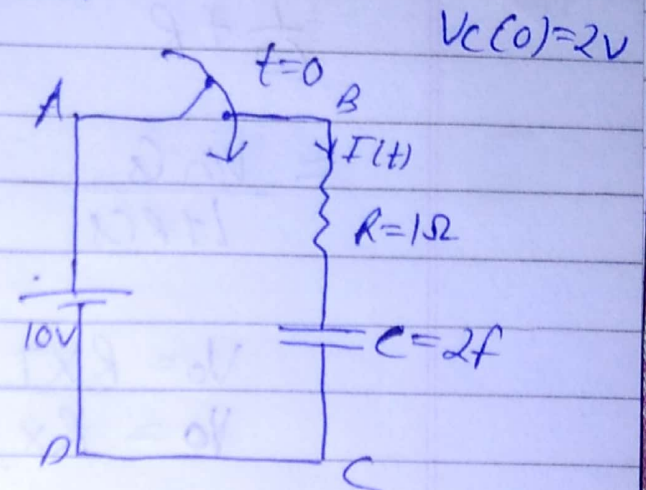
$$I(s) \left[ 1 + \frac{1}{2s} \right] = \frac{10}{s} + 2$$

$$I(s) = \frac{10}{s \left( 1 + \frac{1}{2s} \right)} + \frac{2}{\left( 1 + \frac{1}{2s} \right)}$$

$$I(s) = \frac{2(10 + 2s)}{(2s + 1)}$$

$$= 2 \left[ 1 + \frac{9}{2s + 1} \right]$$

$$I(t) = 2 \left[ 8(t) + \frac{9}{2} e^{-t/2} \right]$$



16/10/19

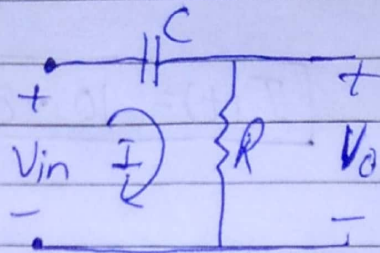
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## Concept of pole and zero and Stability

$$I = \frac{V_{in}}{\frac{1}{Cs} + R}$$

$$= \frac{V_{in} Cs}{1 + RCs}$$



$$V_o = R \times I$$

$$V_o = R \times \frac{V_{in} Cs}{1 + RCs}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{RCs}{RCs + 1}}$$

$$T(s) = \frac{N(s)}{D(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_n}{b_0 s^m + b_1 s^{m-1} + \dots + b_m}$$

$N(s) = 0$  Zero is root of numerator of T/F.

$D(s) = 0$  root pole of T/F

$$T(s) = \frac{Cs}{1 + RCs}$$

$s = 0$  zero

$s = -\frac{1}{RC}$  pole

$$V_o(s) = \frac{(s+1)(s+3)}{s(s^2+4)}$$

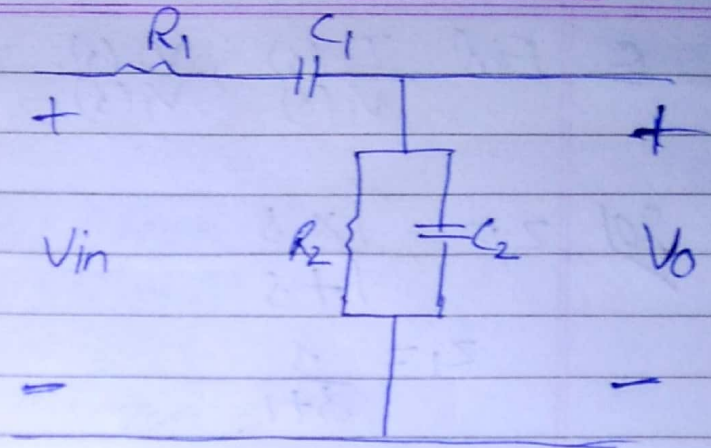
$s = -1, -3$  zero of T/F

$s = 0, \pm 2i$  pole of T/F

(Transfer function)



Q Find  $\frac{V_o}{V_{in}}$

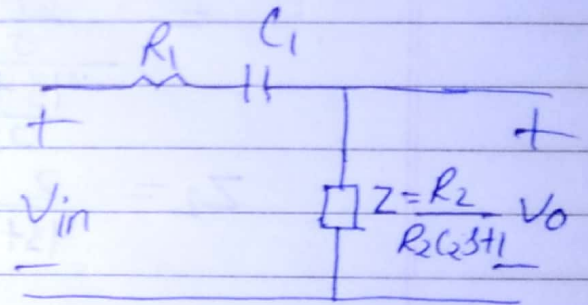


Sol

$$\frac{R_2 \times \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} \Rightarrow \frac{R_2}{R_2 C_2 s + 1}$$

$$i = \frac{V_{in}}{R_1 + \frac{1}{C_1 s} + Z}$$

$$V_o = i \times Z$$



$$\begin{aligned} \frac{V_o}{V_{in}} &= \frac{1}{R_1 + \frac{1}{C_1 s} + Z} \times Z \Rightarrow \frac{1}{R_1 + \frac{1}{C_1 s} + \frac{R_2}{R_2 C_2 s + 1}} \times \frac{R_2}{R_2 C_2 s + 1} \\ &= \frac{(C_1 s)(R_2 C_2 s + 1)}{R_1 (C_1 s)(R_2 C_2 s + 1) + [R_2 C_2 s + 1 + R_2 C_1 s]} \times \frac{R_2}{R_2 C_2 s + 1} \end{aligned}$$

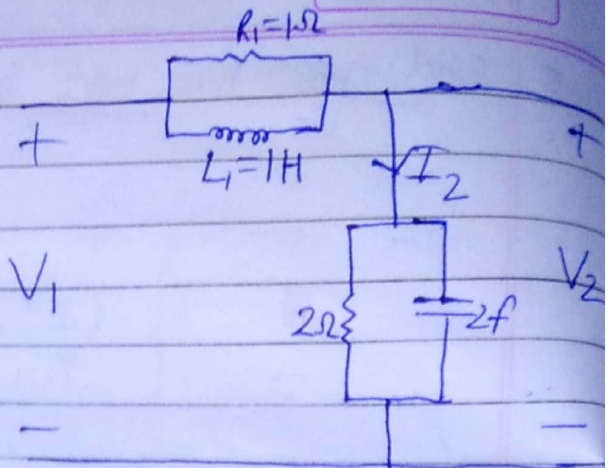
$$\frac{V_o}{V_{in}} = \frac{R_2 C_1 s}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_2 C_1) s + 1}$$

Pole represent by (X)  
zero represent by (O)

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Q Find  $\frac{I_2(s)}{V_1(s)}$ ,  $\frac{V_2(s)}{V_1(s)}$



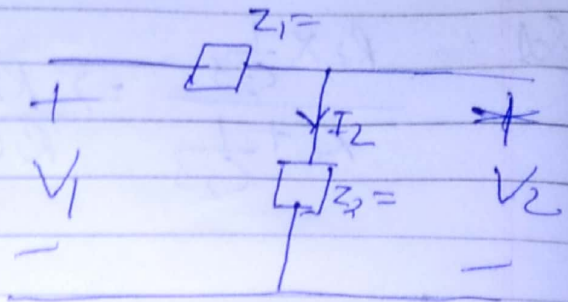
Sol  $Z_1 = \frac{1 \times s}{1+s}$

$Z_1 = \frac{s}{s+1}$

$Z_2 = \frac{2 \times 1}{2s}$   
 $\frac{2+1}{2s}$

$= \frac{1}{s}$   
 $\frac{4s+1}{2s}$

$Z_2 = \frac{2}{4s+1}$



$I_2 = \frac{V_1}{\frac{s}{s+1} + \frac{2}{4s+1}}$

$\frac{I_2}{V_1} = \frac{(s+1)(4s+1)}{s(4s+1) + 2(s+1)}$

$V_2 = I_2 \times Z_2$

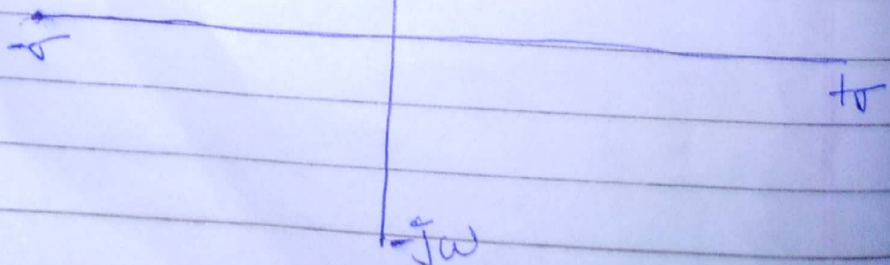
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Pole is the representation of Transfer function  
Pole is represent in S-plane

S-plane

-ve S-plane

+ve S-plane



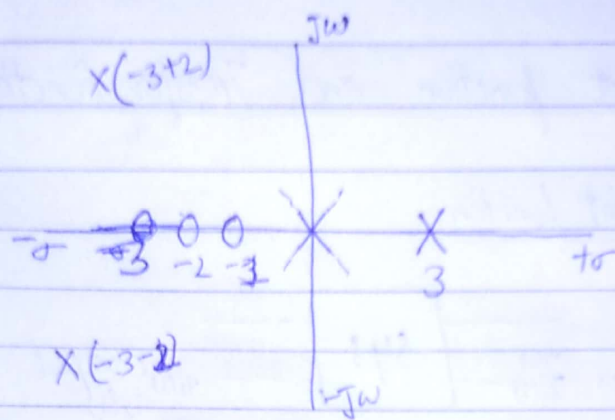


Q. A transfer function  $T(s)$  i.e. given by. Find the pole and zero of T/F and represent in s-plane.

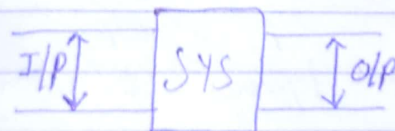
$$T(s) = \frac{(s+1)(s+2)}{s^2(s-3)(s+3)^2+4}$$

Zero = -1, -2 zero of T/F

$D(s)=0 \Rightarrow s=0, 0, 3, -3+2j, -3-2j$  pole



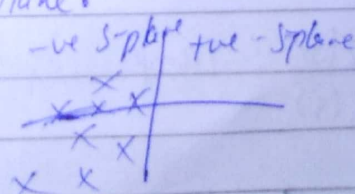
### Stability



For an bounded input if output ~~is~~ <sup>should be</sup> bounded then it is said to be stability.

Bounded I/P, Bounded Output,  
BIBO

\* For an stable system all poles of the T/F should lie in -ve half s-plane.



Eg:-

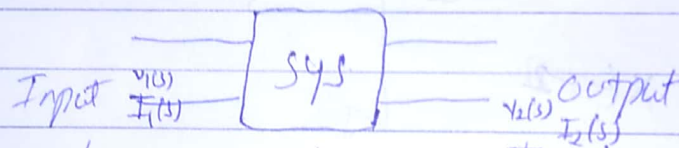
$$T(s) = \frac{1}{(s+1)(s+1)^2+4}(s-1)(s^2+4)$$

Apply Partial fraction

$$= \frac{k_1}{s+1} + \frac{k_2 s}{s^2+4} + \frac{k_3 s + k_4}{(s+1)^2+4} + \frac{k_5}{s-1}$$

Inverse Laplace

$$= k_1 e^{-t} + k_2 \cos 2t + k_3 e^{-t} \cos 2t + k_4 e^{-t} \sin 2t + k_5 e^t$$

21/10/19Driving point function and Transfer functionDriving Point function

ratio of same port quantity in freq. domain

$$\frac{V_1(s)}{I_1(s)}, \frac{I_1(s)}{V_1(s)}, \frac{V_2(s)}{I_2(s)}, \frac{I_2(s)}{V_2(s)}$$

|           |            |           |            |
|-----------|------------|-----------|------------|
| D.P I/P   | D.P I/P    | O/P       | O/P        |
| Impedance | admittance | impedance | admittance |

Transfer function

Output (freq. domain)

Input (freq. domain)

with all initial condition are zero.

$$\frac{V_2(s)}{V_1(s)} \text{ Transfer voltage gain}$$

$$\frac{I_2(s)}{I_1(s)} \text{ Transfer current gain}$$

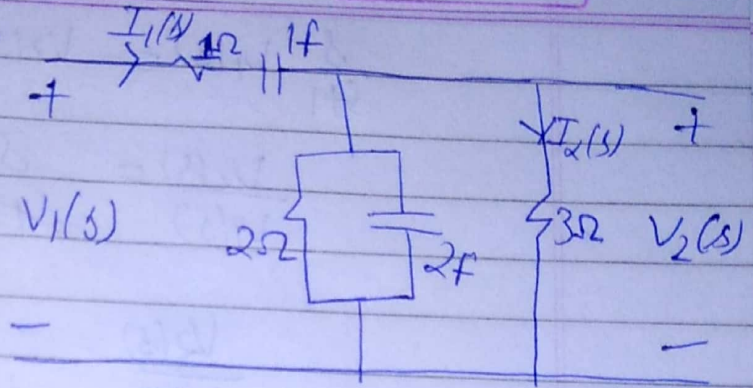
$$\frac{V_2(s)}{I_1(s)} \text{ Transfer impedance}$$

$$\frac{I_2(s)}{V_1(s)} \text{ Transfer admittance}$$

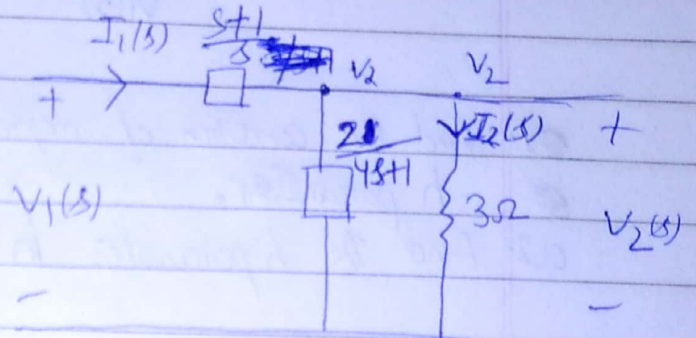
Transfer means initial current zero



Q Calculate the transfer voltage gain & current gain as shown in fig.

