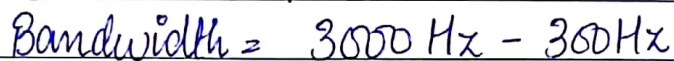
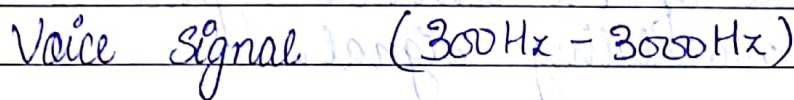


Shivalal



$$f_{20} \approx 93 \text{ KHz} \text{ on } n^2(m+1) \text{ sA} =$$

$$\text{Antenna height} = \frac{\lambda}{4} = \frac{10^5}{4} = 0.25 \times 10^5 \text{ m} = 25 \text{ Km}$$

Modulation is done to reduce the height of antenna.

Need of Modulation:-

- i) Reduce the height of antenna.
- ii) High Range
- iii) Reduce the Interference.
- iv) Multiplexing
- v) High quality signal at receivers end.

⇒ Amplitude modulation:-

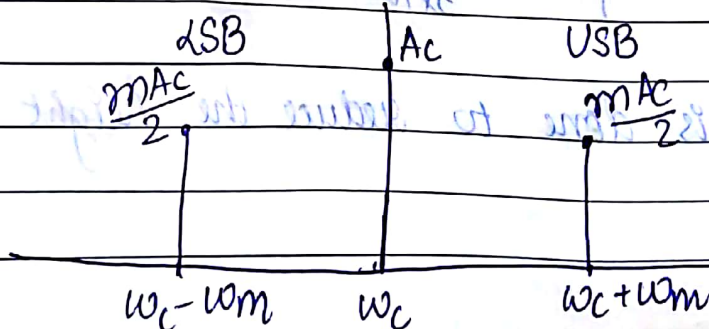
Amplitude of the carrier signal is varied by the modulating signal.

$$\begin{aligned} Y &= (A_c + m(t)) \sin \omega_c t \\ &= (A_c + A_m \sin \omega_m t) \sin \omega_c t \\ &= A_c \left(1 + \frac{A_m}{A_c} \sin \omega_m t \right) \sin \omega_c t \end{aligned}$$

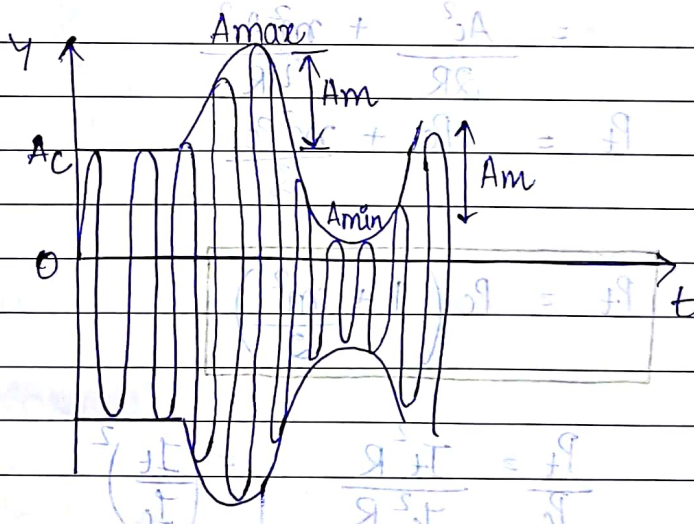
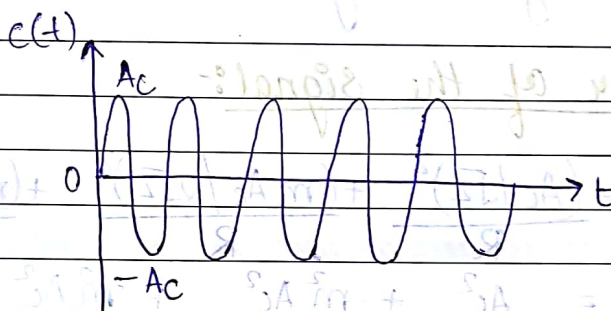
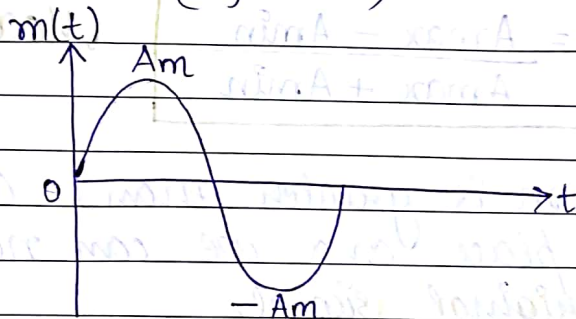
$m = \frac{A_m}{A_c}$ = Modulation Index m lies from 0 to 1

$$\begin{aligned} &= A_c (1 + m \sin \omega_m t) \sin \omega_c t \\ &= A_c \sin \omega_c t + A_c m \sin \omega_m t \sin \omega_c t \\ &= A_c \sin \omega_c t + \frac{2mA_c}{2} \sin \omega_m t \sin \omega_c t \end{aligned}$$

$$\begin{aligned} &= A_c \sin \omega_c t + \frac{mA_c}{2} \cos(\omega_c - \omega_m)t \\ &\quad - \frac{mA_c}{2} \cos(\omega_c + \omega_m)t \end{aligned}$$



$$\begin{aligned}\text{Bandwidth} &= (\omega_c + \omega_m) - (\omega_c - \omega_m) \\ &= 2\omega_m \text{ rad/sec} \\ &= (2f_m \text{ Hz})\end{aligned}$$



$$A_m = \frac{A_{\max} - A_{\min}}{2}$$

$$A_c = A_{\max} - A_m$$

$$A_c = A_{\max} - \left(\frac{A_{\max} - A_{\min}}{2} \right)$$

$$A_c = \frac{A_{\max} + A_{\min}}{2}$$

$$m = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

$$m = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

Modulation index

When m is greater than 1, overmodulation takes place and we can not retrieve or recover the original signal.

Power of the signal:-

$$P_t = \frac{(A_c/\sqrt{2})^2}{R} + \frac{(mA_c/2\sqrt{2})^2}{R} + \frac{(mA_c/2\sqrt{2})^2}{R}$$

$$= \frac{A_c^2}{2R} + \frac{m^2 A_c^2}{8R} + \frac{m^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R} + \frac{m^2 A_c^2}{4R}$$

$$P_t = P_c + \frac{m^2 P_c}{2}$$

$$P_t = P_c \left(1 + \frac{m^2}{2} \right)$$

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = \left(\frac{I_t}{I_c} \right)^2$$

$$\left(\frac{I_t}{I_c} \right)^2 = 1 + \frac{m^2}{2}$$

$$I_t = I_c \sqrt{1 + \frac{m^2}{2}}$$

⇒ Amplitude modulation by complex modulating signal:-

$$m(t) = A_{m1} \sin \omega_{m1} t + A_{m2} \sin \omega_{m2} t + A_{m3} \sin \omega_{m3} t$$

$$c(t) = A_c \sin \omega_c t$$

$$X = (A_c + m'(t)) \sin \omega_c t$$

$$\begin{aligned} &= (A_c + A_{m1} \sin \omega_{m1} t + A_{m2} \sin \omega_{m2} t + A_{m3} \sin \omega_{m3} t) \sin \omega_c t \\ &= A_c \sin \omega_c t + A_{m1} \sin \omega_{m1} t \sin \omega_c t + A_{m2} \sin \omega_{m2} t \sin \omega_c t \\ &\quad + A_{m3} \sin \omega_{m3} t \sin \omega_c t \end{aligned}$$

$$= A_c \sin \omega_c t + \frac{A_c A_{m1}}{A_c} \sin \omega_{m1} t \sin \omega_c t + \frac{A_c A_{m2}}{A_c} \sin \omega_{m2} t \sin \omega_c t$$

$$+ \frac{A_c A_{m3}}{A_c} \sin \omega_{m3} t \sin \omega_c t$$

$$= A_c \sin \omega_c t + \frac{2 m_1 A_c}{2} \sin \omega_{m1} t \sin \omega_c t + \frac{2 m_2 A_c}{2} \sin \omega_{m2} t$$

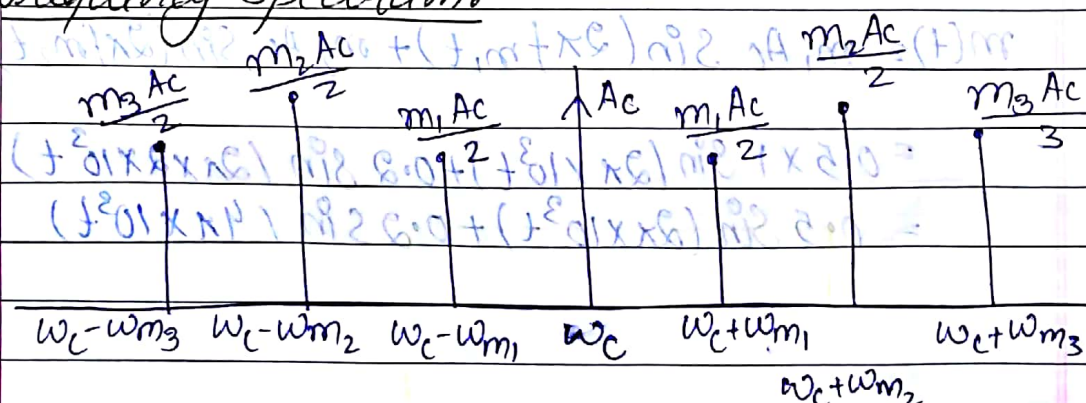
$$\sin \omega_c t + \frac{2 m_3 A_c}{2} \sin \omega_{m3} t \sin \omega_c t$$

$$= A_c \sin \omega_c t + \frac{m_1 A_c}{2} \cos(\omega_c - \omega_{m1}) t - \frac{m_1 A_c}{2} \cos(\omega_c + \omega_{m1}) t$$

$$+ \frac{m_2 A_c}{2} \cos(\omega_c - \omega_{m2}) t - \frac{m_2 A_c}{2} \cos(\omega_c + \omega_{m2}) t$$

$$+ \frac{m_3 A_c}{2} \cos(\omega_c - \omega_{m3}) t - \frac{m_3 A_c}{2} \cos(\omega_c + \omega_{m3}) t$$

Frequency Spectrum:-



$$\text{Bandwidth} = (\omega_c + \omega_{m3}) - (\omega_c - \omega_{m3})$$

$$= 2\omega_{m3} \text{ rad/sec}$$

$$= 2f_{m3} \text{ Hz}$$

Total modulation index

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2}$$

$$P_t = P_c + P_{LSB} + P_{USB}$$

Ques

An arbitrary modulating signal consisting of two modulating frequencies of 1KHz and 2KHz, modulated with a carrier signal having peak amplitude of 1 volt and frequency 1MHz with modulation index 0.5 and 0.2 respectively. Write the resultant expression for Am signal. Calculate its bandwidth in Hz and draw the frequency spectrum.

Sol.

$$m(t) = A_{m1} \sin \omega_{m1} t + A_{m2} \sin \omega_{m2} t$$

$$A_c = 1V$$

$$m_1 = 0.5$$

$$m_2 = 0.2$$

$$f_{m1} = 1 \text{ KHz}$$

$$f_{m2} = 2 \text{ KHz}$$

$$f_c = 1 \text{ MHz}$$

$$m(t) = m_1 A_c \sin(2\pi f_{m1} t) + m_2 A_c \sin(2\pi f_{m2} t)$$

$$= 0.5 \times 1 \sin(2\pi \times 10^3 t) + 0.2 \sin(2\pi \times 2 \times 10^3 t)$$

$$= 0.5 \sin(2\pi \times 10^3 t) + 0.2 \sin(4\pi \times 10^3 t)$$

$$c(t) = A_c \sin \omega_c t$$

$$= 1 \times \sin(2\pi \times 10^6 t)$$

$$= \sin(2\pi \times 10^6 t)$$

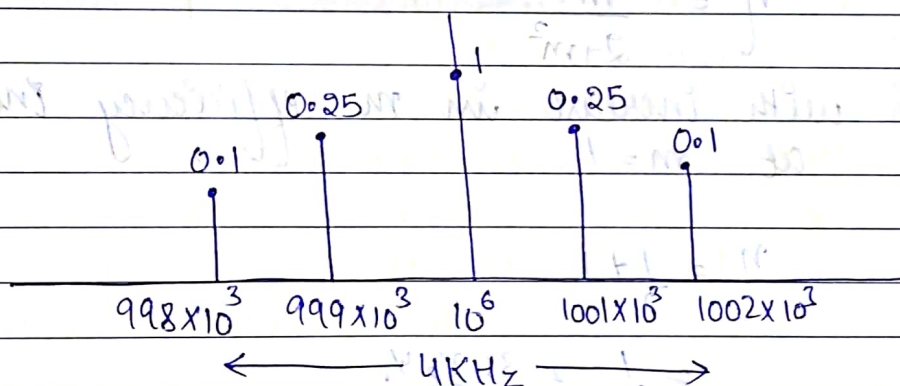
$$y = (1 + m(t)) \sin(2\pi \times 10^6 t)$$

$$= [1 + 0.5 \sin(2\pi \times 10^3 t) + 0.2 \sin(4\pi \times 10^3 t)] \sin(2\pi \times 10^6 t)$$

$$= \sin(2\pi \times 10^6 t) + \frac{0.5}{2} [\cos 2\pi t (10^6 - 10^3) - \cos 2\pi t (10^6 + 10^3)]$$

$$+ \frac{0.2}{2} [\cos 2\pi t (10^6 - 10^3 \times 2) - \cos 2\pi t (10^6 + 2 \times 10^3)]$$