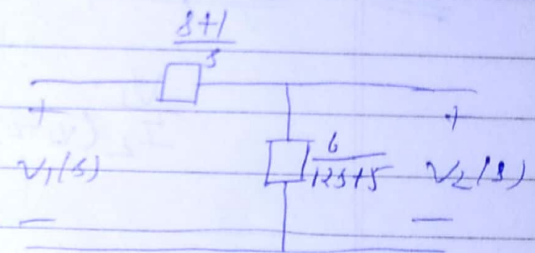


Sol  $I_2(s) = \frac{2}{4s+1} I_1(s)$



$$\frac{I_2(s)}{I_1(s)} = \frac{\frac{2}{4s+1}}{2 + \frac{12s}{4s+1}}$$

$$\frac{I_2(s)}{I_1(s)} = \frac{2}{12s+5}$$



$$\frac{V_1 - V_2}{\frac{s+1}{5}} + \frac{V_2}{\frac{2}{4s+1}} + \frac{V_2}{3} = 0$$

$$\frac{V_1(s) - V_2(s)}{\frac{s+1}{5}} = \frac{V_2(s)}{\frac{6}{12s+5}}$$

$$\frac{5 \times V_1(s)}{s+1} = \frac{V_2(s)}{\frac{6}{12s+5}} + \frac{V_2(s)}{\frac{s+1}{5}}$$

$$= \frac{6s}{(12s+5)s}$$

$$\frac{5 \times V_1(s)}{s+1} = \frac{12s+5}{6} V_2(s) + \frac{V_2(s)}{s+1} \times 5$$

$$\frac{5}{s+1} V_1(s) = \frac{12s+5}{6} V_2(s) + \frac{5s}{s+1} V_2(s)$$

$$\frac{s}{s+1} V_1(s) = V_2(s) \frac{[12s+5(s+1)+6s]}{6(s+1)}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{s[6(s+1)]}{s+1[12s+5(s+1)+6s]}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{6s}{12s+5(s+1)+6s} = \frac{6s}{13s+5}$$

- Q1 Find the condition of reciprocity & ~~symmetry~~ symmetry in h-parameter.
- Q2 Find the h-parameter in term of T-parameter

Soln Reciprocity

$$\frac{V_1}{I_2 (V_2=0)} = \frac{V_2}{I_1 (V_1=0)}$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

from eq (2)  $V_2=0$

$$I_2 = h_{21} I_1$$

$$\frac{I_1}{I_2} = \frac{1}{h_{21}}$$

$$V_1 = \frac{h_{11} I_2}{h_{21}} + h_{12} V_2$$

$$\frac{V_1}{I_2 (V_2=0)} = \frac{h_{11}}{h_{21}}$$

from eq (1)  $V_1=0$

$$h_{12} V_2 = -h_{11} I_1$$

$$\frac{V_2}{I_1} = -\frac{h_{11}}{h_{12}}$$



$$\frac{V_2}{I_1 (V_1=0)} = -\frac{h_{11}}{h_{12}}$$

$$\frac{V_1}{I_2} = \frac{V_2}{I_1}$$

$$\frac{h_{11}}{h_{21}} = -\frac{h_{11}}{h_{12}}$$

$$\boxed{h_{12} = -h_{21}}$$

Symmetry

$$\frac{V_1}{I_1 (I_2=0)} = \frac{V_2}{I_2 (I_1=0)}$$

from eq (2)  $I_1=0$

$$I_2 = h_{22} V_2$$

$$\frac{V_2}{I_2 (I_1=0)} = \frac{1}{h_{22}}$$

$$V_2 = \frac{-h_{21} I_1}{h_{22}}$$

$$V_1 = h_{11} I_1 + \frac{h_{12} (-h_{21}) I_1}{h_{22}}$$

$$V_1 = \left[ \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} \right] I_1$$

$$\frac{V_1}{I_1 (I_2=0)} = \frac{|h|}{h_{22}}$$

$$\frac{|h|}{h_{22}} = \frac{1}{h_{22}}$$

$$\boxed{|h|=1}$$

Sol (2)

$$V_1 = AV_2 - BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

$$(V_1, I_2) = f(I_1, V_2)$$

from eq (2)

$$DI_2 = -I_1 + CV_2$$

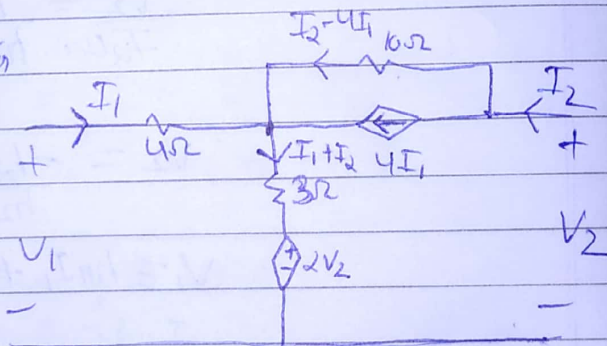
$$I_2 = \frac{-I_1}{D} + \frac{CV_2}{D}$$

$$V_1 = AV_2 - B \left[ \frac{-I_1}{D} + \frac{CV_2}{D} \right]$$

$$= AV_2 + \frac{BI_1}{D} - \frac{BCV_2}{D}$$

$$V_1 = \left( A - \frac{BC}{D} \right) V_2 + \frac{B}{D} I_1$$

Q Find the Y-parameter



$$V_1 = 4I_1 + 3I_1 + 3I_2 + 2V_2$$

$$V_1 = 7I_1 + 3I_2 + 2V_2$$

$$(I_1, I_2) = f(V_1, V_2)$$

$$+V_2 = 10(I_2 - 4I_1) + 3I_1 + 3I_2 + 2V_2$$

$$-V_2 = 13I_2 - 37I_1$$

$$I_2 = \frac{-V_2 + 37I_1}{13}$$



$$V_1 = 7I_1 + 3 \left[ \frac{-V_2 + 37I_1}{13} \right] + 2V_2$$

$$= 7I_1 + \frac{111I_1}{13} - \frac{3V_2}{13} + 2V_2$$

$$V_1 = I_1 \left[ 7 + \frac{111}{13} \right] - V_2 \left[ \frac{3}{13} - 2 \right]$$

~~$$I_1 = \frac{13V_1 + 7V_2}{202} \times \frac{23}{13}$$~~

$$\frac{202I_1}{13} = V_1 - \frac{23}{13} V_2$$

$$I_1 = \frac{13V_1}{202} - \frac{23}{202} V_2$$

Q KVL in ABCDA

~~$$2I(t) + 3 \frac{dI(t)}{dt} - 10 = 0$$~~

$$2I(t) + 3 \frac{dI(t)}{dt} = 10$$

$$2I(s) + 3[sI(s) - 1] = \frac{10}{s}$$

$$2I(s) + 3sI(s) = \frac{10}{s} + 3$$

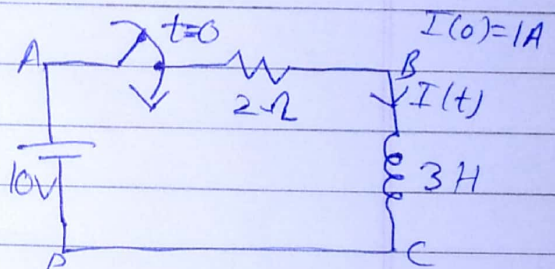
$$I(s) [2 + 3s] = \frac{10}{s} + 3$$

$$I(s) = \frac{10}{s(3s+2)} + \frac{3}{3s+2}$$

$$= \frac{10}{3} \frac{1}{s(3s+2)} + \frac{3}{3(s+\frac{2}{3})} = \frac{10}{3} \left[ \frac{A}{s} + \frac{B}{s+\frac{2}{3}} \right]$$

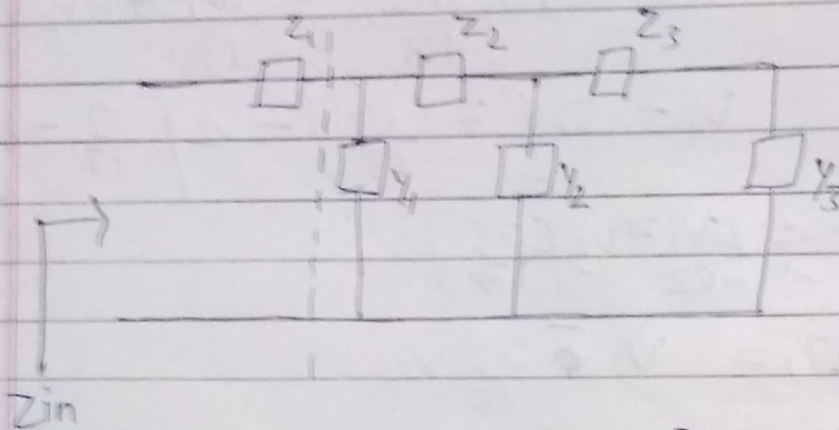
$$+ \frac{1}{s+\frac{2}{3}}$$

$$= 5 \left[ \frac{1}{s} - \frac{1}{s+\frac{2}{3}} \right] + \frac{1}{s+\frac{2}{3}}$$



08/11/19

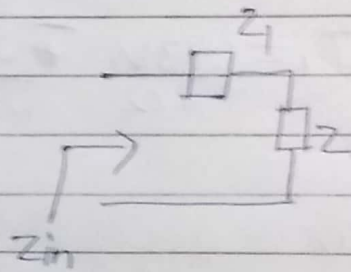
## Impedance or Ladder network



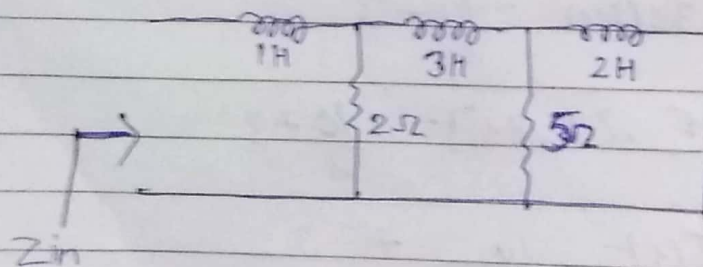
$$Z_{in} = Z_1 + Z$$

$$Z_{in} = Z_1 + \frac{1}{Y_1 + Y'}$$

$$= Z_1 + \frac{1}{Y_1 + \frac{1}{Z_2 + \frac{1}{Y_2 + \frac{1}{Z_3}}}}$$



Q Find the total impedance



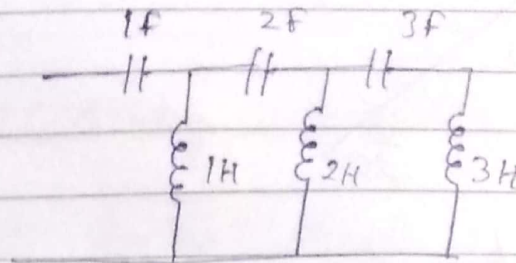
Ans

$$Z_{in} = 1 + \frac{1}{\frac{1}{2} + \frac{1}{3 + \frac{1}{\frac{1}{5} + \frac{1}{2}}}}$$



$$= \frac{8 + 2(12s^2 + 25s)}{6s^2 + 29s + 10}$$

$$= \frac{6s^3 + 29s^2 + 10s + 24s^2 + 50s}{6s^2 + 29s + 10} \Rightarrow \frac{6s^3 + 41s^2 + 60s}{6s^2 + 29s + 10}$$



80]

$$Z_{in} = \frac{1}{s} + \frac{1}{\frac{1}{s} + \frac{1}{\frac{1}{2s} + \frac{1}{\frac{1}{2s} + \frac{1}{\frac{1}{3s} + \frac{1}{3s}}}}}$$

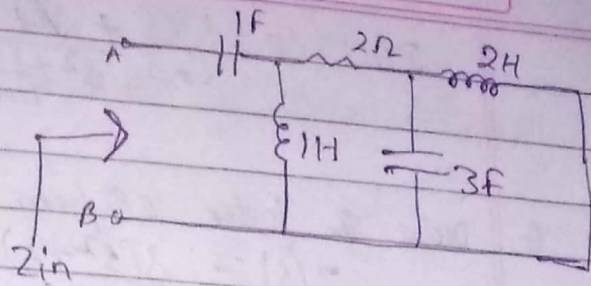
$$= \frac{1}{s} + \frac{1}{\frac{1}{s} + \frac{1}{\frac{1}{2s} + \frac{1}{\frac{2s + 3s}{2s \cdot 2}}}}$$

$$= \frac{1}{s} + \frac{1}{\frac{1}{s} + \frac{1}{\frac{1}{2s} + \frac{2}{4s + 3s}}} \Rightarrow \frac{1}{s} + \frac{1}{\frac{1}{s} + \frac{1}{\frac{1 + 2}{2s \cdot 7s}}}$$

$$= \frac{1}{s} + \frac{1}{\frac{1}{s} + \frac{14s^2}{11s}} \Rightarrow \frac{1}{s} + \frac{1}{\frac{11s + 14s^3}{11s^2}}$$

$$= \frac{1}{s} + \frac{11s^2}{11s + 14s^3}$$

$$Z_{in} = \frac{11s + 14s^3 + 11s^3}{s(11s + 14s^3)} = \frac{11s + 25s^3}{11s^2 + 14s^4}$$



### Property of LC-driving point function $Z(s)/Y(s)$

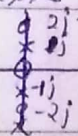
- (i) LC driving point function is the ratio of odd to even or even to odd term.

$$Z(s) = \frac{N(s)}{D(s)} = \frac{a_0 s^5 + a_1 s^3 + a_2 s}{b_0 s^4 + b_1 s^2 + b_2}$$

- (ii) All the coefficient in the polynomial of  $(s)$  are real and +ve.

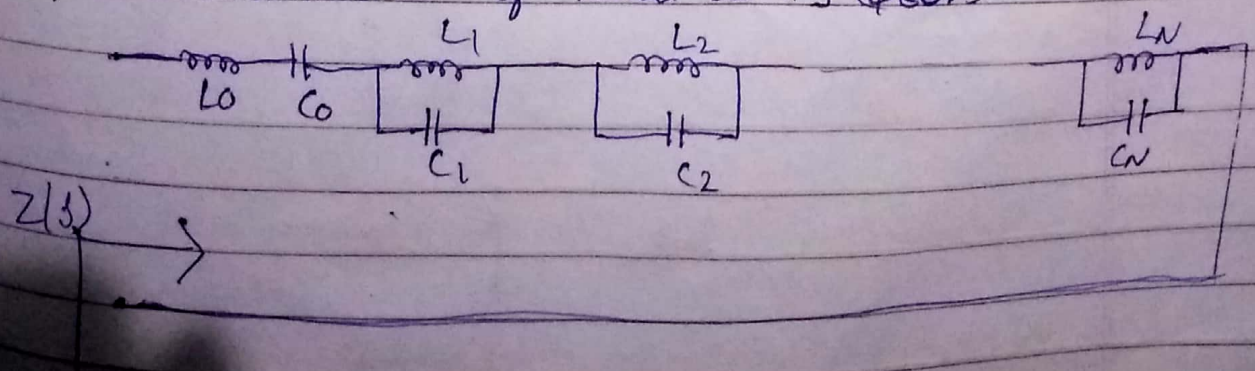
$$= \frac{2s^2 - 3}{s^3 + 3s}$$

- (iii) All the pole, zero, lie on the imaginary axis, in alternative form, in conjugate form.



- (iv) There must be a zero or pole at the origin.

### Foster I<sup>st</sup> form for LC driving point function [Series combination of Parallel element] (SCOP)





$$Z(s) = L_0 s + \frac{1}{C_0 s} + \frac{s/K_1}{s^2 + \frac{1}{L_1 C_1}} + \frac{s/K_2}{s^2 + \frac{1}{L_2 C_2}} + \dots$$

$$\frac{L_1 s \times \frac{1}{C_1 s}}{L_1 s + \frac{1}{C_1 s}} = \frac{8/C_1}{s^2 + 1/4 C_1}$$

Q Draw the Foster I<sup>st</sup> form for the function given by:-

$$Z(s) = \frac{s(s^2 + 2)}{(s^2 + 1)(s^2 + 3)}$$

$$= \frac{As}{(s^2 + 1)} + \frac{Bs}{s^2 + 3} \Rightarrow \frac{s/2}{s^2 + 1} + \frac{s/2}{s^2 + 3}$$

$$s^3 + 2s = s^3(A+B) + s(3A+B)$$

$$A+B=1$$

$$A=1/2$$

$$3A+B=2$$

$$B=1/2$$

$$L_0 \rightarrow X$$

$$C_0 \rightarrow X$$

$$\boxed{C_1 = 2F}$$

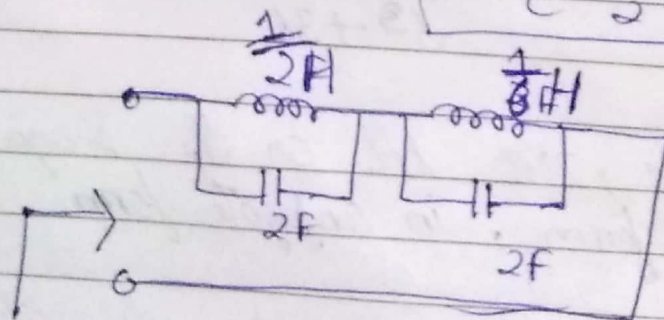
$$\frac{1}{L_1 C_1} = 1$$

$$\boxed{L_1 = \frac{1}{C_1} = \frac{1}{2} H}$$

$$\boxed{C_2 = 2F}$$

$$\frac{1}{L_2 C_2} = 3$$

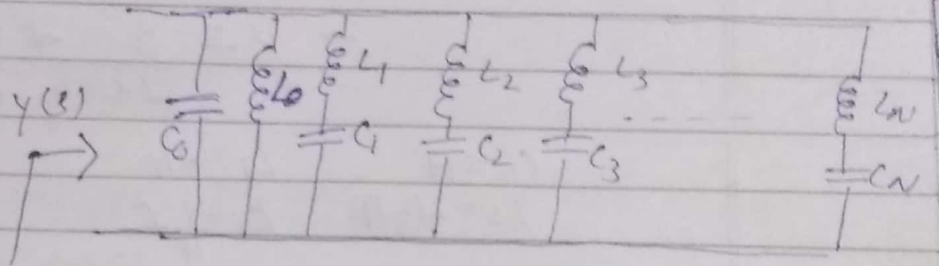
$$\boxed{L_2 = \frac{1}{3C_2} = \frac{1}{6} H}$$



Q  $Z(s) = \frac{(s^2 + 1)(s^2 + 3)}{s(s^2 + 2)}$

Foster's II<sup>nd</sup> form (PCSE)  $Y(s)$

Parallel combination of series element



$$Y(s) = C_0 s + \frac{1}{L_0 s}$$

$$Y(s) = \frac{1}{sL_1 + \frac{1}{C_1 s}} = \frac{C_1 s}{L_1 C_1 s^2 + 1} = \frac{s/L_1}{s^2 + 1/L_1 C_1}$$

$$Y(s) = C_0 s + \frac{1}{L_0 s} + \frac{s/L_1}{s^2 + \frac{1}{L_1 C_1}} + \frac{s/L_2}{s^2 + \frac{1}{L_2 C_2}} + \dots$$

Q Draw the Foster II<sup>nd</sup> form.

$$Z(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)}$$

Sol

$$Y(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)}$$

$$= s + \frac{2s^2+3}{s(s^2+2)}$$

$$\begin{array}{r} s^3+2s \sqrt{s^4+4s^2+3} \quad | \quad s \\ \underline{s^4+2s^2} \phantom{+3} \\ 2s^2+3 \end{array}$$

$$\frac{2s^2+3}{s(s^2+2)} = \frac{A}{s} + \frac{Bs}{s^2+2}$$

$$2s^2+3 = A s^2 + 2A + B s^2$$

$$2s^2+3 = s^2(A+B) + 2A$$



$$\frac{2s^2+3}{s(s^2+2)} = \frac{3/2}{s} + \frac{s/2}{s^2+2}$$

$$= s + \frac{3}{2s} + \frac{s/2}{s^2+2}$$

$$C_0 = 1F$$

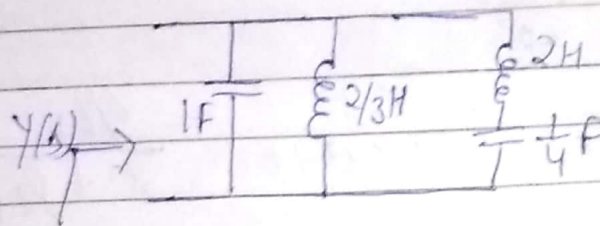
$$L_1 = 2H$$

$$\frac{1}{L_1 C_1} = 2$$

$$L_0 = 2/3 H$$

$$C_1 = \frac{1}{4} F$$

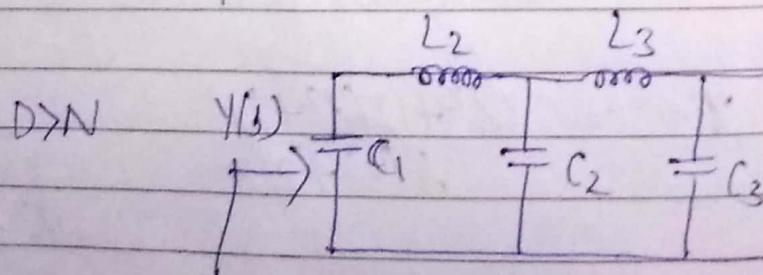
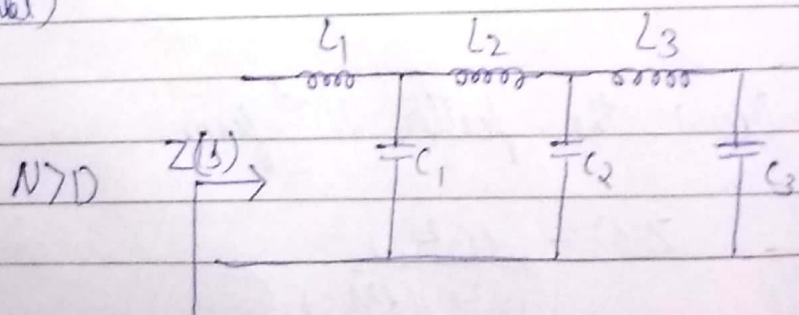
$$C_1 = \frac{1}{4} F$$



Cauer First form  $Z(s)$  or  $Y(s)$

(L-series)

(C-parallel)



$$Z(s) = L_1 s + \frac{1}{C_1 s + 1} \cdot \frac{L_2 s + 1}{C_2 s + 1} \cdot \frac{L_3 s + 1}{C_3 s + 1}$$

Q. For the  $Z(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)}$  Find Cauer  $T^{\text{st}}$  form

Sol  $Y(s) = \frac{s^4 + 4s^2 + 3}{s^3 + 2s}$

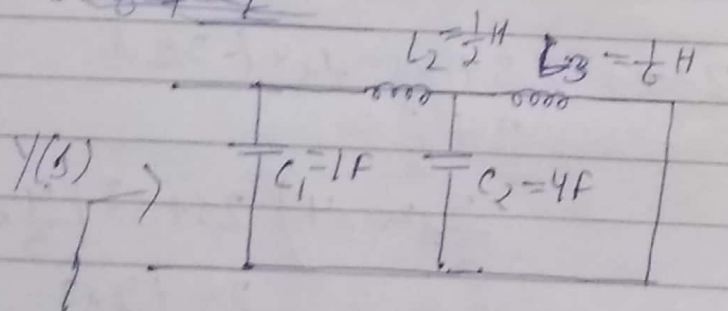
$$\begin{array}{r} s^3 + 2s \overline{) s^4 + 4s^2 + 3} \quad Y(s) \\ \underline{s^4 + 2s^2} \phantom{+ 3} \\ 2s^2 + 3 \end{array}$$

$$\begin{array}{r} 2s^2 + 3 \overline{) s^3 + 2s} \quad Z(s) \\ \underline{s^3 + 3s} \phantom{+ 3} \\ -\frac{1}{2}s \end{array}$$