Frequency spectrum



$$\text{Bandwidth} = 2f_m$$

$$= 2 \times 2$$

$$= 4 \text{ KHz}$$

$$\text{Total modulation index} = \sqrt{m_1^2 + m_2^2}$$

$$= \sqrt{0.5^2 + 0.2^2}$$

Efficiency

$$P_t = P_c \left(1 + \frac{m^2}{2} \right)$$

• $P_c \rightarrow$ carrier power

• $\frac{m^2}{2} P_c \rightarrow$ sideband power

$$\eta = \frac{P_c m^2 / 2}{P_t} = \frac{P_c m^2 / 2}{P_c \left(1 + \frac{m^2}{2} \right)}$$

$$= \frac{m^2 / 2}{1 + \frac{m^2}{2}}$$

$$\eta = \frac{m^2}{2 + m^2}$$

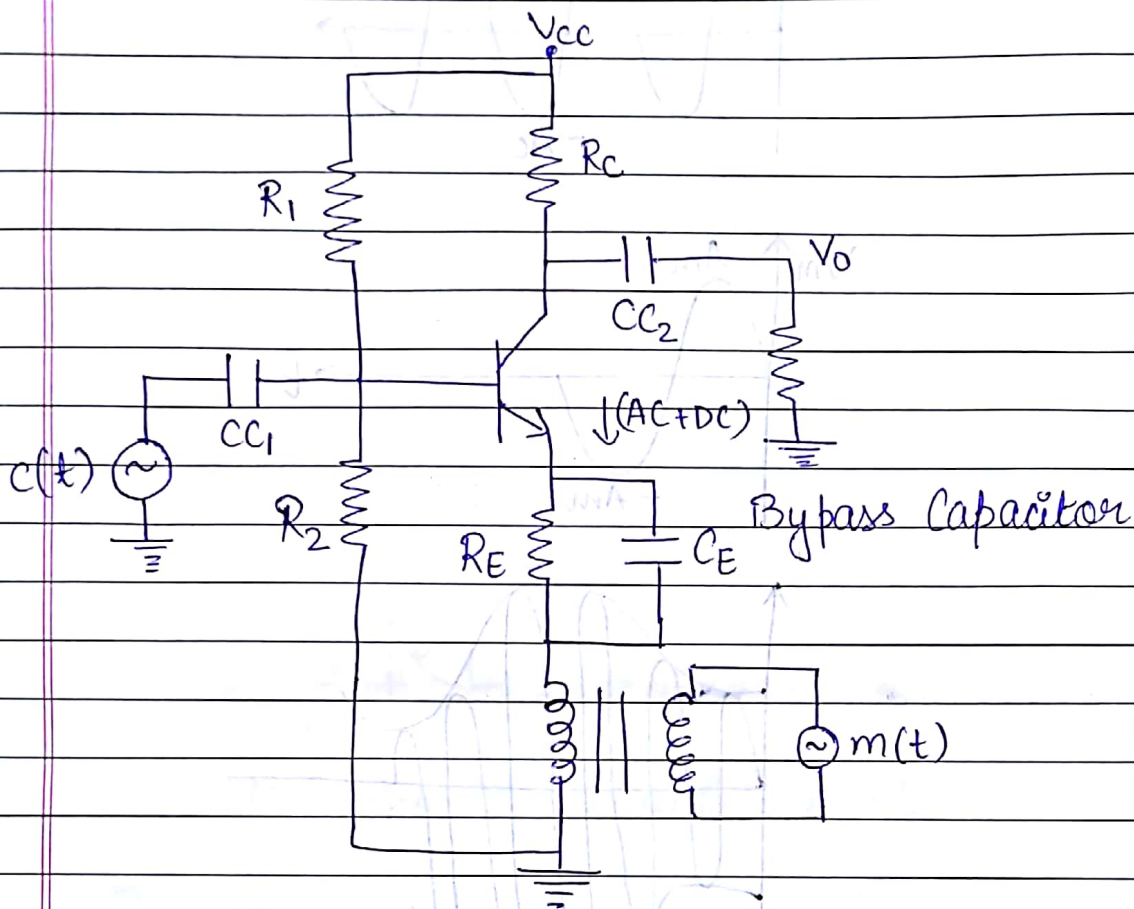
with increase in m , efficiency increases.
at $m = 1$

$$\eta = \frac{1 + 1}{2 + 1}$$

$$= \frac{1}{3} = 33.33\%$$

$m = 1$ = 100%

Amplitude modulation modulator circuit :-



Carrier $\rightarrow$ high frequency signal

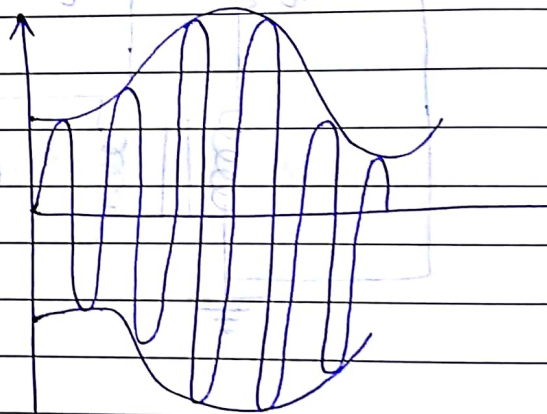
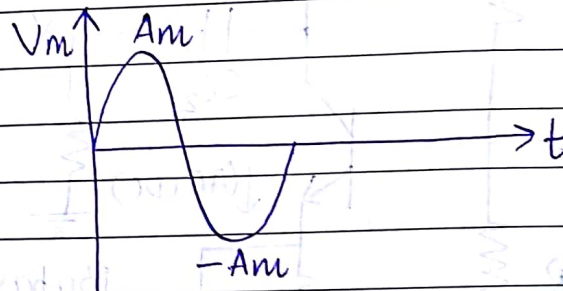
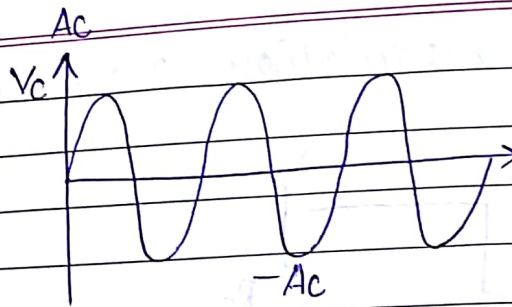
modulating $\rightarrow$ low frequency signal

$E \rightarrow$ modulating signal

$T \rightarrow$

$c(t) \rightarrow$ carrier signal

$m(t) \rightarrow$ modulating signal



$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$

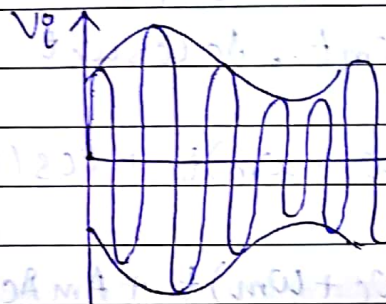
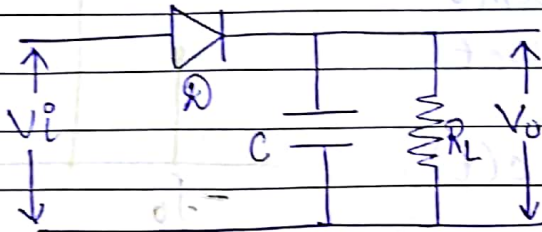
$$\begin{aligned} \frac{R_2}{R_1} &= \frac{V_2/I_2}{V_1/I_1} \\ &= \left(\frac{V_2}{V_1} \right) \left(\frac{I_1}{I_2} \right) \end{aligned}$$

$$\frac{R_2}{R_1} = \frac{N_2}{N_1} \times \frac{N_2}{N_1}$$

$$\frac{R_2}{R_1} = \left(\frac{N_2}{N_1} \right)^2 \Rightarrow \boxed{R_2 = \left(\frac{N_2}{N_1} \right)^2 R_1}$$

AM Demodulator circuit :-

Envelope detector :-

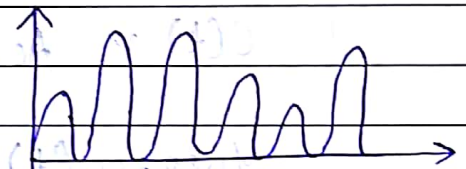


diode $\rightarrow$ only conducts in forward bias
 capacitor $\rightarrow$ gets the modulating signal

$$\tau = R_L C$$

i) τ is large, discharge time is large

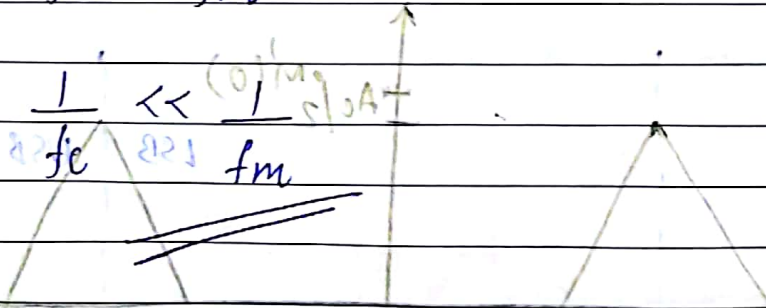
$$\frac{1}{f_c} \ll \tau \ll \frac{1}{f_m}$$



ii) τ is small, fast discharge

$$f_c \gg f_m$$

$$\frac{1}{f_c} \ll \frac{1}{f_m}$$



Double Sideband Suppressed carrier:- (DSB - SC)

$$m(t) = A_m \cos \omega_m t$$

$$c(t) = A_c \cos \omega_c t$$

$$y_{\text{DSB-SC}}(t) = m(t) c(t)$$

$$= (m(t) A_c) \cos \omega_c t$$

$$= \frac{2}{2} A_m \cos \omega_m t \cdot A_c \cos \omega_c t$$

$$= \frac{A_m A_c}{2} [\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$$

$$= \frac{A_m A_c}{2} \cos(\omega_c + \omega_m)t + \frac{A_m A_c}{2} \cos(\omega_c - \omega_m)t$$

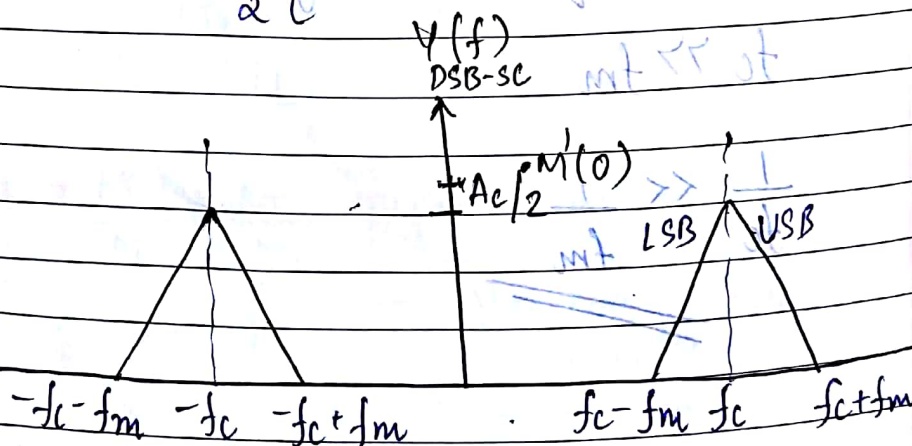
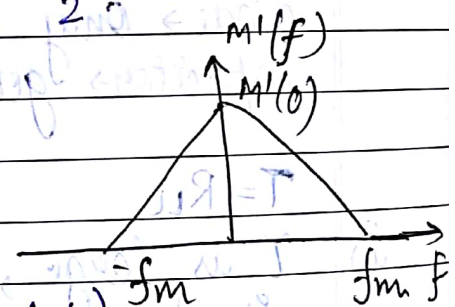
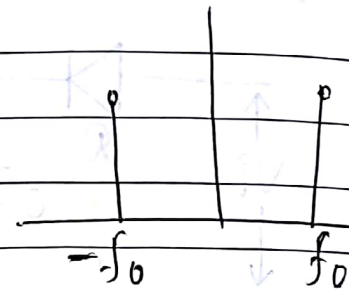
$$y(t) = A_c m'(t) \cos \omega_c t$$

$$c(t) = A_c \cos \omega_c t$$

$$y_{\text{DSB-SC}}(t) = A_c m'(t) \cos(2\pi f_c t)$$

$$y_{\text{DSB-SC}}(f) = A_c M'(f) \times \left[\frac{1}{2} \delta(f - f_c) + \delta(f + f_c) \right]$$

$$y_{\text{DSB-SC}}(f) = \frac{A_c}{2} [M'(f - f_c) + M'(f + f_c)]$$



Demodulation:-

$$y(t) = A_c m'(t) \cos 2\pi f_c t$$



$c(t) \leftarrow$ Coherently Carrier Signal.

$$x(t) = y(t) \cdot c(t)$$

$$= A_c m'(t) \cos 2\pi f_c t \cdot \cos(2\pi f_c t)$$

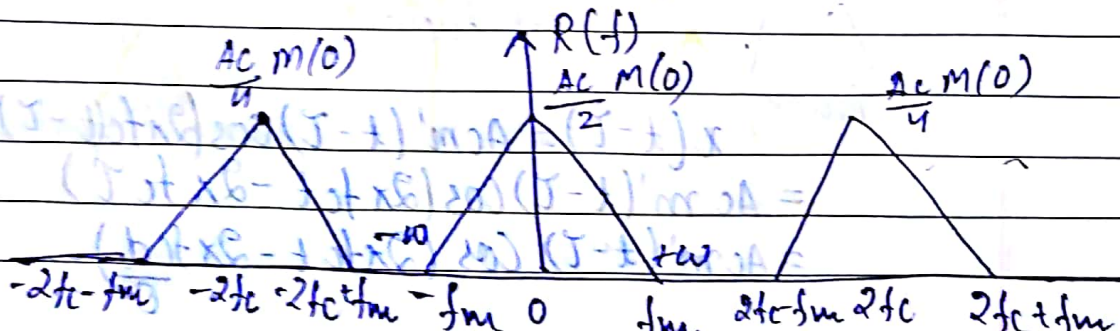
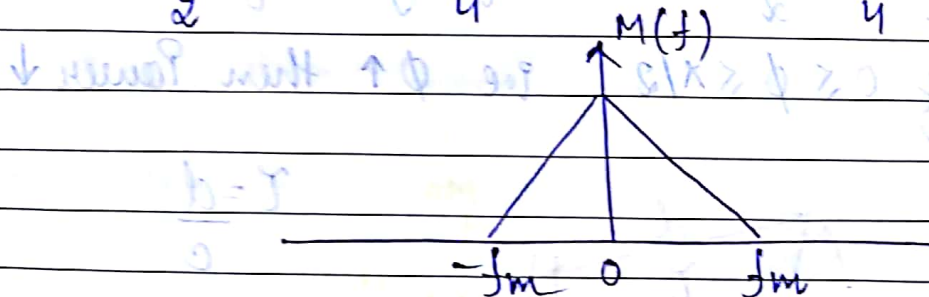
$$= A_c m'(t) \cos^2 2\pi f_c t$$

$$= A_c m'(t) \frac{[1 + \cos 4\pi f_c t]}{2}$$

$$= \frac{A_c m'(t)}{2} + \frac{A_c m'(t) \cos 4\pi f_c t}{2}$$

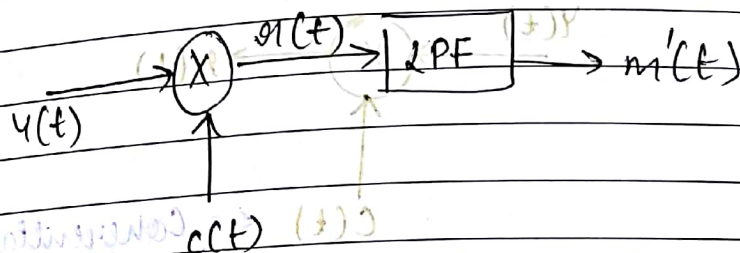
$$R(f) = \frac{A_c}{2} M'(f) + \frac{A_c}{2} M'(f) * \left[\frac{1}{2} \delta(f - 2f_c) + \frac{1}{2} \delta(f + 2f_c) \right]$$

$$R(f) = \frac{A_c}{2} M'(f) + \frac{A_c}{4} M'(f - 2f_c) + \frac{A_c}{4} M'(f + 2f_c)$$



$$\begin{aligned}
 f_c &\gg f_m \\
 2f_c &\gg 2f_m \\
 2f_m &< 2f_c \\
 f_m &< 2f_c - f_m \\
 f_m &< \omega < 2f_c - f_m
 \end{aligned}$$