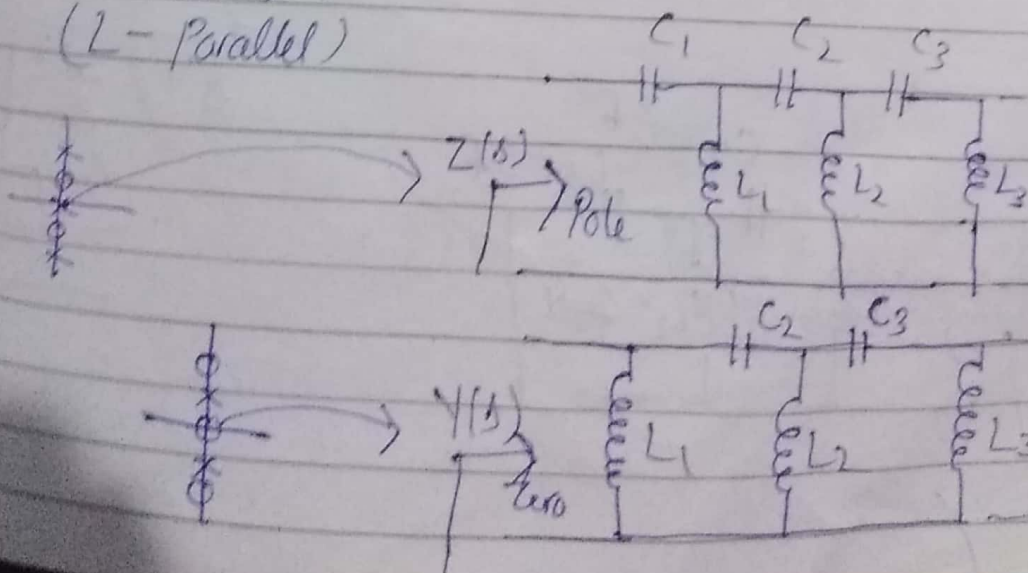
$$\begin{array}{r} -\frac{1}{2}s \overline{) 2s^2 + 3} \quad Y(s) \\ \underline{-s^2 - \frac{3}{2}} \\ 2s^2 \end{array}$$

$$\begin{array}{r} 2s^2 \overline{) s^3 + 2s} \quad Z(s) \\ \underline{s^3 + 3s} \phantom{+ 3} \\ -\frac{1}{2}s \end{array}$$

$$\begin{array}{r} -\frac{1}{2}s \overline{) -\frac{1}{2}s} \quad Y(s) \\ \underline{-\frac{1}{2}s} \\ 0 \end{array}$$



Cauer  $II^{\text{nd}}$  form  
(C-Series)  
(L-Parallel)





$$Z(s) = \frac{1}{C_1 s} + \frac{1}{\frac{1}{L_1 s} + \frac{1}{\frac{1}{C_2 s} + \frac{1}{L_2 s}}}$$

Q  $Z(s) = \frac{3(s^2+2)}{(s^2+1)(s^2+3)}$

Draw the Cauer II<sup>nd</sup> form

Sol  $Y(s) = \frac{s^4 + 4s^2 + 3}{s^3 + 2s}$

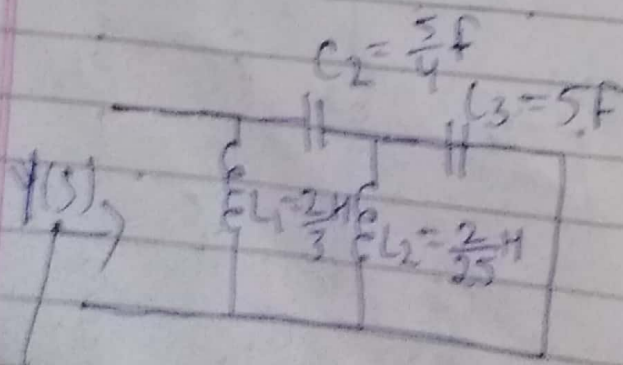
$$Y(s) = \frac{1}{L_1 s} + \frac{1}{\frac{1}{C_2 s} + \frac{1}{L_2 s} + \dots}$$

$$2s + s^3 \overline{) 3 + 4s^2 + s^4} \quad \begin{matrix} 3/2 s \\ 3 + 3/2 s^2 \end{matrix} \quad Y(s)$$

$$\frac{5}{2}s^2 + s^4 \overline{) 2s + s^3} \quad \begin{matrix} 4/5 s \\ 2s + 4/5 s^3 \end{matrix} \quad Z(s)$$

$$\frac{1}{5}s^3 \overline{) \frac{5}{2}s^2 + s^4} \quad \begin{matrix} 0.5/2 s \\ \frac{5}{2}s^2 \end{matrix} \quad Y(s)$$

$$s^4 \overline{) \frac{s^3}{5}} \quad \begin{matrix} 1/5 s \\ s^3/5 \end{matrix} \quad \frac{1}{5s}$$



$$Q \quad Z(s) = \frac{2(s^2 + 1/2)(s^2 + 1)}{s(s^2 + 3/2)}$$

### Tutorial

Q Draw the Foster I<sup>st</sup> & II<sup>nd</sup>, Cauer I<sup>st</sup> & II<sup>nd</sup> form from the function  $Z(s) = \frac{s(s^2 + 3/2)(s^2 + 7/2)}{(s^2 + 1/2)(s^2 + 5/2)}$

Sol Foster I<sup>st</sup> form

$$= \frac{s(s^4 + \frac{7}{2}s^2 + \frac{3}{2}s^2 + 5)}{s^4 + \frac{5}{2}s^2 + \frac{1}{2}s^2 + 3}$$

$$= \frac{s^5 + \frac{7}{2}s^3 + \frac{3}{2}s^3 + 5s}{s^4 + \frac{5}{2}s^2 + \frac{1}{2}s^2 + 3}$$

$$= \frac{2s^5 + 10s^3 + 10s}{2s^4 + 6s^2 + 6}$$

$$= \frac{s(s^4 + \frac{7}{2}s^2 + \frac{3}{2}s^2 + \frac{21}{4})}{s^4 + \frac{5}{2}s^2 + \frac{1}{2}s^2 + \frac{5}{4}}$$

$$s^4 + \frac{5}{2}s^2 + \frac{1}{2}s^2 + \frac{5}{4}$$

$$= \frac{s^5 + \frac{7}{2}s^3 + \frac{3}{2}s^3 + \frac{21}{4}s}{s^4 + \frac{6}{2}s^2 + \frac{5}{4}}$$

$$s^4 + \frac{6}{2}s^2 + \frac{5}{4}$$

$$Z(s) = \frac{4s^5 + 20s^3 + 21s}{4s^4 + 12s^2 + 5}$$



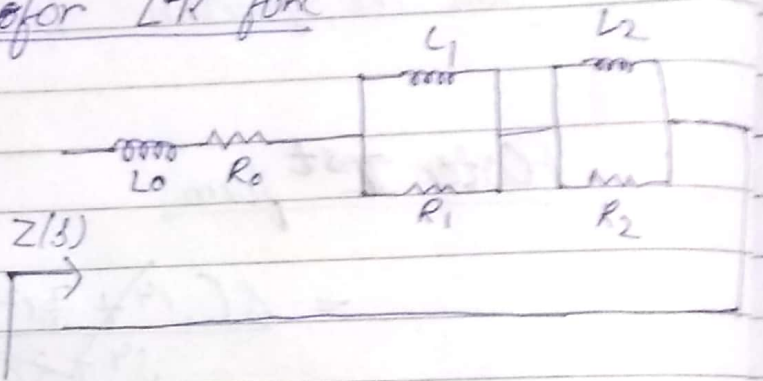
20/11/19

Property of L-R driving point impedance func.

- ① ~~Add~~ pole and zero lie in -ve real axis.
- ② In alternative form.
- ③ Nearest at the origin there must be a zero. [including zero]

Foster's I<sup>st</sup> form for L-R func<sup>n</sup>

$$Z(s) = L_0 s + R_0$$



$$= \frac{L_1 s \times R_1}{L_1 s + R_1} = \frac{s L_1}{s + R_1/L_1}$$

$$Z(s) = L_0 s + R_0 + \frac{s R_1}{s + R_1/L_1} + \frac{s R_2}{s + R_2/L_2} + \dots$$

\* For the foster I<sup>st</sup> form  $\frac{Z(s)}{s}$

Q.  $Z(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$ . Draw the foster's I<sup>st</sup> form.

Sol.  $\frac{Z(s)}{s} = \frac{(s+1)(s+3)}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$

$$A = 3/8$$

$$B = 1/4$$

$$C = 3/8$$

$$= \frac{3/8}{s} + \frac{1/4}{s+2} + \frac{3/8}{s+4}$$

$$Z(s) = \frac{3/8}{s} + \frac{1/4}{s+2} + \frac{3/8}{s+4}$$

$$Z(s) = \frac{1}{s} + \frac{R_1}{s+R_1/L_1} + \frac{R_2}{s+R_2/L_2}$$

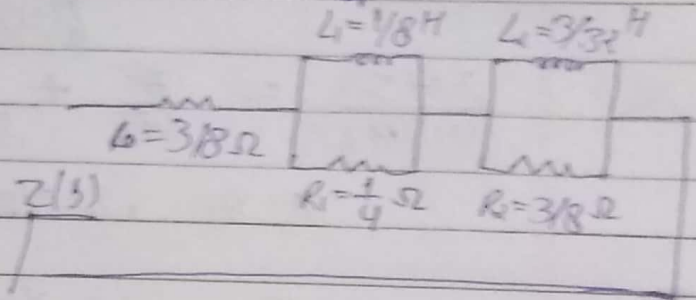
$$R_0 = 3/8 \Omega$$

$$R_1 = 1/4 \Omega$$

$$R_2 = 3/8 \Omega$$

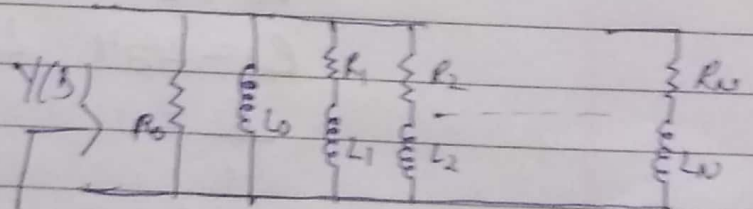
$$\frac{R_1}{L_1} = 2$$

$$L_1 = \frac{1}{8} H \quad L_2 = \frac{3}{32} H$$



Foster II<sup>nd</sup> form for L-R

$$Y(s) = \frac{1}{R_0} + \frac{1}{L_0 s}$$



$$= \frac{1}{R_1 + L_1 s} = \frac{1/L_1}{s + R_1/L_1}$$

$$Y(s) = \frac{1}{R_0} + \frac{1}{L_0 s} + \frac{1/L_1}{s + R_1/L_1} + \frac{1/L_2}{s + R_2/L_2}$$

10

Draw the foster II<sup>nd</sup> form for L-R function

$$Y(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}$$



Sol

$$\begin{array}{r} s^2 + 4s + 3 \overline{) s^2 + 6s + 8} \quad 1 \\ \underline{s^2 + 4s + 3} \phantom{0} \\ 2s + 5 \end{array}$$

$$= 1 + \frac{2s+5}{(s+1)(s+3)}$$

$$\frac{2s+5}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$$

$$A = 3/2, B = 1/2$$

$$1 + \frac{2s+5}{(s+1)(s+3)} = 1 + \frac{3/2}{s+1} + \frac{1/2}{s+3}$$

$$Y(s) = \frac{1}{R_0} + \frac{1}{L_0 s} + \frac{1/L_1}{s + \frac{R_1}{L_1}} + \frac{1/L_2}{s + \frac{R_2}{L_2}}$$

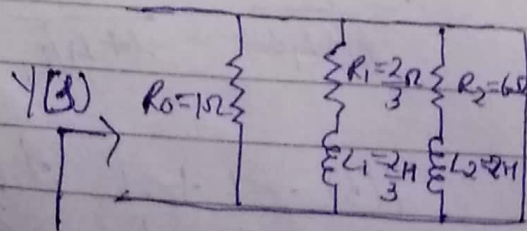
$$R_0 = 1 \Omega$$

$$L_1 = 2/3 H, L_2 = 2 H$$

$$\frac{R_1}{L_1} = 1$$

$$\frac{R_2}{L_2} = 3 \Rightarrow R_2 = 6 \Omega$$

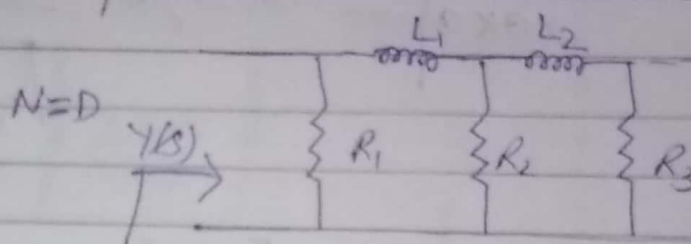
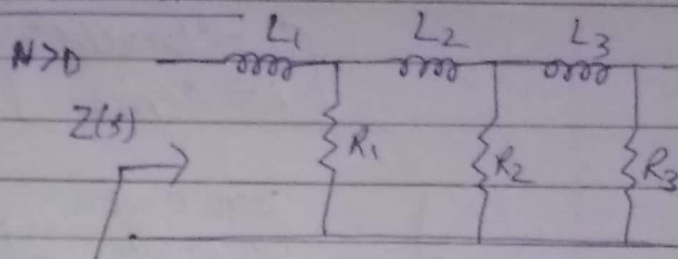
$$R_1 = \frac{2}{3} \Omega$$