⇒ Costa



$Acm'(t)$

If there is some phase difference ϕ

$$\begin{aligned}
 y(t) &= Acm'(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) \\
 &= \frac{Acm'(t)}{2} [\cos(4\pi f_c t + \phi) + \cos \phi] \\
 &= \frac{Acm'(t)}{2} \cos \phi + \frac{Acm'(t)}{2} \cos(4\pi f_c t + \phi)
 \end{aligned}$$

VCO =

$x_1(t)$

$x_1(t)$

$x_2(t) =$

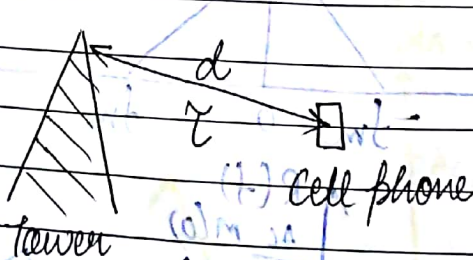
Case 1: If $\phi \neq 0$, Power of signal of low pass filter output = $\frac{Ac^2}{8} P_m \cos^2 \phi$

$x_3(t)$

Case 2: If $\phi = \frac{\pi}{2}$, Power of signal of LPF o/p = 0

$x_3(t)$

Case 3: If $0 < \phi < \pi/2$ i.e. $\phi \uparrow$ then Power $\downarrow$



$$\tau = \frac{d}{c}$$

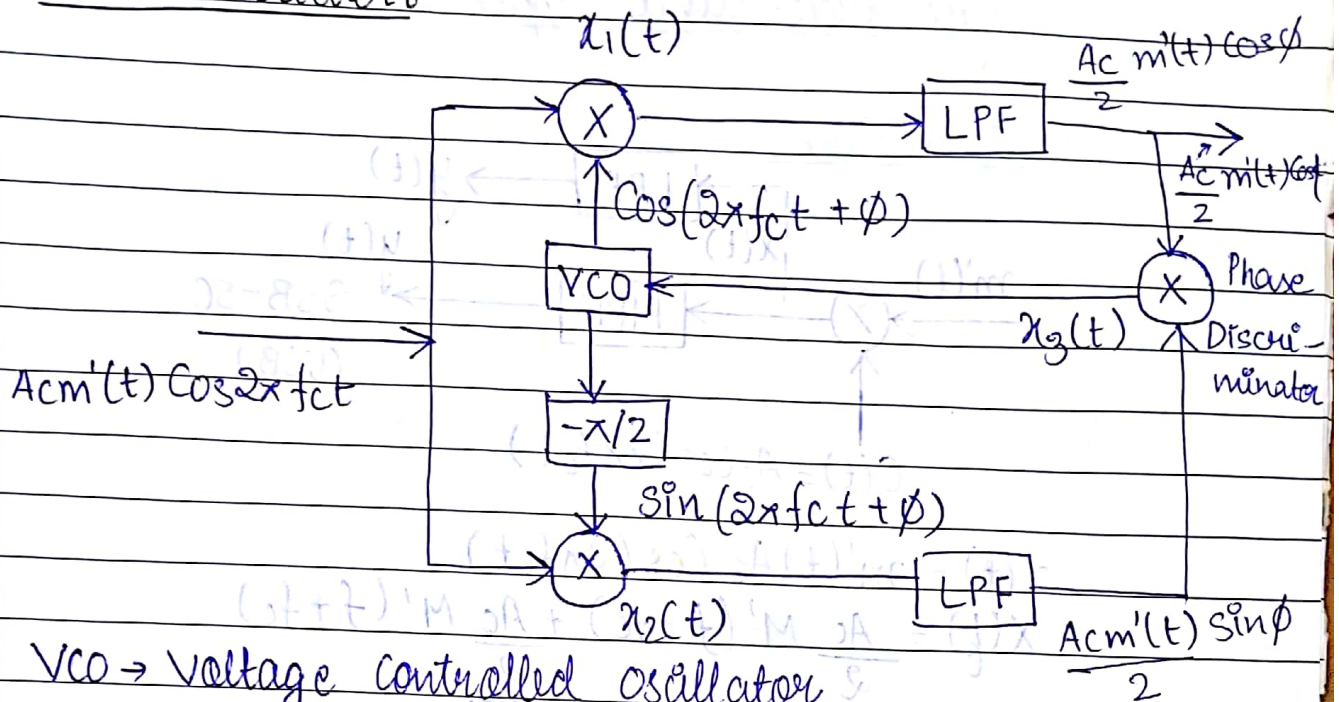
Cell phone

tower

$$\begin{aligned}
 x(t-\tau) &= Acm'(t-\tau) \cos(2\pi f_c(t-\tau)) \\
 &= Acm'(t-\tau) \cos(2\pi f_c t - 2\pi f_c \tau) \\
 &= Acm'(t-\tau) \cos(2\pi f_c t - 2\pi f_c \frac{d}{c})
 \end{aligned}$$

Bam

⇒ Costas Receiver:



$$x_1(t) = Acm'(t) \cos 2\pi f_c t \cos(2\pi f_c t + \phi)$$

$$x_1(t) = \frac{Acm'(t)}{2} \cos \phi + \frac{Acm'(t)}{2} \cos(4\pi f_c t + \phi)$$

$$x_2(t) = Acm'(t) \cos(2\pi f_c t) \sin(2\pi f_c t + \phi)$$

$$= \frac{Acm'(t)}{2} \sin(4\pi f_c t + \phi) + \frac{Acm'(t)}{2} \sin \phi$$

$$x_3(t) = \left[\frac{Acm'(t)}{2} \right]^2 \cos \phi \sin \phi$$

$$x_3(t) = \frac{Ac^2 m'^2(t)}{4} \cos \phi \sin \phi$$

$$-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$$

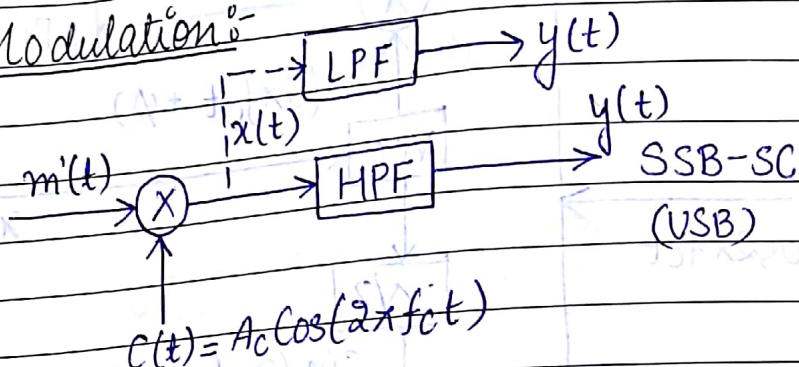
$$\text{If } \phi > 0, x_3(t) = +ve$$

$$\text{If } \phi < 0, x_3(t) = -ve$$

$$\text{Bandwidth of DSB-SC} = 2f_m$$

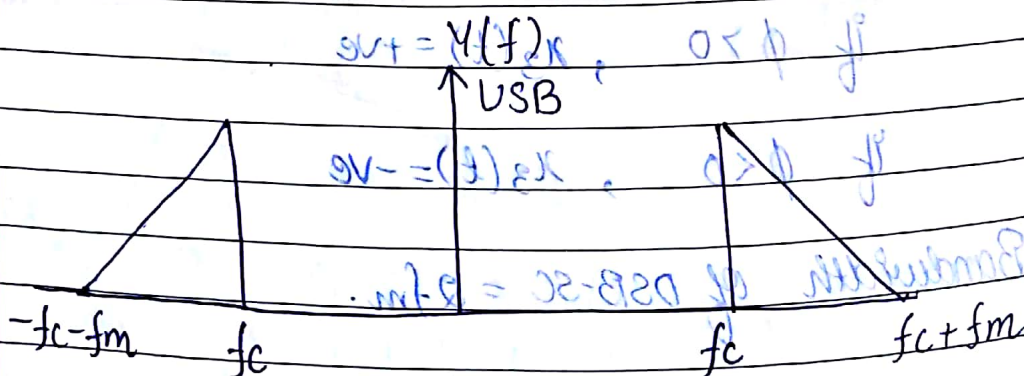
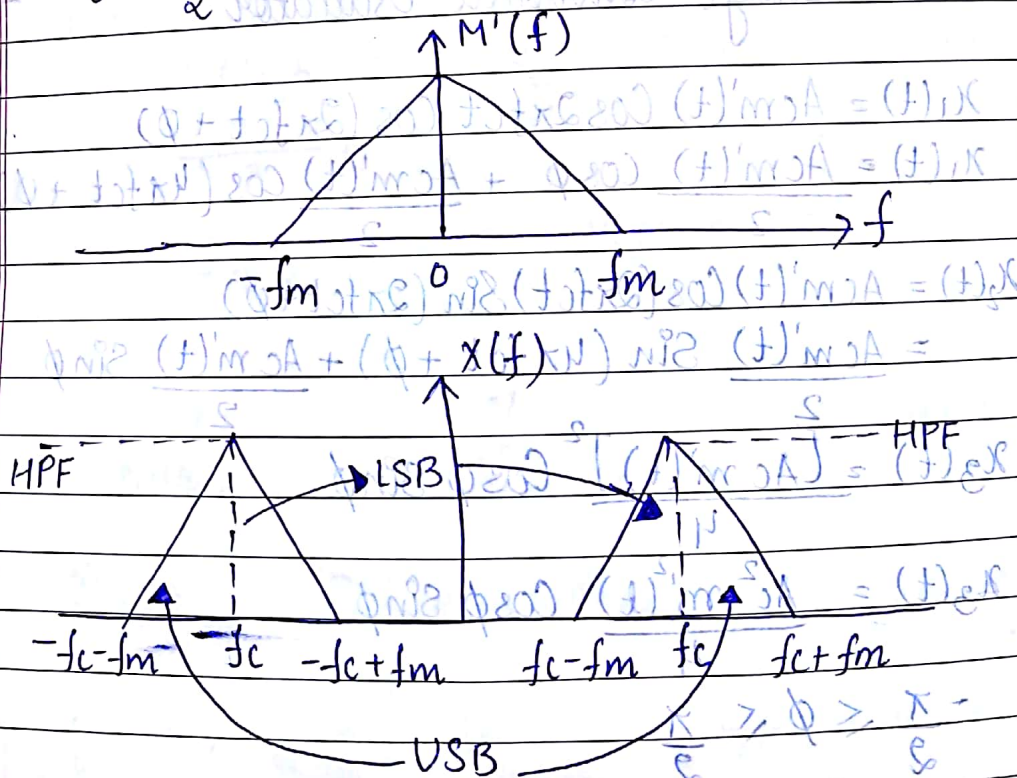
Single Side-band Suppressed carrier:- (SSB-SC)

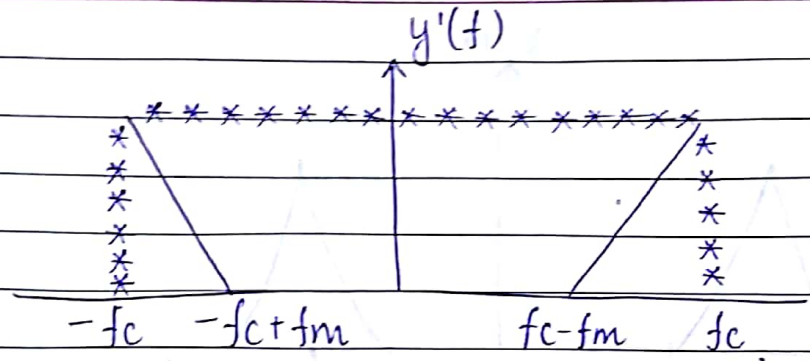
Modulation:-



$$x(t) = m'(t) A_c \cos(2\pi f_c t)$$

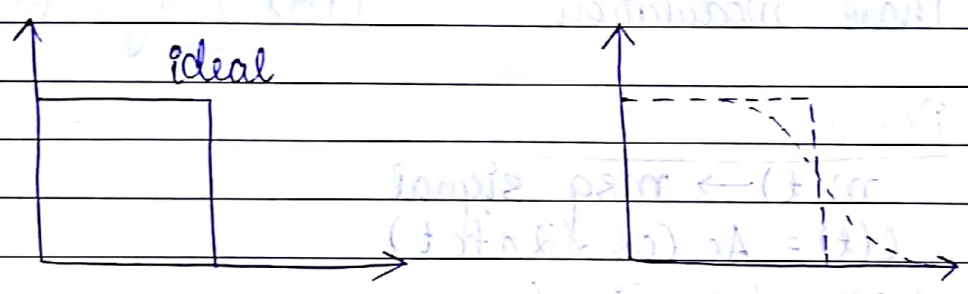
$$X(f) = \frac{A_c}{2} M'(f - f_c) + \frac{A_c}{2} M'(f + f_c)$$



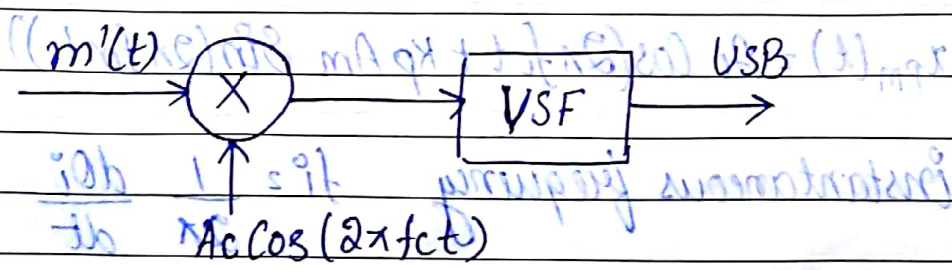
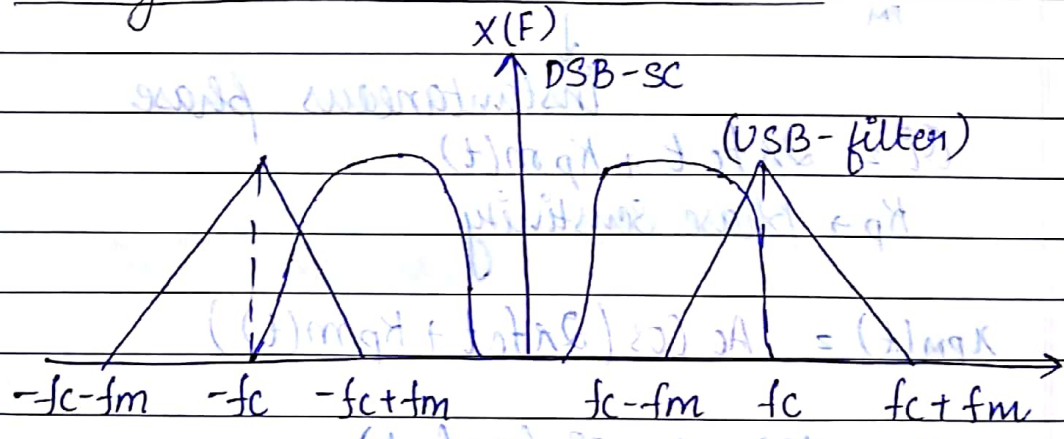


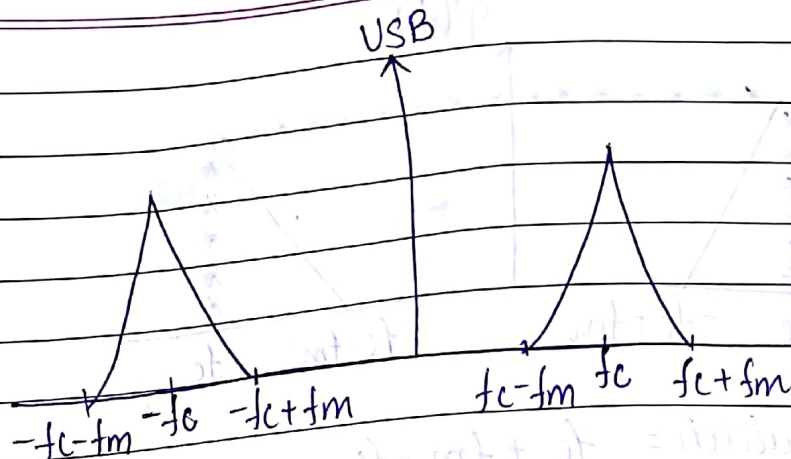
Bandwidth = $f_c + f_m - f_c$
 $= \underline{\underline{f_m}}$

Spectral Efficiency will increase by 2 times.
 of DSBSC.