22/11/19

Cover I<sup>st</sup> form for L-R Ckt

[L-series  
R-Parallel]



$$Z(s) = \frac{L_1 s + 1}{\frac{1}{R_1} + \frac{1}{L_2 s + 1} + \frac{1}{R_2} + \dots}$$

Q Draw the Cover 2<sup>nd</sup> form  $Z(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$

Sol

$$Y(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)} \Rightarrow \frac{s^2 + 4s + 3}{s^2 + 6s + 8}$$

$$\frac{s^2 + 4s + 3}{s^2 + 6s + 8} \left( 1 \rightarrow Y(s) \right)$$

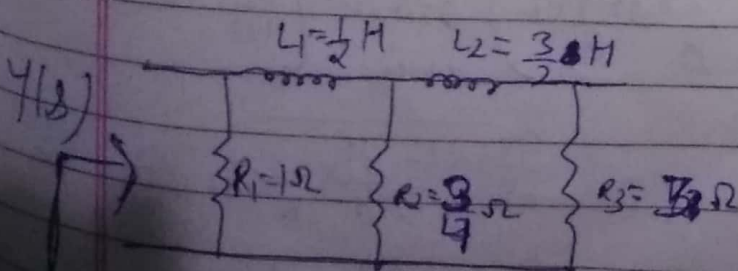
$$\frac{2s+5}{s^2+6s+8} \left( \frac{s^2+4s+3}{s^2+5s} \right) \left( \frac{s}{2} \right)$$

$$\frac{3s+3}{2} \left( \frac{2s+5}{2s+4} \right) \left( \frac{4}{3} \right)$$

$$\frac{2}{2} \left( \frac{3s+3}{2} \right) \left( \frac{3}{2} \right)$$

$$\frac{3s+3}{2} \left( \frac{3}{2} \right)$$

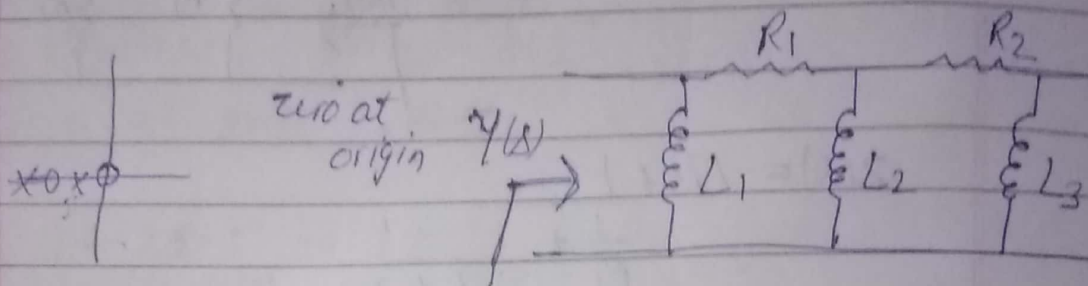
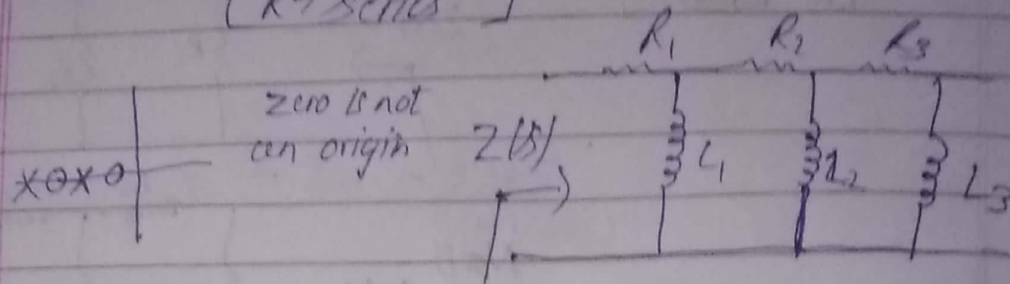
$$\frac{3}{2} \left( \frac{3}{2} \right)$$





Cover II<sup>nd</sup> form for L-R

$\left[ \begin{array}{l} L \rightarrow \text{Parallel} \\ R \rightarrow \text{Series} \end{array} \right]$



$$Z(s) = R_1 + \frac{1}{\frac{1}{L_1 s} + \frac{1}{R_2 + \frac{1}{\frac{1}{L_2 s} + \frac{1}{R_3 + \dots}}}}$$

Q. Draw the Cover II<sup>nd</sup> form  $Z(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$

$$\frac{8+6s+s^2}{3+4s+s^2} \left( \frac{3/8}{s+2} \right) \quad Z(s)$$

$$\frac{14s+5s^2}{8} \left( \frac{64}{14s} \right) \quad Y(s)$$

$$\frac{44s+32}{14} \left( \frac{14s+5s^2}{8} \right) \left( \frac{14 \times 14}{44 \times 8} \right) \quad Z(s)$$

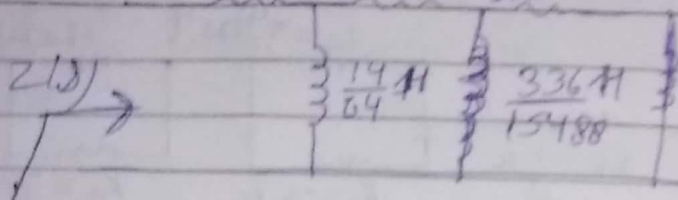
$$\frac{192s^2}{8 \times 352} \left( \frac{44s+32}{14} \right) \left( \frac{352 \times 44}{24 \times 14} \right)$$

$$82 \overline{) \begin{array}{r} 24 \\ 192 \\ \hline 52 \end{array}} \quad \begin{array}{r} 24 \\ 352 \\ \hline 218 \end{array}$$

$$\begin{array}{r} 24 \\ 352 \\ \hline \end{array}$$

X

$$R_1 = 318 \Omega, R_2 = \frac{196}{352} \Omega, R_3 = \frac{24}{352} \Omega$$



$$Z(s) = \frac{s(2s+5)}{(s+\frac{1}{2})(s+7)}$$

Tutorial

Q

$$Z(s) = \frac{(s+\frac{1}{2})(3s+7)}{(s+\frac{5}{2})(4s+13)}$$

Draw Foster I<sup>st</sup>, II<sup>nd</sup>, Cauer I<sup>st</sup> & II<sup>nd</sup> for L-R ckt.

Sol

Foster I<sup>st</sup> form

$$\frac{Z(s)}{s} = \frac{(s+\frac{1}{2})(3s+7)}{s(s+\frac{5}{2})(4s+13)} = \frac{A}{s} + \frac{B}{s+\frac{5}{2}} + \frac{C}{4s+13}$$

$$(s+\frac{1}{2})(3s+7) = A(s+\frac{5}{2})(4s+13) + Bs(4s+13) + C(s+\frac{1}{2})s$$

$$A = \frac{7}{65}, B = -\frac{2}{15}, C = \frac{121}{39}$$

$$= \frac{7/65}{s} + \frac{-2/15}{s+\frac{5}{2}} + \frac{121/39}{4s+13}$$

$$Z(s) = \frac{7}{65} + \frac{-2/15 s}{s+\frac{5}{2}} + \frac{121/39 s}{4s+13}$$



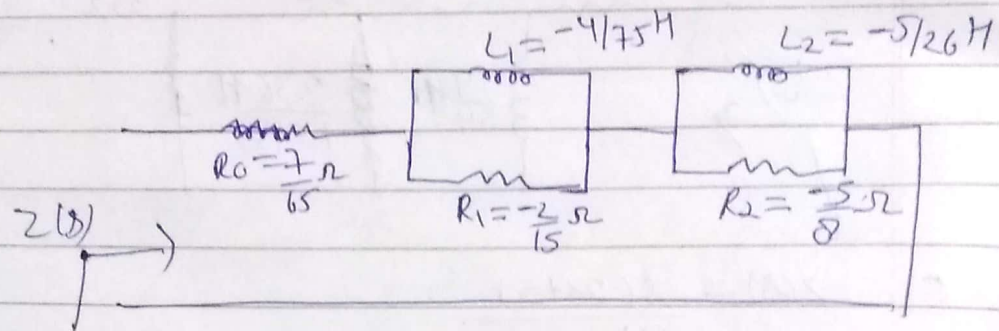
$$R_0 = \frac{7}{65} \Omega$$

$$\frac{R_1}{L_1} = \frac{5}{2}$$

$$R_1 = -\frac{2}{15} \Omega$$

$$L_1 = \frac{2R_1}{5} = \frac{-2 \times 2}{15 \times 5} = -\frac{4}{75} \text{ H}$$

$$R_2 = -\frac{5}{8} \Omega \quad R_3 = \frac{121}{304} \Omega \quad L_2 = -\frac{5}{26} \text{ H}$$



Foster II<sup>nd</sup> form

$$Y(s) = \frac{(s + 5/2)(4s + 13)}{(s + 1/2)(3s + 7)} = \frac{8s^2 + 46s + 65}{6s^2 + 17s + 7}$$

$$\frac{6s^2 + 17s + 7}{6s^2 + 17s + 7} \cdot \frac{8s^2 + 46s + 65}{6s^2 + 17s + 7} \left( \frac{8}{6} \right)$$

$$\frac{70s + 167}{3}$$

$$Y(s) = \frac{8}{6} + \frac{70/3s + 167/3}{(s + 1/2)(3s + 7)}$$

$$\frac{70/3s + 167/3}{(s + 1/2)(3s + 7)} = \frac{A}{s + 1/2} + \frac{B}{3s + 7}$$

$$A = 8, \quad B = -2/3$$

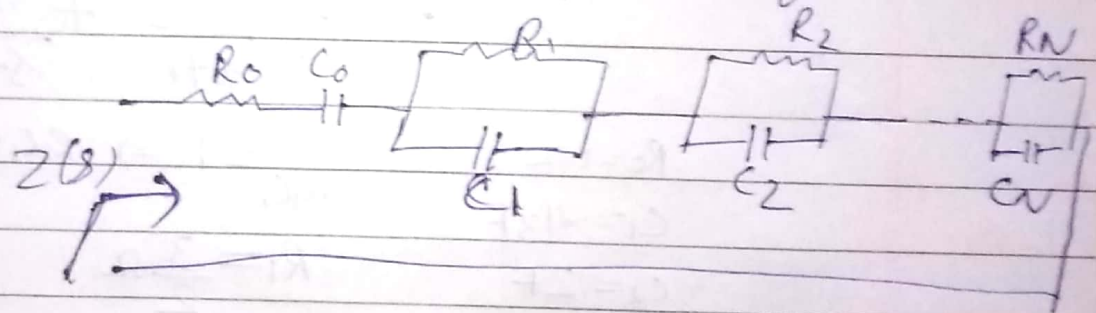
$$Y(s) = \frac{8}{6} + \frac{8}{s+1} + \frac{-4/3}{3s+7}$$

## 25/11/19 Property of RC impedance function

- 1) Property of RC impedance function same as RL impedance function except nearest at the origin there must be a pole.