Vestigial sideband modulation :-





Angle modulation:-

- a) Frequency modulation (FM) } Two types
b) Phase modulation (PM) } Angle modulation

a) Phase modulation:-

$m(t) \rightarrow$ msg signal

$$c(t) = A_c \cos(2\pi f_c t)$$

Modulated signal

$$x(t) = A_c \cos(\phi(t))$$

PM

Instantaneous phase

$$\phi(t) = 2\pi f_c t + K_p m(t)$$

$K_p \rightarrow$ phase sensitivity

$$x_{PM}(t) = A_c \cos(2\pi f_c t + K_p m(t))$$

if $m(t) = A_m \sin(2\pi f_m t)$

$$x_{PM}(t) = A_c \cos(2\pi f_c t + K_p A_m \sin(2\pi f_m t))$$

Instantaneous frequency $f_i = \frac{1}{2\pi} \frac{d\phi}{dt}$

$$= \frac{1}{2\pi} \frac{1}{dt} [2\pi f_c t + K_p m(t)]$$

$$= \frac{1}{2\pi} \times 2\pi f_c + \frac{d}{dt} [K_p A_m \sin(2\pi f_m t)]$$

$$f_i = f_c + K_p A_m f_m \cos(2\pi f_m t)$$

maximum frequency deviation:-

$$\Delta f = \max |f_i - f_c|$$

$$= \max |f_c + K_p A_m \sin(\cos(2\pi f_m t)) - f_c|$$

$$\Delta f = K_p A_m f_m$$

$$\text{Modulation Index } (\beta) = \frac{\Delta f}{f_m} = K_p A_m$$

maximum phase deviation:-

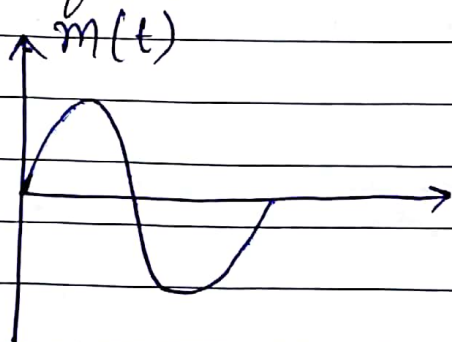
$$\Delta \phi = \max |\phi_i(t) - 2\pi f_c t|$$

$$= \max |K_p A_m \sin 2\pi f_m t|$$

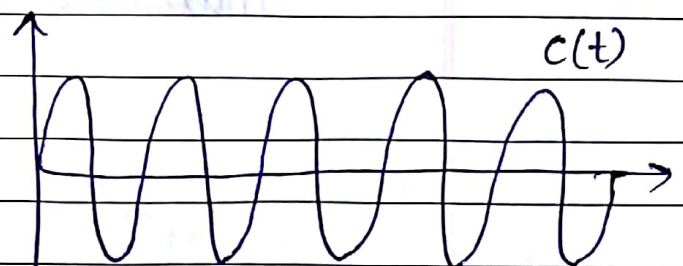
$$\Delta \phi = K_p A_m$$

maximum phase deviation = modulation index

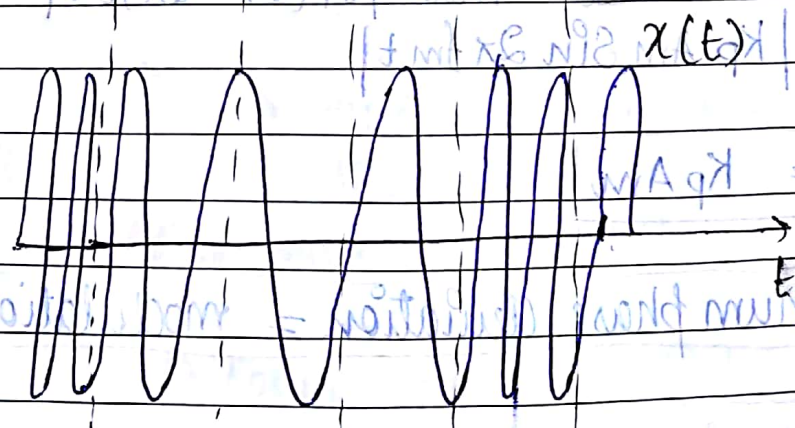
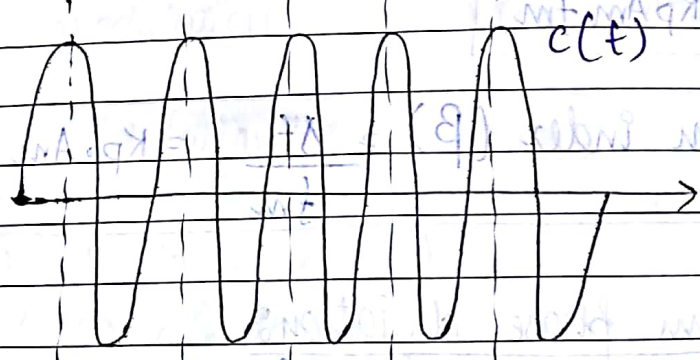
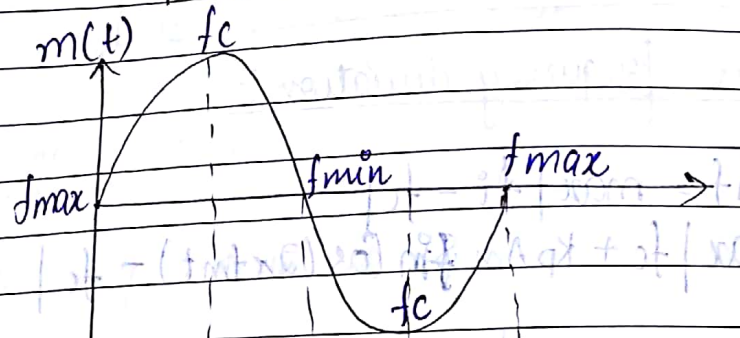
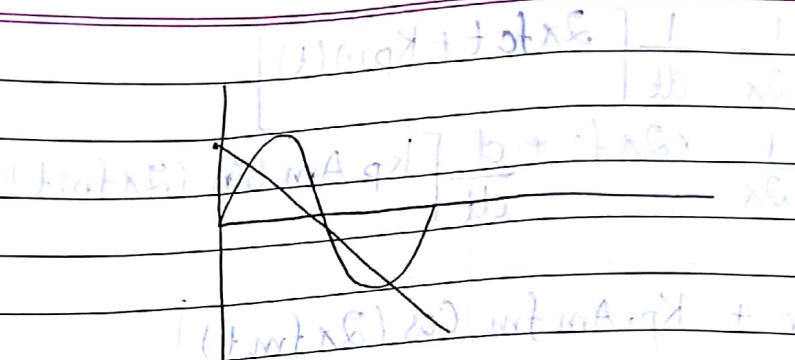
waveforms:-



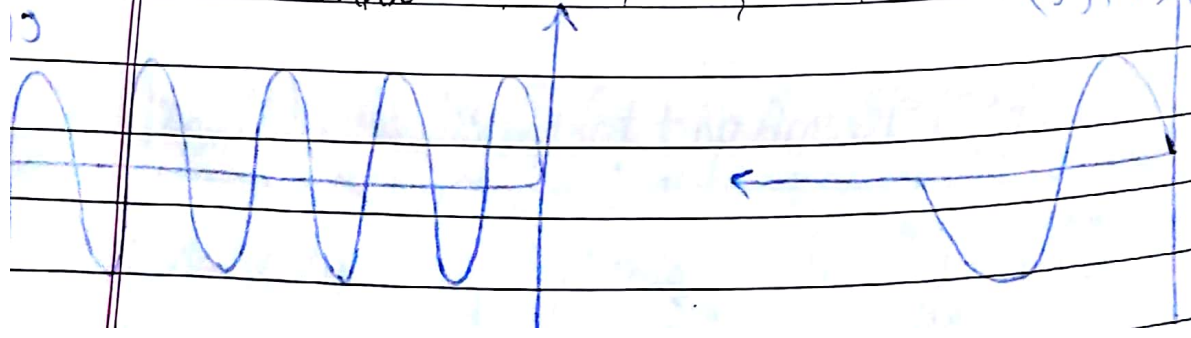
message signal



carrier signal



max min



b) Frequency modulation :-

$$c(t) = A_c \cos(2\pi f_c t)$$

$m(t) \rightarrow$ msg signal

$$y_{FM}(t) = A_c \cos(\theta_i(t))$$

$$f_i = f_c + K_f m(t)$$

$K_f \rightarrow$ frequency sensitivity

$$\theta_i(t) = 2\pi \int_0^t f_i(t) dt$$

$$= 2\pi \int_0^t (f_c + K_f m(t)) dt$$

$$= 2\pi f_c t + 2\pi K_f \int_0^t m(t) dt$$

$$y_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

If $m(t) = A_m \cos(2\pi f_m t)$

$$y_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi K_f \int_0^t A_m \cos(2\pi f_m t) dt \right]$$

$$f_i = f_c + K_f A_m \cos(2\pi f_m t)$$

maximum frequency deviation :-

$$\Delta f = \max |f_i - f_c|$$

$$= \max |K_f A_m \cos(2\pi f_m t)|$$

$$\Delta f = K_f A_m$$

modulation index

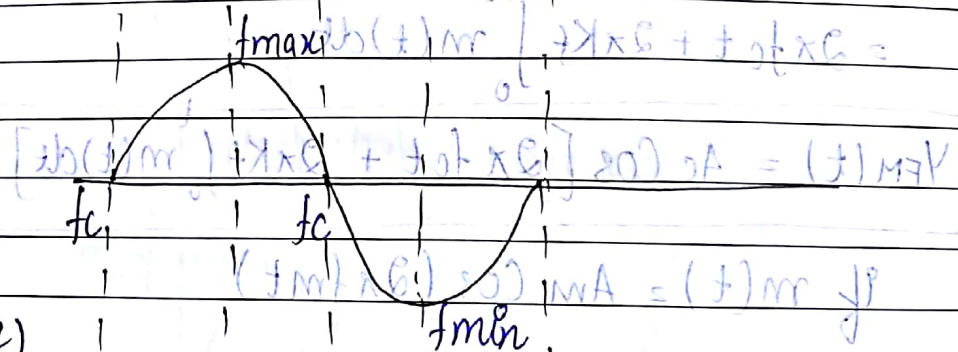
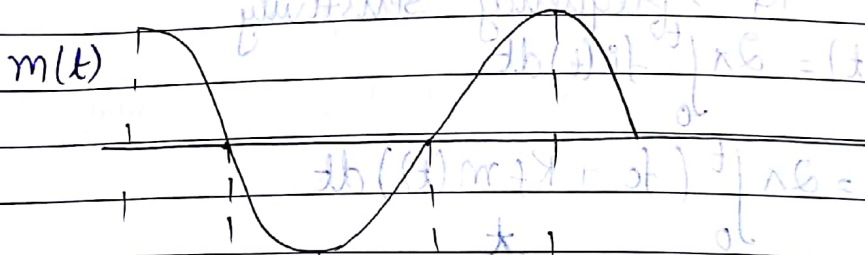
$$\beta_{FM} = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m}$$

$$y_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi K_f \int_0^t A_m \cos 2\pi f_m t dt \right]$$

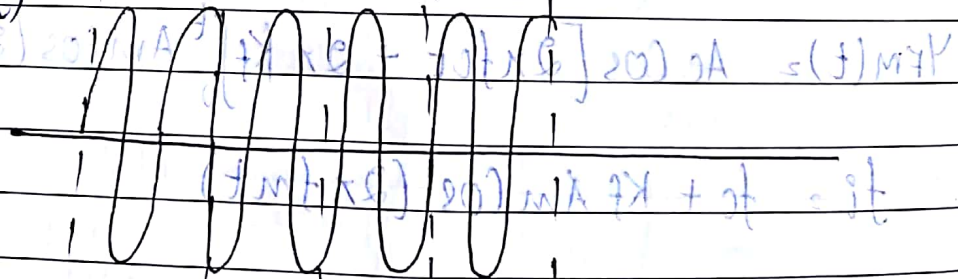
$$= A_c \cos \left[2\pi f_c t + \frac{2\pi K_f A_m}{2\pi f_m} \sin 2\pi f_m t \right]$$

$$y_{FM} = A_c \cos \left[2\pi f_c t + \frac{K_f A_m}{f_m} \sin 2\pi f_m t \right]$$

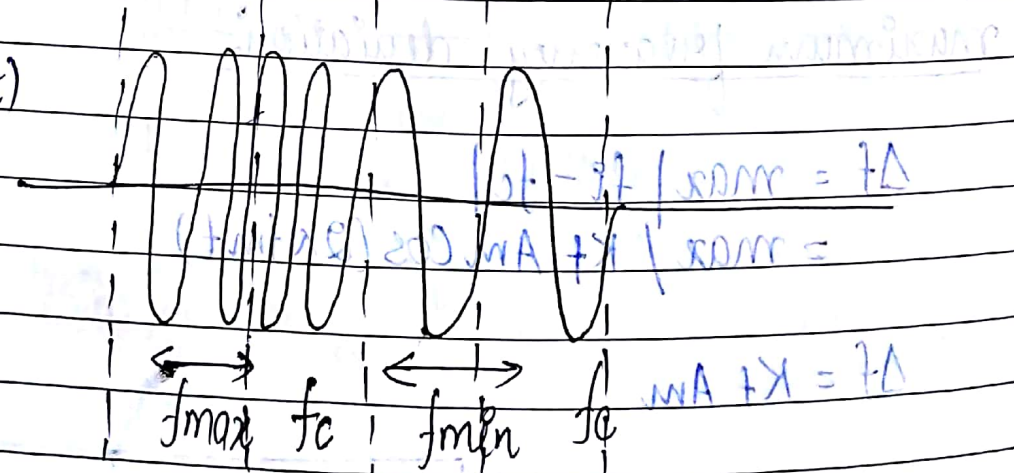
$m(t)$



$c(t)$



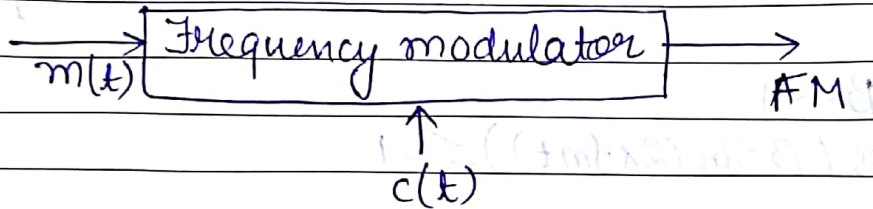
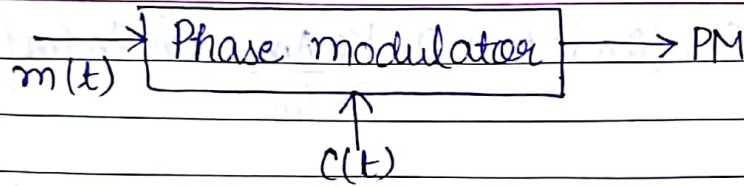
$y(t)$



f_{max} f_c f_{min} f_c

$$\Delta f = \frac{K_f A_m}{f_m} = 75 \text{ kHz}$$

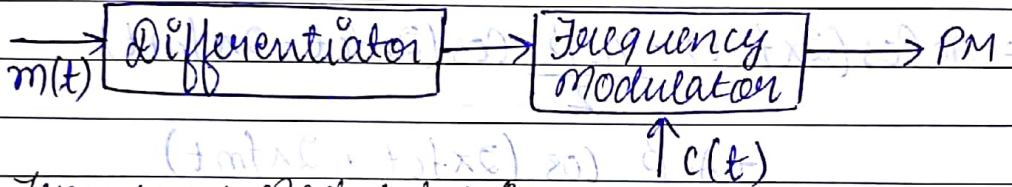
modulation index



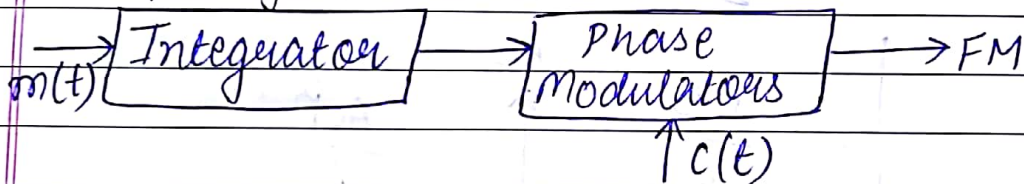
$$x_m(t) = A_c \cos(2\pi f_c t + K_p m(t))$$

$$x_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

* Phase Modulator:-



* Frequency Modulator:-



Frequency modulated signal:-

$$x_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi K_f \int m(t) dt \right]$$

$$P_t = \frac{A_c^2}{2R} = P_c \times \beta^2$$

Narrowband FM

$$\beta \ll 1$$

$$m(t) = A_m \cos(2\pi f_m t)$$

$$x_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi K_f \int_0^t A_m \cos(2\pi f_m t) dt \right]$$

$$= A_c \left[\cos(2\pi f_c t + \frac{2\pi K_f A_m}{2\pi f_m} \sin 2\pi f_m t) \right]$$

$$x_{FM}(t) = A_c \left[\cos 2\pi f_c t + \beta \sin(2\pi f_m t) \right]$$

$$\beta \ll 1$$

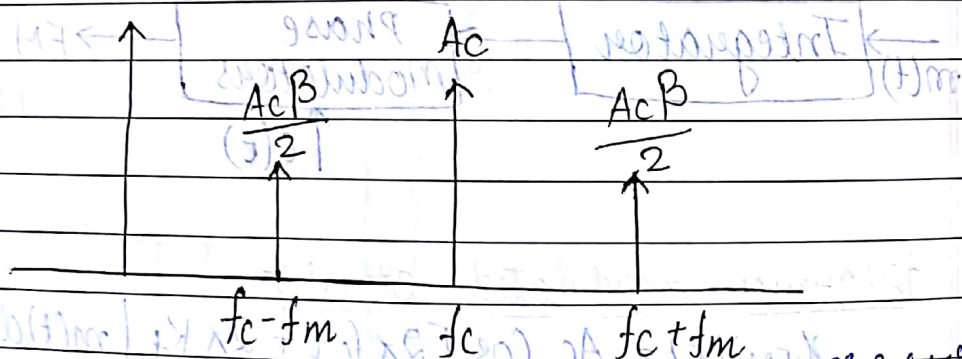
$$\cos(\beta \sin(2\pi f_m t)) \approx 1$$

$$\sin(\beta \sin(2\pi f_m t)) \approx \beta \sin 2\pi f_m t$$

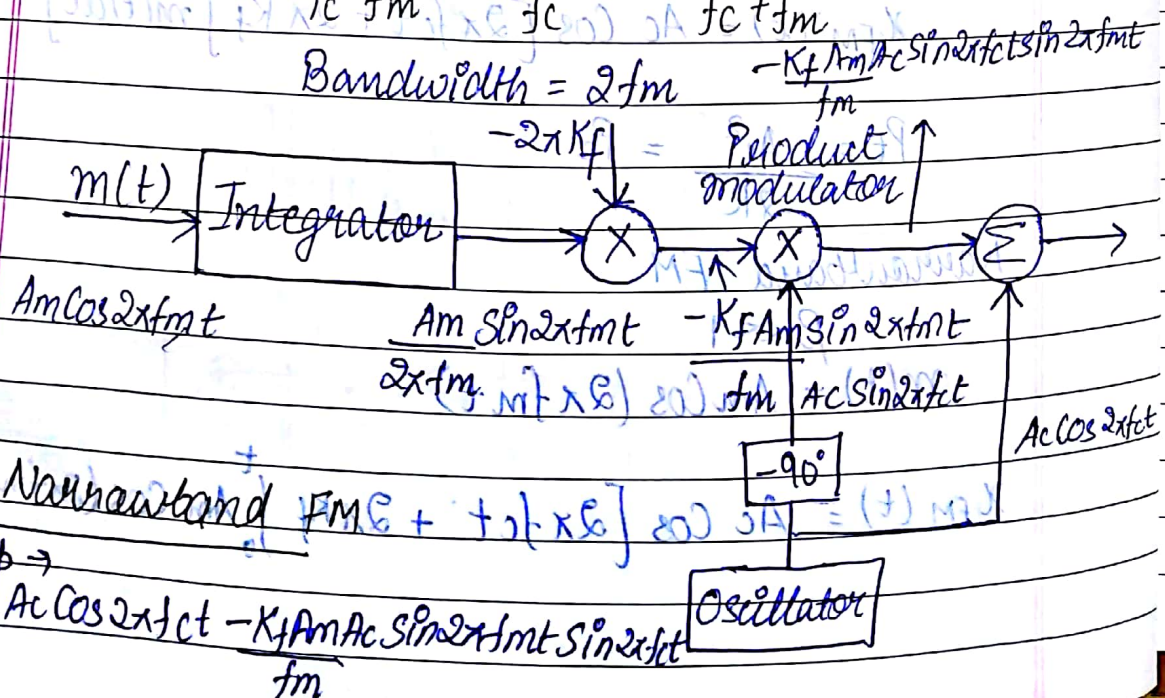
$$= A_c \cos(2\pi f_c t) - \frac{2A_c \beta \sin(2\pi f_m t) \sin(2\pi f_c t)}{2}$$

$$= A_c \cos(2\pi f_c t) - \frac{A_c \beta}{2} \cos(2\pi f_c t - 2\pi f_m t)$$

$$+ \frac{A_c \beta}{2} \cos(2\pi f_c t + 2\pi f_m t)$$



$$\text{Bandwidth} = 2f_m$$



Wideband FM :-

$$\beta \gg 1$$

$$x_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$= \operatorname{Re} \left[A_c e^{j(2\pi f_c t + \beta \sin(2\pi f_m t))} \right]$$

$$= \operatorname{Re} \left[A_c e^{j\beta \sin(2\pi f_m t)} \cdot e^{j(2\pi f_c t)} \right]$$

$$S_b(t) = A_c e^{j\beta \sin[2\pi f_m(t + K/f_m)]}$$

$$= A_c e^{j\beta \sin[2\pi f_m t + 2\pi K]} = A_c e^{j\beta \sin(2\pi f_m t)}$$

$$= A_c e^{j\beta \sin(2\pi f_m t)}$$

$$S_b(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}$$

$$C_n = \frac{1}{T} \int_0^T S_b(t) e^{-j2\pi n f_m t} dt$$

Discrete Fourier Series

$$C_n = \frac{1}{T} \int_0^T A_c e^{j\beta \sin 2\pi f_m t} e^{-j2\pi n f_m t} dt$$

$$= \frac{1}{T} \int_0^T A_c e^{j[\beta \sin(2\pi f_m t) - 2\pi n f_m t]} dt$$

Assume $2\pi f_m t = x$

$$C_n = \frac{1}{T} \int_0^{2\pi} A_c e^{j[\beta \sin x - n x]} \frac{dx}{2\pi f_m}$$

$$\text{SHM } 1.801 - \text{SHM } 1.82$$

$$J_n(\beta) = J_{-n}(\beta) \quad n = \text{even}$$

$$J_n(\beta) = -J_{-n}(\beta) \quad n = \text{odd}$$

$$J_{-n}(\beta) = -J_n(\beta)$$

$$C_n = A_c \int_0^{2\pi} \frac{1}{2\pi} e^{j(\beta \sin x - nx)} dx$$

Bessel's function of I kind
nth order.

$$C_n = A_c J_n(\beta)$$

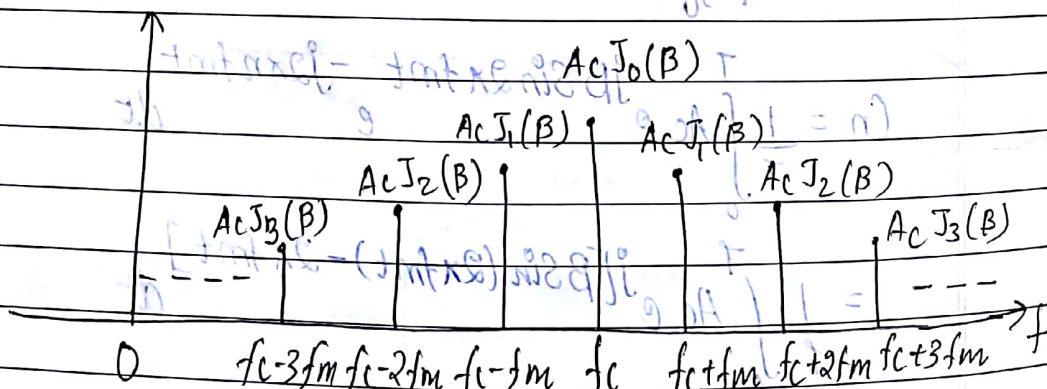
$$S_b(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi n f_m t}$$

$$y(t) = \text{Re} \left[\sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t} \right]$$

$$= \text{Re} \left[\sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi t (f_c + n f_m)} \right]$$

$$y(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos [2\pi t (f_c + n f_m)]$$

$$y(t) = A_c J_0(\beta) \cos(2\pi f_c t) + A_c J_1(\beta) \cos(2\pi t (f_c + f_m)) - A_c J_1(\beta) \cos(2\pi t (f_c - f_m)) + A_c J_2(\beta) \cos(2\pi t (f_c + 2f_m)) + A_c J_2(\beta) \cos(2\pi t (f_c - 2f_m)) - A_c J_3(\beta) \cos(2\pi t (f_c + 3f_m)) - A_c J_3(\beta) \cos(2\pi t (f_c - 3f_m)) + \dots$$



$$\text{Bandwidth} = (f_c + n f_m) - (f_c - n f_m)$$

$$= f_c + n f_m - f_c + n f_m$$

$$= 2n f_m$$

$$n = \infty$$

$$n = \infty$$

$$88.1 \text{ MHz} - 108.1 \text{ MHz}$$

Carrson's rule:-

$$\text{Bandwidth} = 2 \left(1 + \frac{1}{\beta} \right) \Delta f$$

for narrow band FM

$$\beta \ll 1$$

$$\frac{1}{\beta} \gg 1$$

$$\text{Bandwidth} \approx 2 \left(\frac{1}{\beta} \right) \Delta f$$

$$= \frac{2\Delta f}{\Delta f/f_m} = 2f_m$$

$$\begin{array}{l} \swarrow 95\text{MHz} \quad \searrow 95\text{MHz} + 95\text{KHz} \\ 95\text{MHz} - 95\text{KHz} \quad \underline{\underline{190\text{KHz}}} \\ \text{or } \frac{190\text{KHz}}{2} \end{array}$$

for wideband FM

$$\beta \gg 1, \quad \frac{1}{\beta} \ll 1$$

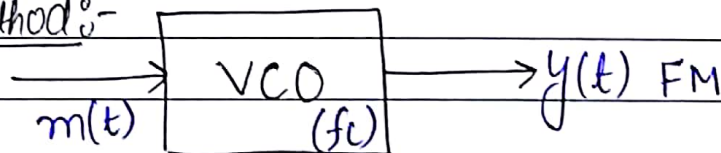
$$\text{Bandwidth} \approx 2\Delta f$$

$$\begin{aligned} \text{B.W.} &= 2(\Delta f + f_m) \\ &= 2(75\text{K} + 20\text{K}) \\ &= 190\text{KHz} \end{aligned}$$

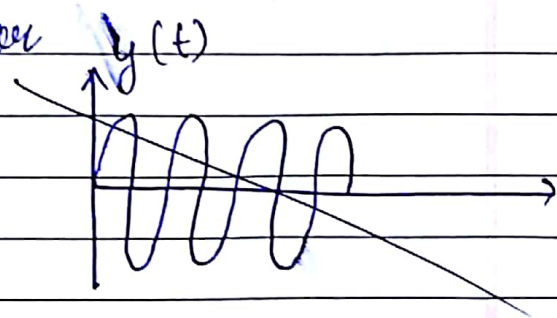
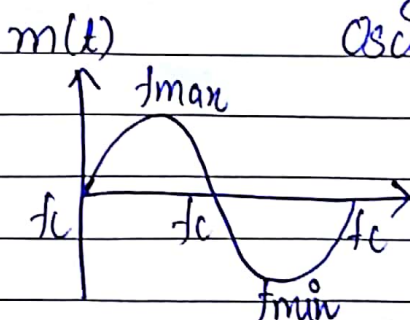
$\Delta f_{\text{max}} = 75\text{KHz}$ fixed by government of India.