~~OXOX~~

### Foster I<sup>st</sup> form for RC impedance function



$$Z(s) = R_0 + \frac{1}{Cs}$$

$$Z(s) = \frac{R_1 \times \frac{1}{C_1 s}}{\frac{R_1 + 1}{C_1 s}} = \frac{\frac{1}{C_1}}{s + \frac{1}{R_1 C_1}}$$

$$Z(s) = R_0 + \frac{1}{Cs} + \frac{\frac{1}{C_1}}{s + \frac{1}{R_1 C_1}} + \frac{\frac{1}{C_2}}{s + \frac{1}{R_2 C_2}} + \dots$$

Partial fraction in RC function is done for  $Z(s)$ .



$$Z(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}$$

$$s^2 + 4s + 3 \overline{) s^2 + 6s + 8} \quad 1$$

$$\underline{2s + 5}$$

$$\frac{(s+2)(s+4)}{(s+1)(s+3)} = 1 + \frac{(2s+5)}{(s+1)(s+3)}$$

$$1 + \frac{2s+5}{(s+1)(s+3)} = 1 + \frac{A}{s+1} + \frac{B}{s+3}$$

$$= 1 + \frac{3/2}{s+1} + \frac{1/2}{s+3}$$

$$R_0 = 1\Omega$$

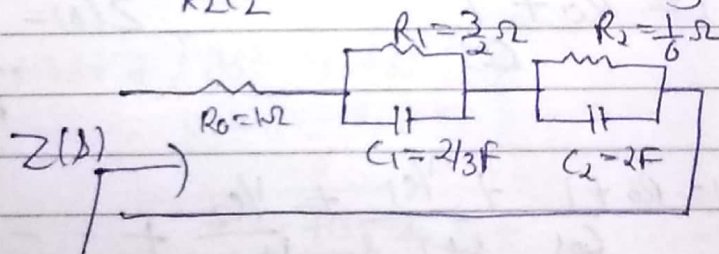
$$C_1 = 2/3 F$$

$$C_2 = 2 F$$

$$\frac{1}{R_1 C_1} = 1 \Rightarrow R_1 C_1 = 1$$

$$R_1 = \frac{3}{2} \Omega$$

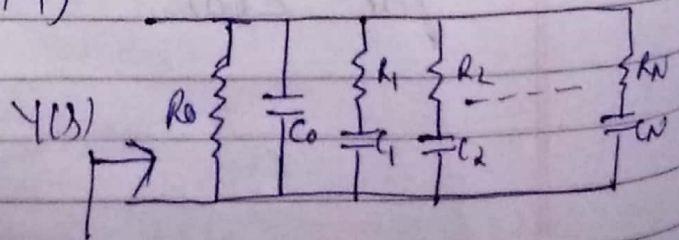
$$\frac{1}{R_2 C_2} = 3 \Rightarrow R_2 C_2 = \frac{1}{3} \Rightarrow R_2 = \frac{1}{6} \Omega$$



Foster II<sup>nd</sup> form

$$Y(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

$$Y(s) = \frac{1}{R_0} + Gs$$



$$Y(s) = \frac{1}{R_0} + Gs + \frac{s/R_1}{s + 1/R_1 C_1} + \frac{s/R_2}{s + 1/R_2 C_2}$$

Partial fraction in Foster II<sup>nd</sup> form is done for  $\frac{Y(s)}{s}$

$$\frac{Y(s)}{s} = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$$

$$\frac{(s+1)(s+3)}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$A = 3/8, B = 1/4, C = 3/8$$

$$\frac{Y(s)}{s} = \frac{3/8}{s} + \frac{1/4}{s+2} + \frac{3/8}{s+4}$$

$$\begin{aligned} Y(s) &= \frac{3/8 s}{s} + \frac{1/4 s}{s+2} + \frac{3/8 s}{s+4} \\ &= \frac{3}{8} + \frac{1/4 s}{s+2} + \frac{3/8 s}{s+4} \end{aligned}$$

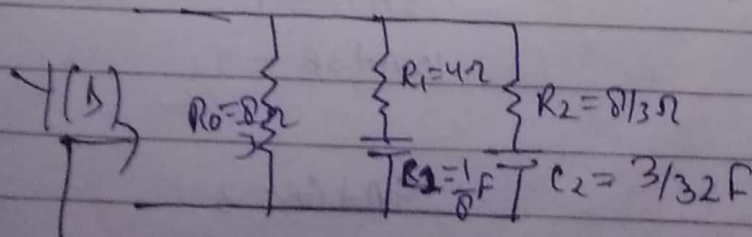
$$R_0 = 8/3 \Omega$$

$$R_1 = 4 \Omega$$

$$R_2 = 8/3 \Omega$$

$$R_1 C_1 = \frac{1}{2} \Rightarrow C_1 = \frac{1}{8} F$$

$$R_2 C_2 = \frac{1}{4} \Rightarrow C_2 = \frac{3}{32} F$$





$$Q \quad Z(s) = \frac{1}{8} \frac{(2s+3)(s+6)}{(s+5/2)}$$

sol) Foster 1st form

$$\begin{aligned} \frac{(2s+3)(s+6)}{s(s+5/2)} &= 2 + \frac{20s+36}{(2s^2+5s)} \\ &= 2 + \frac{10s+18}{s(s+5/2)} \end{aligned}$$

### Tutorial

$$Q \quad Z(s) = \frac{8(s^2+5/2)}{(2s^2+3)(3s^2+10)}$$

sol) Foster 1st form for LC

$$Z(s) = \frac{s(s^2+5/2)}{(2s^2+3)(3s^2+10)} = \frac{As}{2s^2+3} + \frac{Bs}{3s^2+10}$$

$$s(s^2+5/2) = As(3s^2+10) + Bs(2s^2+3)$$

$$s^3 + \frac{5}{2}s = 3As^3 + 10As + 2Bs^3 + 3Bs$$

$$\frac{s^3 + 5s}{2} = s^3(3A+2B) + s(10A+3B)$$

$$3A+2B=1 \quad \times 3$$

$$10A+3B=5/2 \quad \times 2$$

$$9A+6B=3$$

$$20A+6B=5$$

$$11A=2$$

$$A = \frac{2}{11}$$

$$3 \times \frac{2}{11} + 2B = 1$$

$$2B = 1 - \frac{6}{11} = \frac{5}{11} \Rightarrow B = \frac{5}{22}$$

$$= \frac{2s}{s^2+3} + \frac{5s}{s^2+10}$$

$$= \frac{5/11}{s^2+5/2} + \frac{53/66}{s^2+10/3}$$

$$C_1 = 11F$$

$$C_2 = \frac{66}{5}F$$

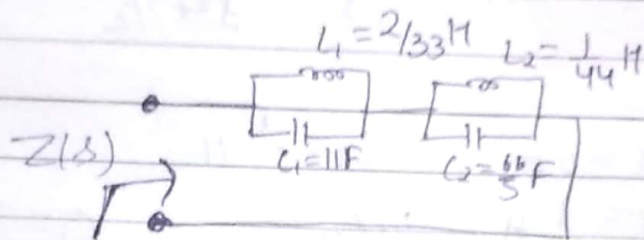
$$L_1 C_1 = \frac{2}{3}$$

$$L_1 = \frac{2}{33}H$$

$$L_2 C_2 = \frac{3}{10}$$

$$L_2 = \frac{3 \times 5}{10 \times 66} = \frac{1}{44}H$$

$$L_2 = \frac{1}{44}H$$



Foster II<sup>nd</sup> form

$$Y(s) = \frac{(2s^2+3)(3s^2+10)}{8(s^2+5/2)} \Rightarrow \frac{6s^4+29s^2+30}{s^3+\frac{5}{2}s}$$

$$= \frac{12s^4+58s^2+60}{2s^3+5s}$$

$$2s^3+5s \overline{) 12s^4+58s^2+60} \quad 6s$$

$$12s^4+30s^2$$

$$\underline{28s^2+60}$$

$$\frac{6s^4+29s^2+30}{s^3+5/2s} = 6s + \frac{28s^2+60}{2s^3+5s}$$

$$= 6s + \frac{14s^2+30}{8(s^2+5/2)}$$

$$\frac{14s^2+30}{8(s^2+5/2)} = \frac{A}{8} + \frac{Bs}{s^2+5/2}$$

$$14s^2+30 = A(s^2+5/2) + Bs^2$$

$$= As^2 + \frac{5A}{2} + Bs^2$$



$$14s^2 + 30 = s^2(A+B) + \frac{5}{s}A$$

$$A+B=14$$

$$\frac{5}{s}A = 30 \Rightarrow A = 12$$

$$B = 2$$

$$Y(s) = 6s + \frac{12}{s} + \frac{2s}{s^2 + 5/2}$$

$$C_0 = 6F \quad L_1 = \frac{1}{2}H$$

$$L_0 = \frac{1}{12}H \quad L_1 C_2 = \frac{2}{5} \quad C_1 = \frac{4}{5}F$$

27/11/19

R → Series

C → Parallel

Cauer I<sup>st</sup> form for RC function

$$Z(s) = R_1 + \frac{1}{C_1 s + \frac{1}{R_2 + \frac{1}{C_2 s + \frac{1}{R_3 + \frac{1}{C_3 s}}}}}$$

$$C_1 s + \frac{1}{R_2 + \frac{1}{C_2 s + \frac{1}{R_3 + \frac{1}{C_3 s}}}}$$

$$R_2 + \frac{1}{C_2 s + \frac{1}{R_3 + \frac{1}{C_3 s}}}$$

$$C_2 s + \frac{1}{R_3 + \frac{1}{C_3 s}}$$

N=D

Const.

Z(s)

s → ∞

D &gt; N

⊕

When origin has pole

$$Z(s) = \frac{(2s+3)(3s+6)}{s(s+5/2)}$$

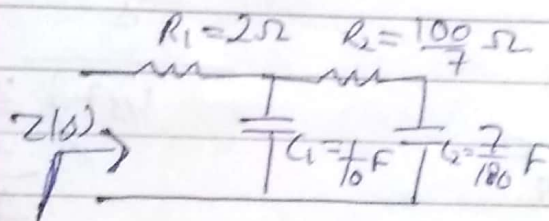
$$Z(s) = \frac{2s^2 + 15s + 18}{s^2 + 5s}$$

$$\begin{array}{r} 2s^2 + 5s \overline{) 4s^2 + 30s + 36} \quad (2 \rightarrow Z(s)) \\ \underline{4s^2 + 10s} \phantom{+ 36} \\ 20s + 36 \end{array}$$

$$\begin{array}{r} 20s + 36 \overline{) 2s^2 + 5s} \quad (10 \rightarrow 10) \\ \underline{20s^2 + 36s} \phantom{+ 36} \\ 10s \phantom{+ 36} \end{array}$$

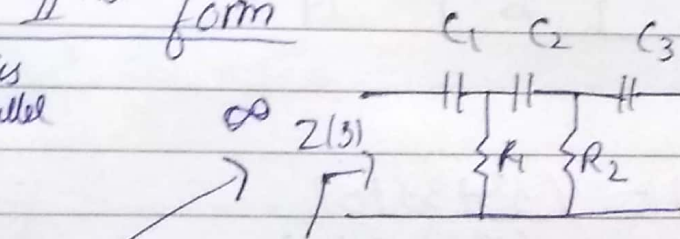
$$\begin{array}{r} 10s \overline{) 20s + 36} \quad (2 \rightarrow 2) \\ \underline{20s} \phantom{+ 36} \\ 36 \end{array}$$

$$\begin{array}{r} 36 \overline{) 36} \quad (1 \rightarrow 1) \\ \underline{36} \\ 0 \end{array}$$

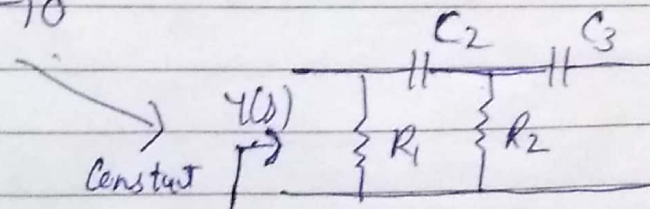


Cover II<sup>nd</sup> form

C → Series  
R → Parallel



$s \rightarrow 0$



$$Z(s) = \frac{1}{C_1 s} + \frac{1}{\frac{1}{R_1} + \frac{1}{\frac{1}{C_2 s} + \frac{1}{\frac{1}{R_2} + \dots}}}$$



Ex:  $Z(s) = \frac{(2s+3)(s+6)}{s(s+\frac{5}{2})}$

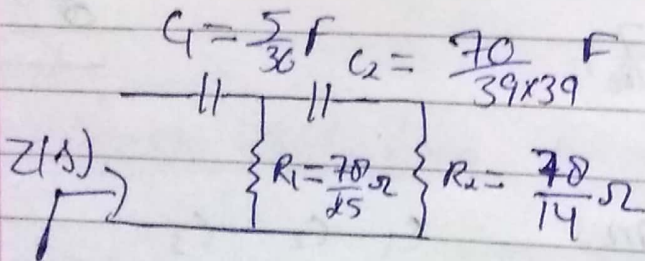
Sol

$$\frac{5s+2s^2}{36+30s+4s^2} \left( \frac{36}{5s} \right)$$

$$\frac{78s+4s^2}{5} \left( \frac{25}{78} \right)$$

$$\frac{28s^2}{39} \left( \frac{78s+4s^2}{5} \right) \left( \frac{39 \times 78}{28 \times 5} \right)$$

$$\frac{28s^2}{39} \left( \frac{28s^2}{39} \right) \left( \frac{28}{4 \times 39} \right)$$



Q  $Z(s) = \frac{(s^2+3s+5)}{(s^3+4s^2+9s)}$

29/11/19

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## Filter

Filter is a ckt which pass a range of freq. of desired signal and stop other freq. of undesired signal.