$\Rightarrow$ FM modulator circuit:-

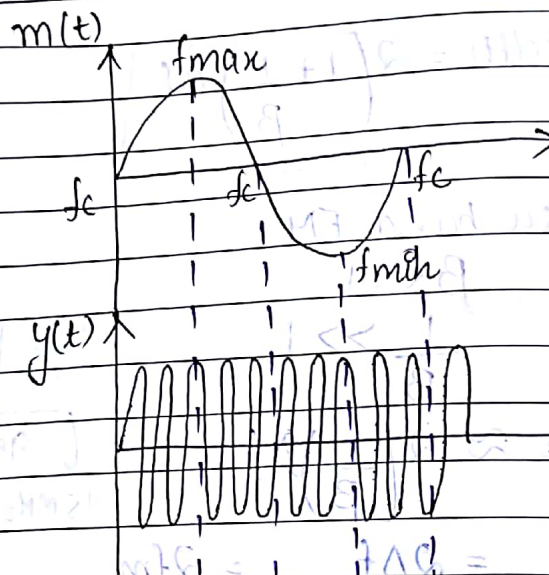
1. Direct method:-



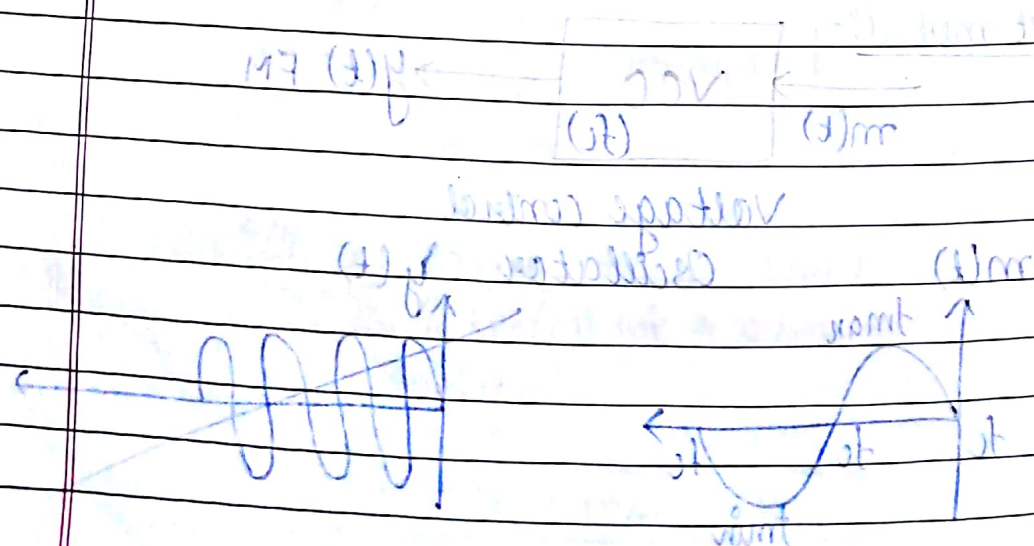
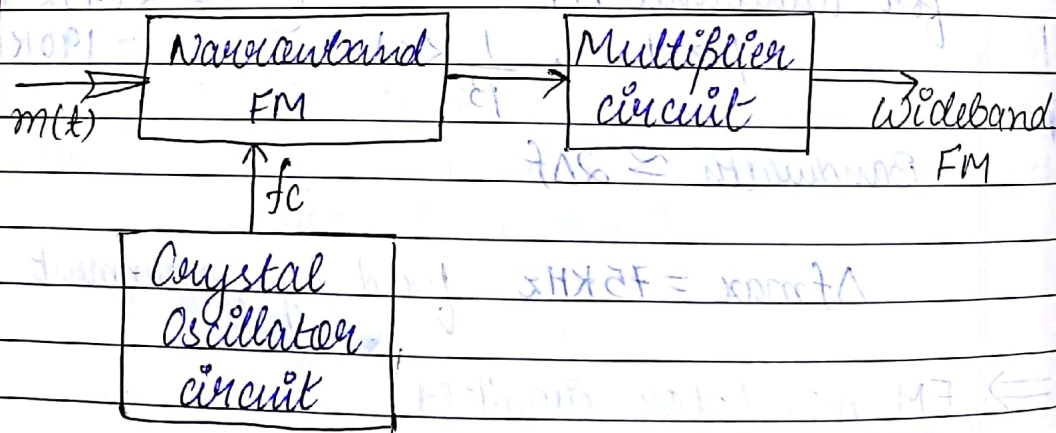
Voltage control
Oscillator



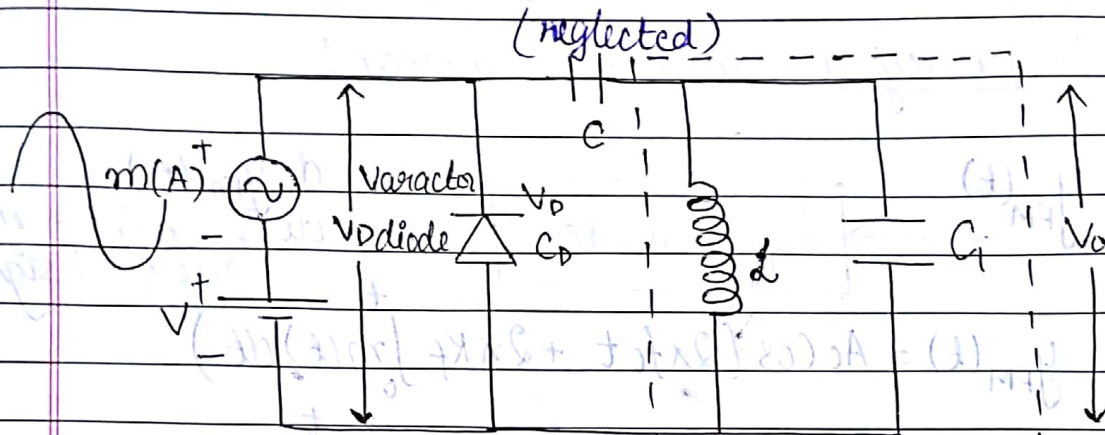
2. Indirect method:-



2. Indirect method:-



⇒ FM modulator using Varactor diode:-



Tank circuit

$$C_D = K(V_D)^{-1/2}$$

$$V_D = m(t) + V$$

$$C_D = K(m(t) + V)^{-1/2}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{L(C_1 + C_D)}}$$

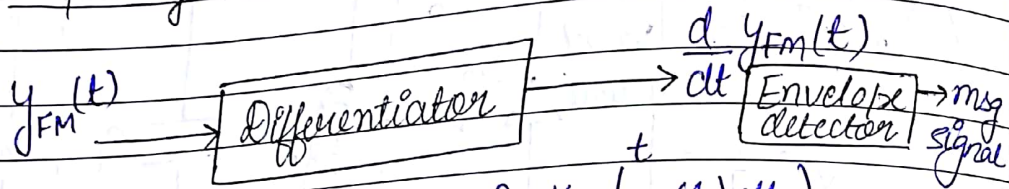
$$\omega = \frac{1}{\sqrt{L(C + K(V_D)^{-1/2})}}$$

$$= (L(C + K(V_D)^{-1/2}))^{-1/2}$$

$$\omega = (L(C + K(m(t) + V)^{-1/2}))^{-1/2}$$

⇒ FM Demodulation:-

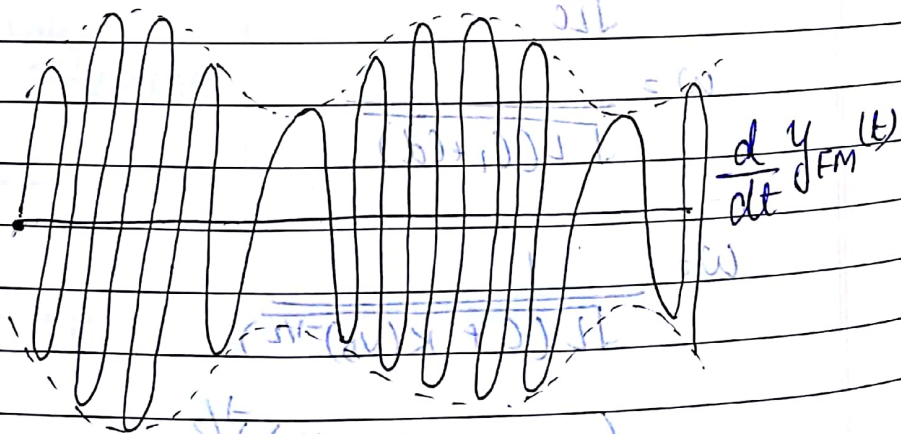
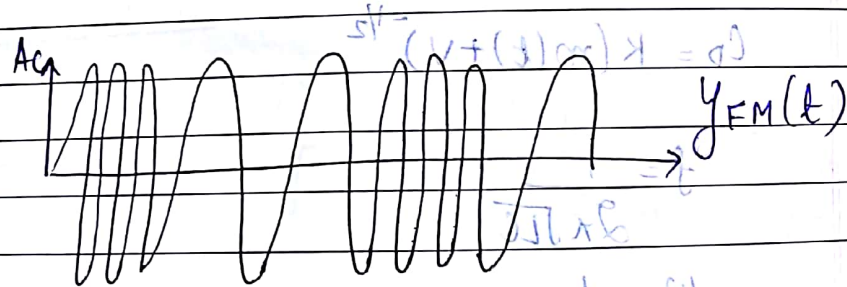
i) Frequency discrimination method:-



$$y_{FM}(t) = A_c \cos \left(2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right)$$

$$\frac{d}{dt} y_{FM}(t) = -A_c \sin \left(2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right) (2\pi f_c + 2\pi K_f m(t))$$

$$= -A_c (2\pi f_c + 2\pi K_f m(t)) \sin \left(2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right)$$



$$\frac{1}{f_c} \left(\frac{f_c + K_f m(t)}{f_c} \right) = 1 + \frac{K_f m(t)}{f_c}$$

modulation index

$$A_m = (A_c + A_m \sin \omega_m t) \sin \omega_c t$$

$m < 1$

$$A_m < A_c$$

$$\max |k_f \cdot m(t)| < f_c$$

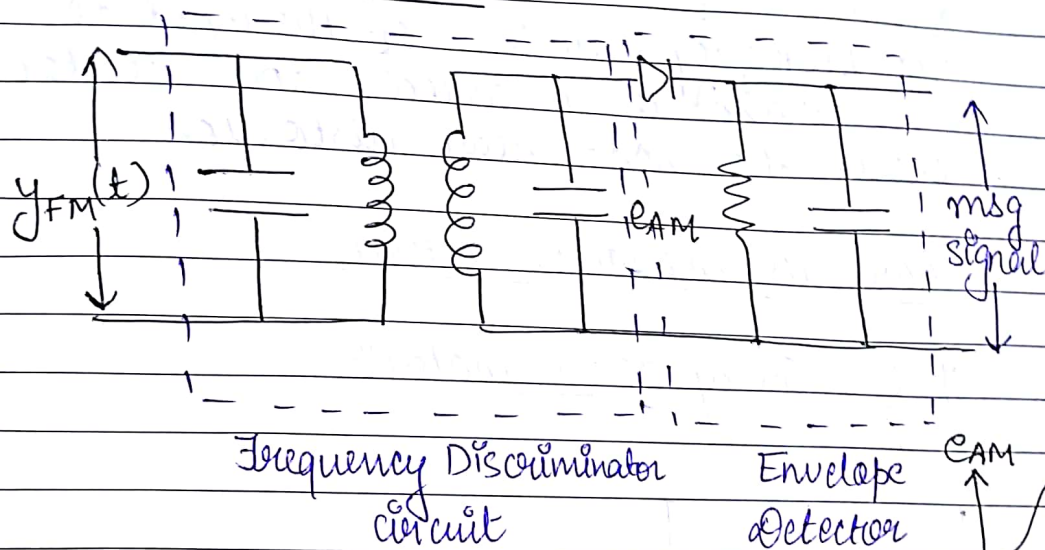
$$\Delta f < f_c$$



75KHz

MHz

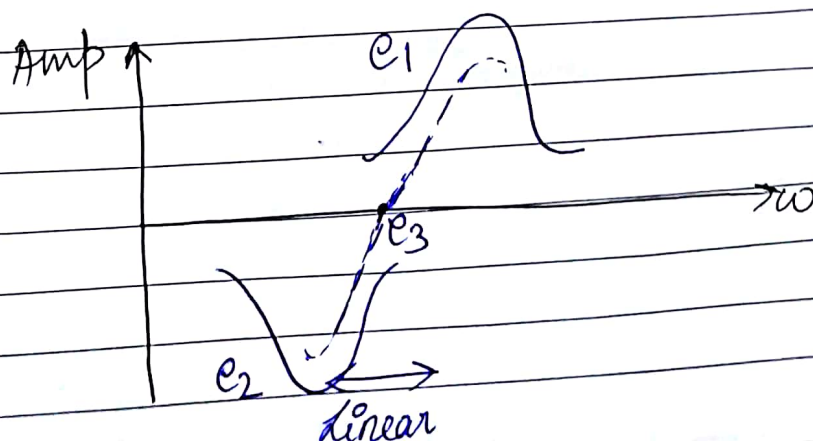
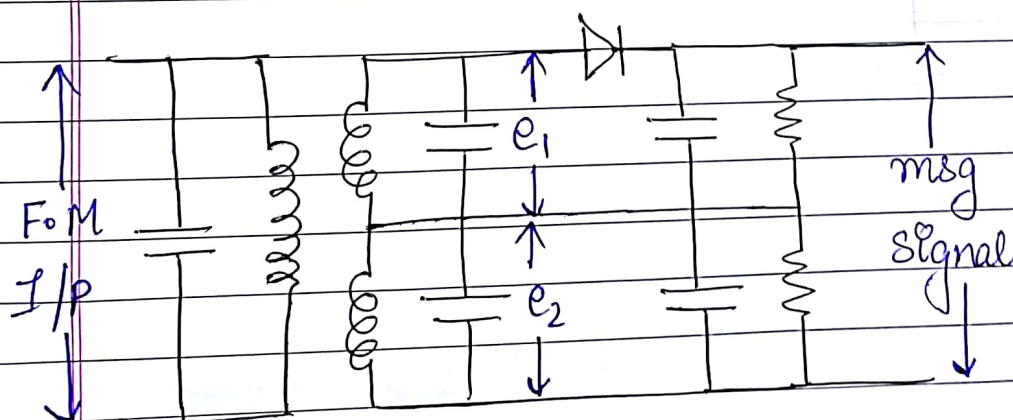
1→ Simple slope detector:-