- Low pass filter
- High pass filter
- Band pass filter
- Band ~~pass~~ Reject filter

### ① Low Pass filter

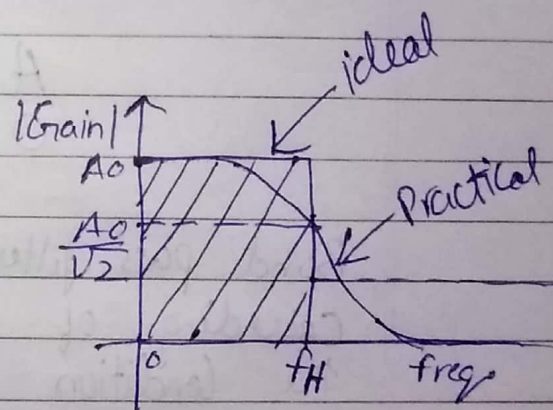
Which pass the low freq. signal and stop the high freq. signal

At ideally

$$A_0 \Rightarrow 0 < f < f_H$$

$$0 \quad f > f_H$$

Cut  
 $f_H =$  cut-off freq. of low pass filter  
 $BW = 0 - f_H$



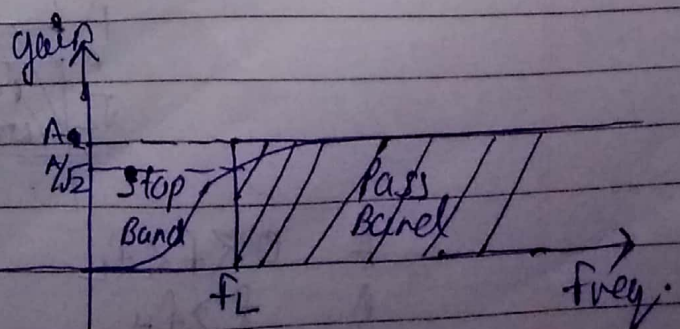
BW → The range of freq. at which gain of the system becomes  $\frac{1}{\sqrt{2}}$  of its maxm value.

### ② High Pass filter

~~$$A_0 \Rightarrow 0 < f < f_L$$~~

$$A_0 \quad 0 < f < f_L$$

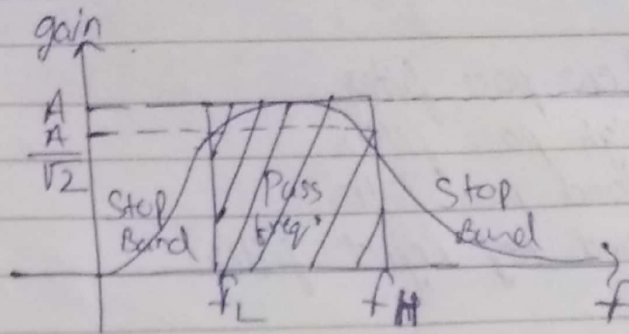
$$0 \quad f > f_L$$



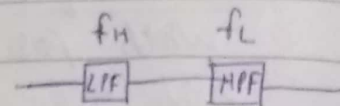


$f_L$  = lowest cut-off freq. at high pass filter.

### ③ Band Pass filter



$$BW = f_H - f_L$$

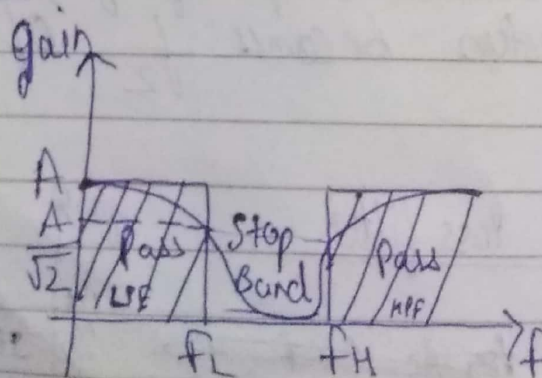


$$A \quad f_L < f < f_H$$

$$0 \quad \text{otherwise}$$

Band pass filter when we design with the cascading of low pass & high pass filter but the condition is highest freq. of low pass filter should be greater than lowest freq. of high pass filter.  $[f_H > f_L]$

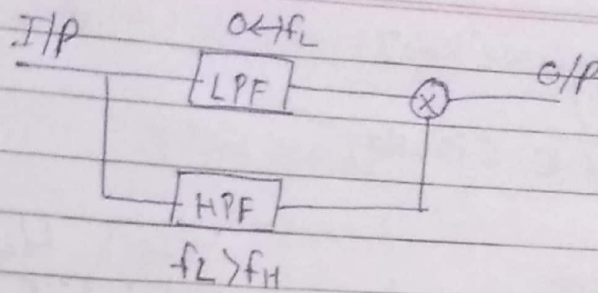
### ④ Band Reject Filter



$$A \quad 0 < f < f_L$$

$$A \quad f > f_H$$

$$0 \quad f_L < f < f_H$$



Filter can be divided into two group

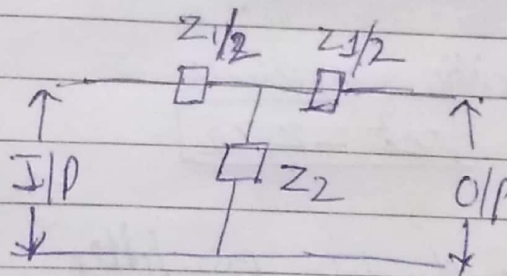
- 1) Active filter
- 2) Passive filter

### Passive Filter Design

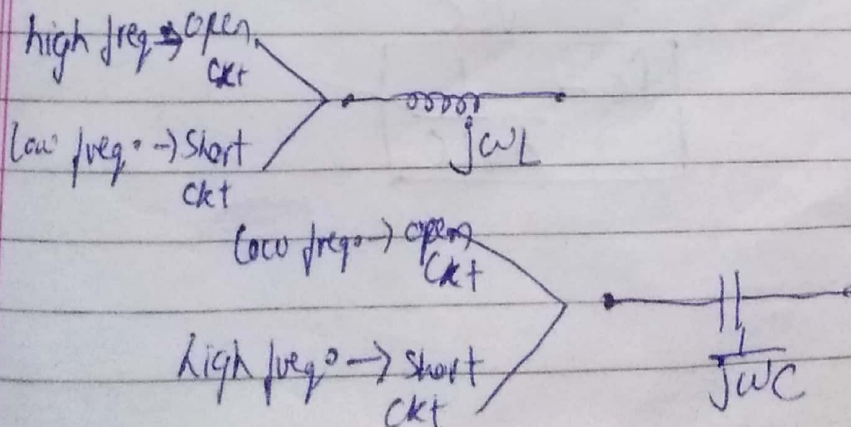
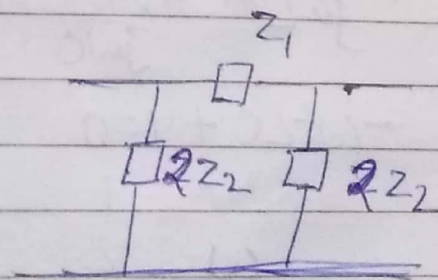
#### Low pass filter

Two standard form is used to design a filter

(i) T-form



(ii)  $\pi$ -form





$$Z_1 = j\omega L \text{ (series)}$$

$$Z_2 = \frac{1}{j\omega C} \text{ (parallel)}$$

$$\frac{Z_1}{2} = j\omega \frac{L}{2}$$

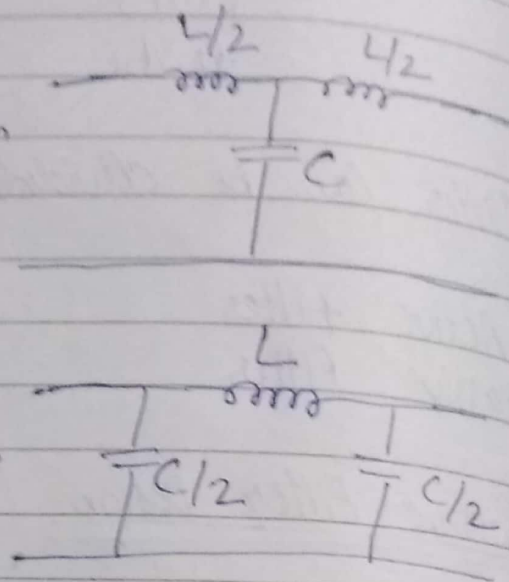
$$2Z_2 = \frac{2}{j\omega C}$$

$$= \frac{2}{j\omega C}$$

low pass

T-join

$\pi$ -form



Cut-off freq:

$$Z_1 + 4Z_2 = 0$$

Characteristic impedance

$$Z_0^2 = Z_1 Z_2$$

Cut-off freq of low pass filter:

$$j\omega L + 4 \times \frac{1}{j\omega C} = 0$$

$$-\omega^2 LC + 4 = 0$$

$$\omega^2 LC = 4$$

$$\omega = \frac{2}{\sqrt{LC}}$$

$$f_c = \frac{1}{\pi \sqrt{LC}}$$

Characteristic impedance of low pass filter

$$Z_0^2 = j\omega L \times \frac{1}{j\omega C}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

$$f_c^2 = \frac{1}{\pi^2 LC}$$

$$L = \frac{1}{\pi^2 f_c^2 C}$$

$$Z_0^2 = \frac{1}{\pi^2 f_c^2 C}$$

$$\boxed{C = \frac{1}{Z_0 \pi f_c}}$$

$$\boxed{L = \frac{Z_0}{\pi f_c}}$$

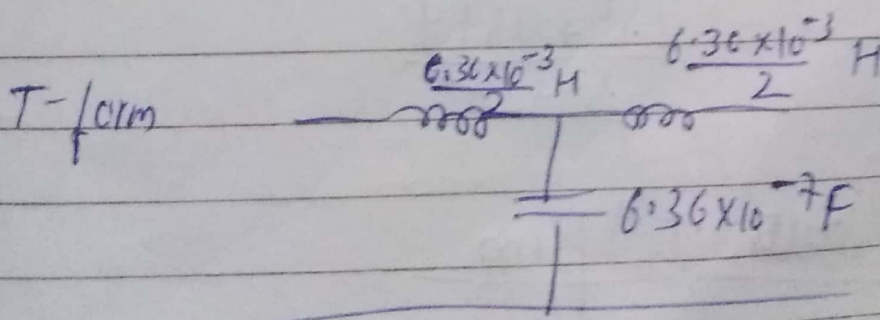
Q Design a low pass filter for a cut-off freq. 5 kHz and characteristic impedance of 100  $\Omega$ .

Sol  $L = \frac{Z_0}{\pi f_c} = \frac{100}{3.14 \times 5 \times 10^3} = \frac{1}{50 \times 3.14}$

$$L = 6.36 \times 10^{-3} \text{ H}$$

$$C = \frac{1}{Z_0 \pi f_c} = \frac{1}{100 \times 3.14 \times 5 \times 10^3} = \frac{1}{5 \times 10^5 \times 3.14}$$

$$C = 6.36 \times 10^{-7} \text{ F}$$





03/12/19

Q1 Design a low pass filter for a cut off freq 5 kHz and characteristic impedance  $3k\Omega$ .