Disadvantage:-

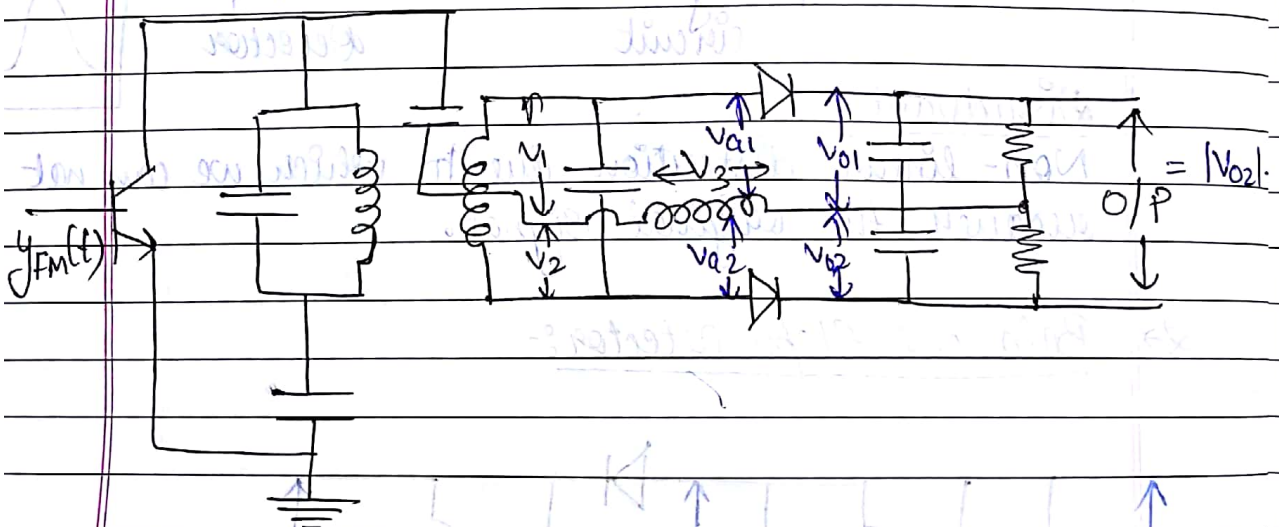
Non-linear distortion due to which we can not recover the original signal.

2→ Balanced Slope Detector:-



Disadvantages:-

The frequency deviation of the input FM signal is more than the balanced slope detector will create the non-linear distortion.

Phase discrimination method:-Foster Seeley discriminator:-

pass
band

↓

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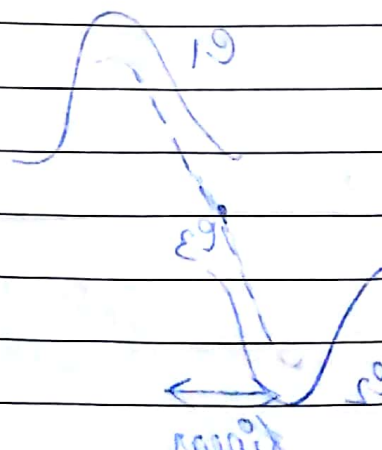
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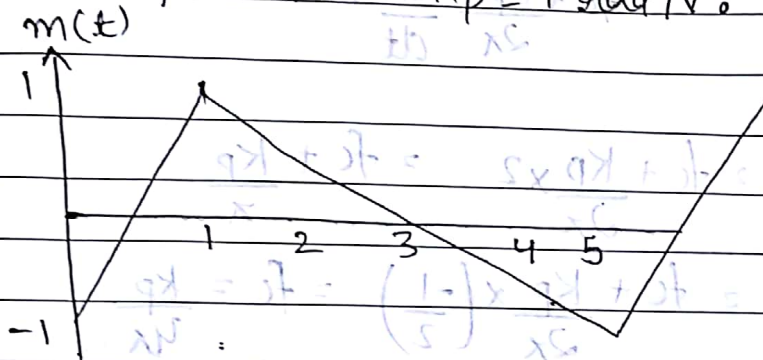
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Ques Determine the ratio of maximum instantaneous frequency and minimum instantaneous freq in PM for modulating signal as shown in fig. given that $f_c = 1 \text{ MHz}$, $K_p = 1 \text{ rad/V}$.



$$\frac{f_{\max}}{f_{\min}} = ?$$

Sol. $(y-1) = \frac{1-0}{1-0} (x-0) \quad \frac{2}{1-0} (x-0)$

$$y-1 = \frac{1}{2} x$$

$$y = \frac{x}{2} + 1 \quad y = \frac{2+x}{2}$$

$$(y-1) = \frac{5-1}{(-2)} (x-1)$$

$$(y-1) = \frac{1+1}{(1-0)} (x-0)$$

$$y-1 = 2x \quad y = 2x+1$$

$$(y-1) = \frac{-1-1}{5-1} (x-1)$$

$$= \left[\frac{-2}{4} (x-1) \right] \quad \frac{-x}{2} + \frac{3}{2} = y$$

$$y = 2x-1$$

$$\frac{-x}{2} + \frac{3}{2} = y$$

$$x_{PM}(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

$$f_i = \frac{1}{2\pi} \frac{d}{dt} [2\pi f_c t + k_p m(t)]$$

$$= f_c + \frac{k_p}{2\pi} \frac{d}{dt} m(t)$$

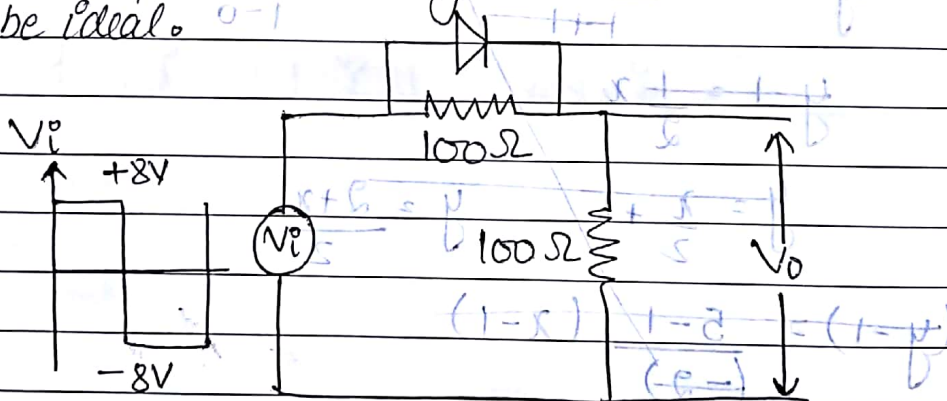
$$f_{i1} = f_c + \frac{k_p}{2\pi} \times 2 = f_c + \frac{k_p}{\pi}$$

$$f_{i2} = f_c + \frac{k_p}{2\pi} \times \left(-\frac{1}{2}\right) = f_c - \frac{k_p}{4\pi}$$

Ques

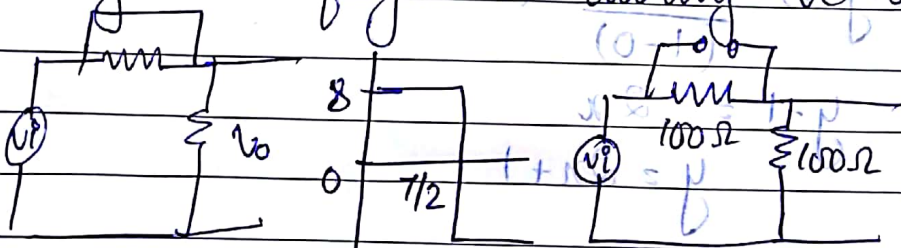
For the circuit shown in figure determine the output dc voltage assume the diode to be ideal.

Sol.



during +ve half cycle -

during -ve half cycle



$V_o = V_i$

$(1-x) = 4V$

$1 - 1 = (1-x)$

$$V_{DC} = \frac{1}{T} \left[\int_0^{T/2} 8 \cdot dt + \int_{T/2}^T 4 \cdot dt \right]$$

$$V_{DC} = \frac{8 \times \frac{T}{2} + 4 \times \frac{T}{2}}{T} = 4V$$

Noise:-

There are two types of Noise:-

- i) External Noise
- ii) Internal Noise

External Noise:-

- i) Atmospheric Noise
- ii) Solar Noise
- iii) Cosmic Noise Range $\rightarrow$ 8MHz to 1.43GHz
- iv) Industrial Noise

Internal Noise:-

- i) Thermal Noise $\rightarrow$ white noise or Johnson noise.
- ii) It is dependant on temp^o and the bandwidth of the signal.
The noise power is given by $P_n = KTB_n$
K is Boltzmann's constant.

$$K = 1.38 \times 10^{-23} \text{ J/K}$$

T = Temperature

$B_n =$ Required Bandwidth

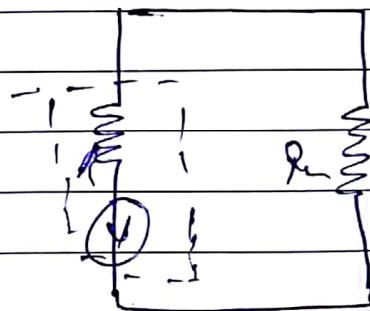
$$P_n = \frac{V^2}{R_L}$$

$$P_n = \left(\frac{V_n}{2} \right)^2 \frac{1}{R}$$

$$V_n^2 = 4R_n R$$

$$= 4KT B_n R$$

$$V_n = \sqrt{4KT B_n R}$$



Ques Calculate the thermal noise power level in dBW at the output of receiver if it has 10MHz bandwidth and temperature is 21°C .

Sol.

$$P_n = KTB_n$$

$$K = 1.38 \times 10^{-23} \text{ J/K}$$

$$T = \text{Temperature}$$

$$B_n = \text{Receiver Bandwidth}$$

$$P_n = \frac{V_r^2}{R}$$

$$P_n = \left(\frac{V_r}{\sqrt{2}} \right)^2$$

$$V_r = \sqrt{2 P_n R}$$

$$= \sqrt{KTB_n R}$$

$$V_r = \sqrt{KTB_n R}$$

Internal Noise:-

a) Thermal Noise:-

$$P_n = KTB_n$$

$$V_n = \sqrt{4KTB_n R}$$

b) Shot noise:- It is due to the random path of the current carriers flowing in a material.

$$I_s^2 = 2I_{dc} q B_n$$

B_n = Required Bandwidth

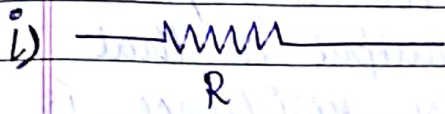
c) Transit time Noise:- The transit time noise occurs at high frequency when transit time of charge carriers crossing the semiconductor junction approaches the signal period.

* It occurs at high frequency.

d) Flicker Noise:- It results from the minute variation in resistance in semiconductor material. Flicker noise is inversely proportional to the frequency and sometimes called as $\frac{1}{f}$ noise or pink noise.

* It occurs at low frequency.

e) Partition Noise:- The partition noise is generated when the current gets divided in two or more paths.

Noise Calculation:-

$$P_n = KTB_n$$

$$V_n^2 = 4KTB_nR$$

ii) Addition of noise due to several sources:-

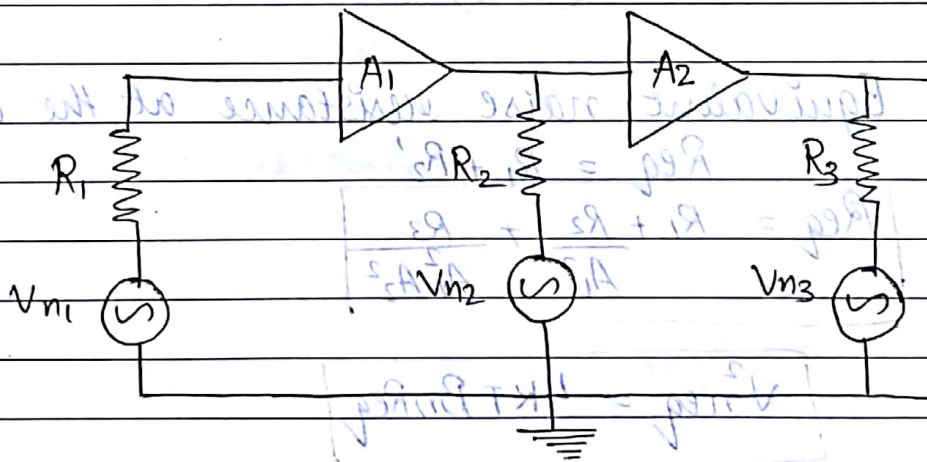
$$V_{n1}^2 = 4KTB_nR_1$$

$$V_{n2}^2 = 4KTB_nR_2$$

$$V_n^2 = V_{n1}^2 + V_{n2}^2$$

$$V_{nm}^2 = V_{ac}^2 + V_{dc}^2$$

$$V_n^2 = \sqrt{4KTB_n(R_1 + R_2)}$$

iii) Addition of noise due to several amplifiers in series:-

$$V_{n3} = \sqrt{4KTB_nR_3}$$

$$V_{n3}' = \sqrt{4KTB_nR_3'}$$

$$\frac{V_{n3}}{A_2} = \frac{\sqrt{4KTB_nR_3}}{A_2}$$

$$= \frac{\sqrt{4KTB_nR_3}}{A_2^2}$$

$$= \sqrt{\frac{4KTB_nR_3}{A_2^2}}$$

$$R_3' = \frac{R_3}{A_2^2}$$

R_3' is the resistance which placed at the input of second stage, It would produce the same noise voltage at the output as that produced by R_3 . When a noise resistance is transferred from output to its input, it must be divided by square of voltage gain of that stage.