Q2 Draw the Cauer II & I form for  $Z(s) = \frac{(2s+1)(s+4)}{s(s+2)}$

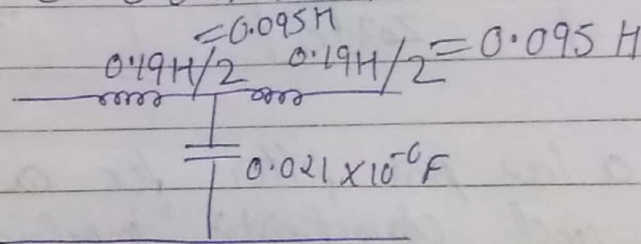
Sol ①  $f = 5 \times 10^3 \text{ Hz}$ ,  $Z_0 = 3 \times 10^3 \Omega$

$$L = \frac{Z_0}{\pi f_c} = \frac{3 \times 10^3}{3.14 \times 5 \times 10^3}$$

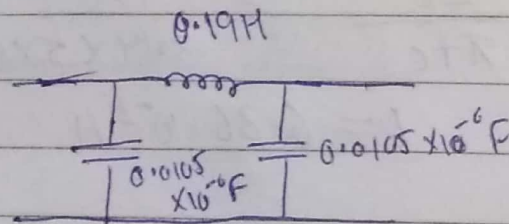
$$L = 0.19 \text{ H}$$

$$C = \frac{1}{Z_0 \pi f_c} = \frac{1}{3 \times 10^3 \times 3.14 \times 5 \times 10^3}$$

$$C = 0.021 \times 10^{-6} \text{ F}$$



$\pi$ -form



Sol ②  $Z(s) = \frac{(2s+1)(s+4)}{s(s+2)}$  RC

$$= \frac{2s^2 + 9s + 4}{s^2 + 2s}$$

Cauer I<sup>st</sup> form

# Assignment → Positive Real function (PRF)

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$$\begin{array}{r} s^2 + 2s \\ 2s^2 + 9s + 4 \end{array} \left( \frac{2}{s} \rightarrow 20 \right)$$

$$2s = 4s$$

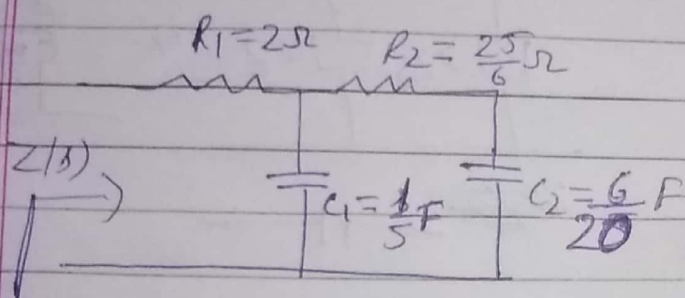
$$10s = 4s$$

$$\begin{array}{r} 5s + 4 \\ s^2 + 2s \\ 2s^2 + 9s + 4 \end{array} \left( \frac{s}{5} \rightarrow 14 \right)$$

$$\begin{array}{r} 6s \\ 5s + 4 \\ 5s + 4 \end{array} \left( \frac{25}{6} \rightarrow 12 \right)$$

$$14 \rightarrow 14$$

$$\begin{array}{r} 4 \\ 6s \\ 5s + 4 \\ 5s + 4 \end{array} \left( \frac{6s}{20} \right)$$



Cauer II<sup>nd</sup> form

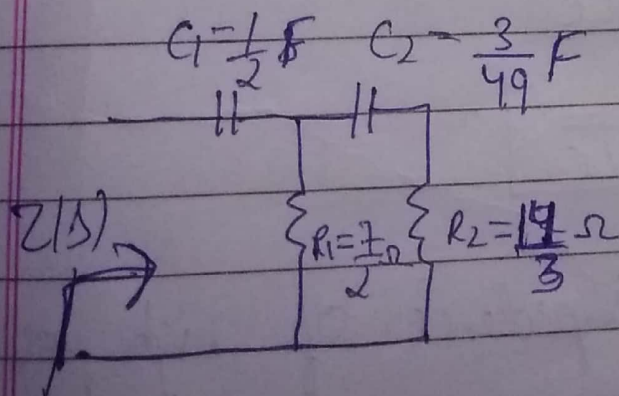
$$\begin{array}{r} 2s + s^2 \\ 4 + 9s + 2s^2 \end{array} \left( \frac{2}{s} \right)$$

$$4 + 2s$$

$$\begin{array}{r} 7s + 2s^2 \\ 2s + s^2 \\ 2s + 4s^2 \end{array} \left( \frac{2}{7} \right)$$

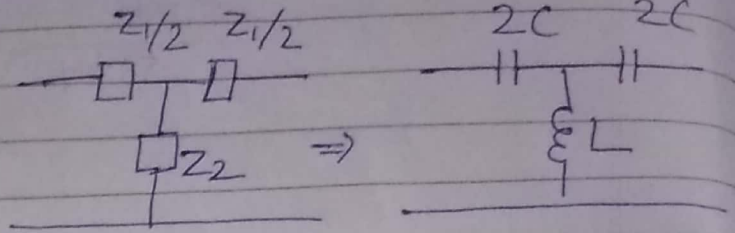
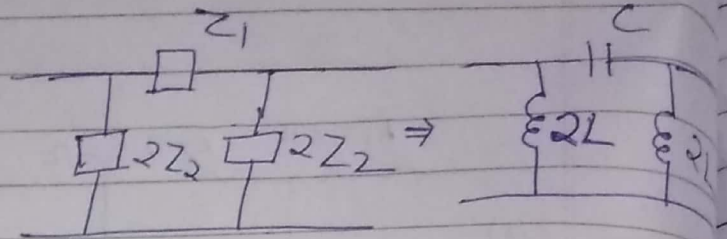
$$\begin{array}{r} 3s^2 \\ 7s + 2s^2 \\ 7s \end{array} \left( \frac{49}{33} \right)$$

$$\begin{array}{r} 2s^2 \\ 3s^2 \\ 3s^2 \end{array} \left( \frac{3}{14} \right)$$





04/12/19

Positiv High Pass filterT-form $\pi$ -form

$$Z_1 = \frac{1}{j\omega C}$$

$$Z_2 = j\omega L$$

Cut-off frequency

$$Z_1 + 4Z_2 = 0$$

$$\frac{1}{j\omega C} + 4j\omega L = 0$$

$$1 - 4\omega^2 LC = 0$$

$$\omega = \frac{1}{2\sqrt{LC}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

Characteristics of impedance of high pass filter

$$Z_0^2 = Z_1 Z_2$$

$$= \frac{1}{j\omega C} \times j\omega L$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$f_c^2 = \frac{1}{16\pi^2 LC}$$

$$Z_0^2 = \frac{L}{C}$$

$$LC = \frac{1}{16\pi^2 f_c^2}$$

$$L = CZ_0^2$$

$$C^2 Z_0^2 = \frac{1}{16\pi^2 f_c^2}$$

$$C^2 = \frac{1}{16\pi^2 f_c^2 Z_0^2}$$

$$C = \frac{1}{4\pi f_c Z_0}$$

$$L = \frac{Z_0}{4\pi f_c}$$

Q Design a low pass filter of 5kHz freq. and high pass filter of cut-off freq. 2kHz for a characteristics of  $1k\Omega$ . with the help of low pass & high pass filter design a band pass freq. of 225kHz.

Sol

$$f_c = 5 \times 10^3 \text{ Hz}$$

$$f_c = 5 \times 10^3 \text{ Hz}$$

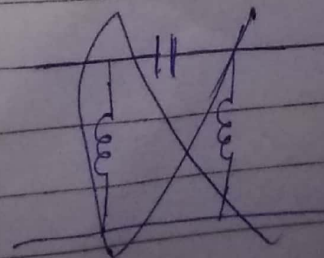
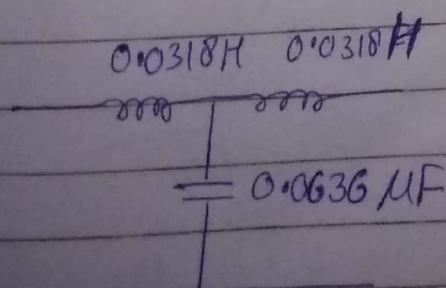
$$Z_0 = 1 \times 10^3 \Omega$$

$$L = \frac{Z_0}{\pi f_c} = \frac{1 \times 10^3}{3.14 \times 5 \times 10^3}$$

$$L = 0.0636 \text{ H}$$

$$C = \frac{1}{Z_0 \pi f_c} = \frac{1}{10^3 \times 3.14 \times 5 \times 10^3}$$

$$C = 0.0636 \times 10^{-6} \text{ F}$$





For high pass

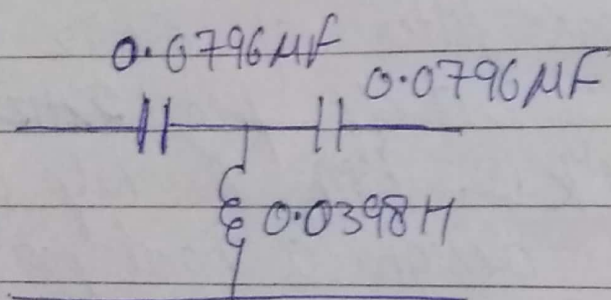
$$f_c = 2 \times 10^3 \text{ Hz} \quad Z_0 = 1 \times 10^3 \text{ } \Omega$$

$$L = \frac{Z_0}{4\pi f_c} = \frac{1 \times 10^3}{4 \times 3.14 \times 2 \times 10^3}$$

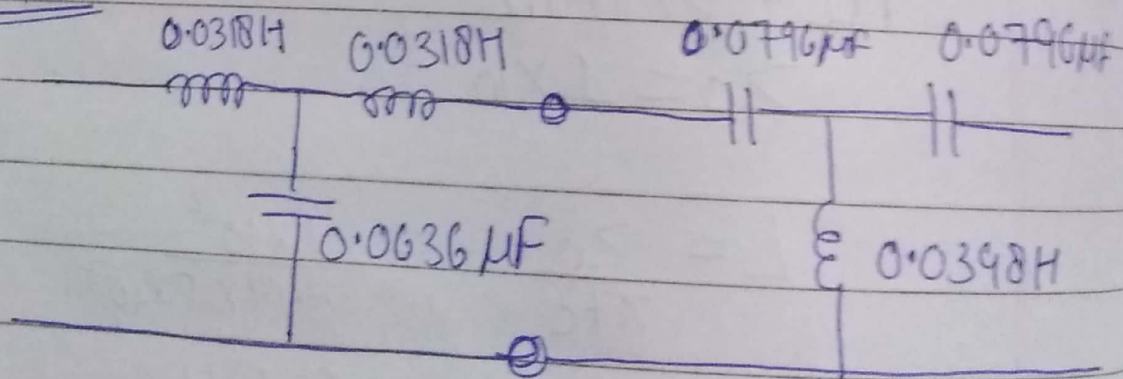
$$L = 0.0398 \text{ H}$$

$$C = \frac{1}{4\pi f_c Z_0} = \frac{1}{4 \times 3.14 \times 2 \times 10^3 \times 10^3}$$

$$C = 0.0398 \text{ } \mu\text{F}$$



Band Pass Filter



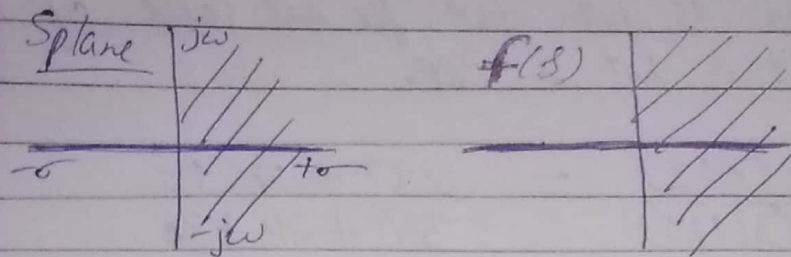
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Positive Real Function [PRF]

- ① When the  $s$  is real and +ve it means  $F(s)$  should be real and +ve.



PRF represent the existence of a <sup>real</sup> function.

Procedure of PRF

$$Z(s) = \frac{(s+1)(s+3)}{(s+2)(s^2+10)}$$

- ② Check whether the func<sup>n</sup>  $Z(s)$  PRF or not.

Sol

$$\frac{s^2 + 4s + 3}{s^3 + 2s^2 + 10s + 20}$$

I All the coefficient are real & +ve.

II All the pole should be lie in -ve half s-plane.

III Optional:- when pole is lie on imaginary axis.

When In the partial fraction of  $Z(s)$  or  $F(s)$ , the residue of complex pole should be real & +ve.

IV Find Even & odd part of  $N(s)$  &  $D(s)$

|                                      |                      |                        |
|--------------------------------------|----------------------|------------------------|
| $M_1 \& N_1 \rightarrow$ Numerator   | Even $M_1 = s^2 + 3$ | Even $M_2 = 2s^2 + 20$ |
| $M_2 \& N_2 \rightarrow$ Denominator | odd $N_1 = 4s$       | odd $N_2 = s^3 + 10s$  |

$$F(\omega) = m_1 m_2 - N_1 N_2 \geq 0$$

for all value of  $\omega$



$$\begin{aligned}
 & (2s^2+20)(s^2+3) - (4s)(s^3+10s) \\
 & 2s^4 + 26s^2 + 60 - 4s^4 - 40s^2 \\
 & -2s^4 - 14s^2 + 60 \\
 & -2\omega^4 + 14\omega^2 + 60
 \end{aligned}$$

$$s = j\omega$$

This function is not +ve for all value of  $\omega$   
 So, it is not a PRF.

$$\frac{s(2s+1)(s+3)}{(s+2)(3s+15)} = \frac{2s^2 + 7s + 3}{3s^2 + 21s + 30}$$

$$M_1 = 2s^2 + 3$$

$$M_2 = 3s^2 + 30$$

$$N_1 = 7s$$

$$N_2 = 21s$$

$$\begin{aligned}
 & = (2s^2+3)(3s^2+30) - (7s)(21s) \\
 & = 6s^4 + 69s^2 + 90 - 147s^2 \\
 & = \cancel{6s^4} + = 6s^4 - 78s^2 + 90
 \end{aligned}$$

$$\frac{s(3s+2)}{(5s+3)(s+4)}$$

Check given the polynomial

are PRF & not.

If PRF draw the Cauer I<sup>st</sup> & II<sup>nd</sup> form if possible.