$$R_{eq}' = R_2 + R_3'$$

$$= R_2 + \frac{R_3}{A_2^2}$$

$$R_2' = \frac{R_{eq}'}{A_1^2}$$

$$= \frac{R_2}{A_1^2} + \frac{R_3}{(A_1 A_2)^2}$$

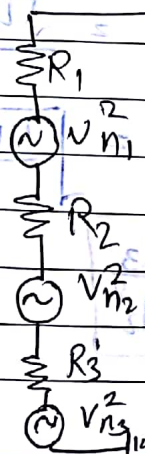
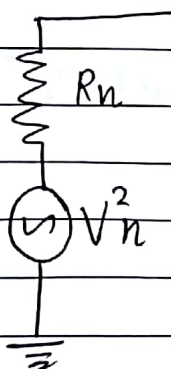
Equivalent noise resistance at the input

$$R_{eq} = R_1 + R_2'$$

$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2}$$

$$V_{n_{eq}}^2 = 4KT B R_{eq}$$

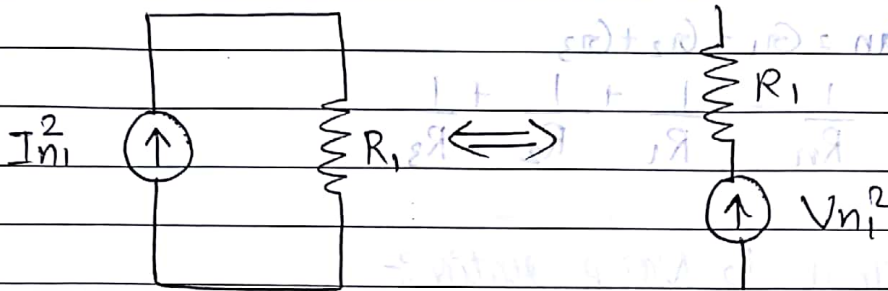
iv) Resistors are in series



$$V_n^2 = V_{n1}^2 + V_{n2}^2 + V_{n3}^2$$

$$R_n = R_1 + R_2 + R_3$$

v) Resistors are in parallel:-

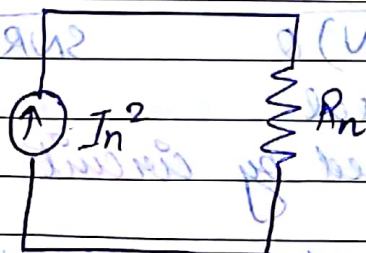
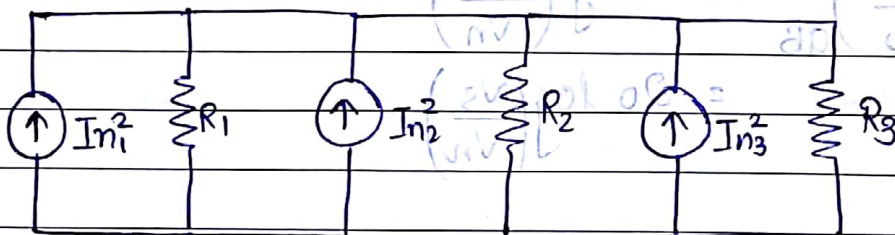


$$V_{n1}^2 = 4KT \cdot B_n R_1$$

$$I_{n1}^2 = \frac{V_{n1}^2}{R_1^2}$$

$$= \frac{4KT B_n}{R_1}$$

$$= 4KT B_n G_1$$



$$I_{n1}^2 = 4KT B n G_1$$

$$I_{n2}^2 = 4KT B n G_2$$

$$I_{n3}^2 = 4KT B n G_3$$

$$I_n^2 = I_{n1}^2 + I_{n2}^2 + I_{n3}^2$$

$$= 4KT B n G_n$$

$$G_n = G_1 + G_2 + G_3$$

$$\frac{1}{R_n} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Signal to Noise ratio :-

$$\frac{P_s}{P_n} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

$$= \frac{V_s^2 / R}{V_n^2 / R}$$

$$= \left(\frac{V_s}{V_n} \right)^2$$

$$\left(\frac{S}{N} \right)_{dB} = 10 \log \left(\frac{V_s}{V_n} \right)^2$$

$$= 20 \log \left(\frac{V_s}{V_n} \right)$$

Noise factor :-

$$F = \frac{(S/N)_i}{(S/N)_o} = \frac{\text{SNR at I/P}}{\text{SNR at O/P}}$$

$$F = 1 \text{ Ideal}$$

Noise produced by circuit is zero.

$$F = \frac{P_{si} / P_{ni}}{P_{so} / P_{no}} = \left(\frac{P_{si}}{P_{so}} \right) \cdot \left(\frac{P_{no}}{P_{ni}} \right)$$

Noise Figure:-

$$NF = 10 \log_{10} F$$

↓

Noise Factor in dB

Ideal value = 0

Figure of merit:-

$$FOM = \frac{(S/N)_{o/p}}{(S/N)_{i/p}}$$

$$(1) -z-1 \quad \text{and} \quad -z-1 = (z+1) \text{ and}$$

$$f(1/2) = 7$$

Ques The 1st stage of two stage amplifier [Gain & B.W = constant] has voltage gain of 10, a 600Ω input resistor, 1600Ω input resistance to the second stage amplifier having gain 20 and output resistance of second stage is $100k\Omega$. Determine the equivalent noise resistance.

Sol. $R_{eq} = 600 + \frac{1600}{(10)^2} + \frac{100 \times 10^3}{10^2 \cdot 20^2} = (1020.5) \Omega$

$$= 6070 + 16 + 2.5$$

$$= 618.2 \Omega \angle 93^\circ = 0.09 + j0.09$$

(B-Bandwidth) = 8

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots$$

$$K_{eq} = \frac{[P]}{[R]}$$

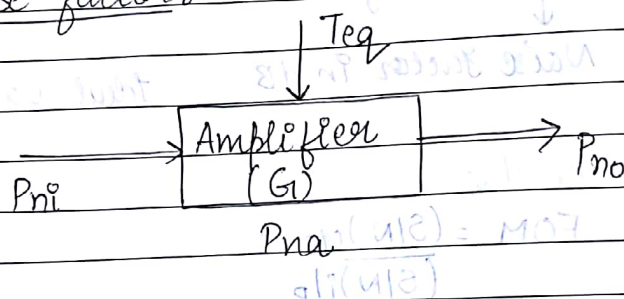
Baumwolle ist keine einjährige Pflanze

$$K \perp B_m = (I - 1)K \perp B_m$$

$$r(1-f) = pT$$

$$T(1-\gamma) e^{-\rho T}$$

Amplifier Noise Contribution in terms of Noise factor:-



$$P_{ni}(\text{total}) = P_{ni} + P_{na} \quad \text{--- (1)}$$

$$P_{no} = G P_{ni}(\text{total})$$

$$F = \frac{(S/N)_i}{(S/N)_o}$$

$$= \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}$$

$$F = \frac{P_{no}}{P_{ni} \times G}$$

$$P_{no} = F P_{ni} G$$

$$P_{ni}(\text{total}) = \frac{P_{no}}{G} = \frac{F G P_{ni}}{G} = F P_{ni}$$

from eq (1)

$$P_{ni} + P_{na} = F P_{ni}$$

$$P_{na} = (F-1) P_{ni}$$

$$P_{na} = (F-1) K T B_n \quad (B = \text{Bandwidth})$$

from $P_n = K T B_n$

Bandwidth is same on both the sides

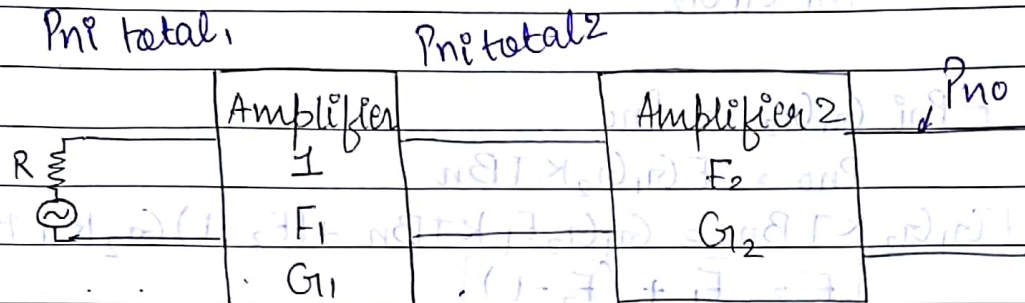
$$K T_a B_n = (F-1) K T B_n$$

$$T_a = (F-1) T$$

$$T_a = (F-1) T$$

T_{eq} = Noise temp^o due to Amplifier

Noise factor of Amplifier Connected in Cascade



$$P_{ni} = KTB_n$$

Total Input noise at Amplifier 1

$$P_{ni\ (total)} = P_{ni} + P_{na}$$

$$= P_{ni} + (F_1 - 1) P_{ni}$$

$$= F_1 P_{ni}$$

$$P_{ni\ (total)}_{I/P} = F_1 KTB_n$$

Total I/P noise power at I/P of Amplifier 2

$$P_{ni\ (total)}_{2I/P} = G_1 F_1 KTB_n + (F_2 - 1) KTB_n$$

Total O/P noise power at O/P of Amplifier 2.

$$P_{no} = G_2 P_{ni\ (total)}_2$$

$$P_{no} = G_1 G_2 F_1 KTB_n + (F_2 - 1) G_2 KTB_n$$

$$F = \frac{(S/N)_i}{(S/N)_o}$$

$$= \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}$$

$$F = \frac{P_{no}}{P_{ni} G_1 G_2}$$

$$F P_{ni} G_1 G_2 = P_{no}$$

$$P_{no} = F G_1 G_2 K T B_n$$

$$F G_1 G_2 K T B_n = G_1 G_2 F_1 K T B_n + (F_2 - 1) G_2 K T B_n$$

$$F = F_1 + \frac{(F_2 - 1)}{G_1}$$

for n no. of Amplifiers connected in cascade

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \dots$$

Equivalent Noise temp. of Amplifier in Cascade :-

$$F - 1 = \frac{F_1 - 1}{G_1} + \frac{F_2 - 1}{G_1 G_2} + \dots + \frac{(F_n - 1)}{G_1 G_2 \dots G_{n-1}}$$

$$(F - 1) = \frac{(F_1 - 1)}{G_1} + \frac{F_2 - 1}{G_1 G_2} + \frac{F_3 - 1}{G_1 G_2 G_3} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}}$$

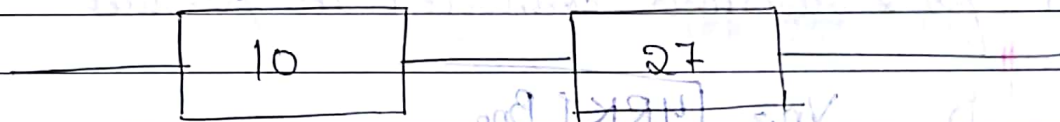
$$\frac{T_{eq}}{T} = \frac{T_1}{T} + \frac{T_2}{T G_1} + \dots + \frac{T_n}{T G_1 G_2 \dots G_{n-1}}$$

$$R_{eq} = R_1 + \frac{R_2}{G_1^2} + \frac{R_3}{G_1^2 G_2^2} + \dots$$

Ques The first stage of 2 stage Amplifier has voltage gain of 10, a 600Ω input resistance. A 1600Ω eq. noise resistance and $27K\Omega$ output resistor. For the second stage these values are 25, $81K\Omega$, $10K\Omega$ and $1M\Omega$ respectively. Find the equivalent Input noise resistance of this 2 stage amplifier.

Sol $G_1 = 10$ $R_{i1} = 600\Omega$ $R_{eq} = 1600\Omega$
 $R_{o1} = 27K\Omega$

$$R_{eq} = R_1 + \frac{R_2}{A_1^2} + \frac{R_3}{A_1^2 A_2^2}$$

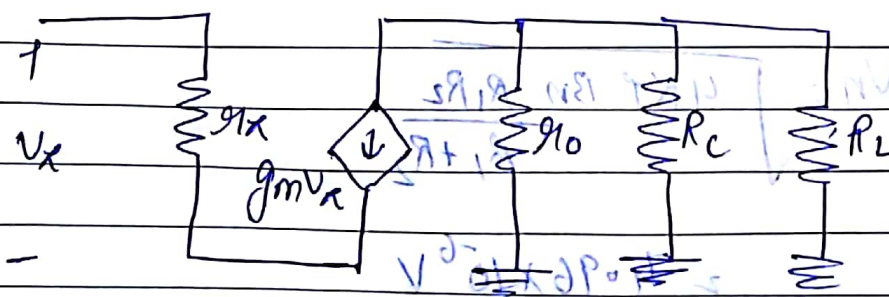
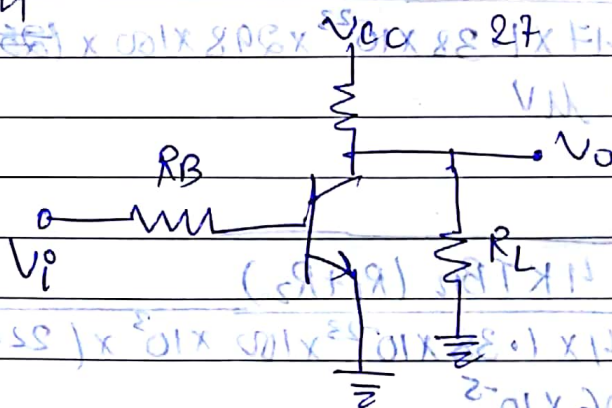


Amplifier 1 Amplifier 2

$$R_{i1} = 600\Omega$$

$$R_{eq1} = 1600\Omega$$

$$\begin{array}{ccccc} R_o & R_{eq2} & R_{i2} & R_o \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 27 & 10 & 81 & 1M\Omega \end{array}$$



$R_C \parallel r_o \parallel R_L$ and $r_o \parallel R_L$ by this $R_{o1} \parallel R_{eq2}$.

$$R_1 = R_1 + R_{eq1}$$

$$R_2 = (R_{01} \parallel R_{eq2}) + R_{in2}$$

$$R_3 = R_0$$

$$G_1 = 10$$

$$G_2 = 25$$

$$R_{eq} = R_1 + \frac{R_2}{G_1^2} + \frac{R_3}{G_1^2 G_2^2}$$

$$R_{eq} = \cancel{1103K\Omega} \quad 227K\Omega$$

Ques → Two resistors $22K\Omega$ and $47K\Omega$ are at room temp determine the thermal noise voltage for a bandwidth of $100KHz$ i) for each resistor
ii) for 2 resistors connected in series
iii) for 2 resistors connected in parallel

Sol.

$$i) \quad V_{n1} = \sqrt{4RKTB_{no}} \\ = \sqrt{4 \times 22 \times 1.38 \times 10^{-23} \times 100 \times (25+273)} \times 10^6 \\ = 6.015 \mu V$$

$$V_{n2} = \sqrt{4RKTB_{no}} \\ = \sqrt{4 \times 47 \times 1.38 \times 10^{-23} \times 298 \times 100 \times (25+273)} \times 10^6 \\ = 8.792 \mu V$$

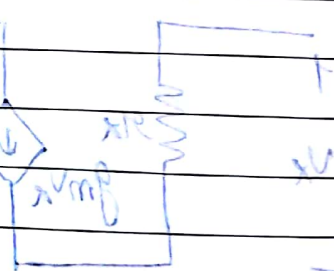
ii)

In series

$$V_n = \sqrt{4KTB_n (R_1 + R_2)} \\ = \sqrt{4 \times 1.38 \times 10^{-23} \times 100 \times 10^3 \times (22+47)} \\ = 1.06 \times 10^{-5}$$

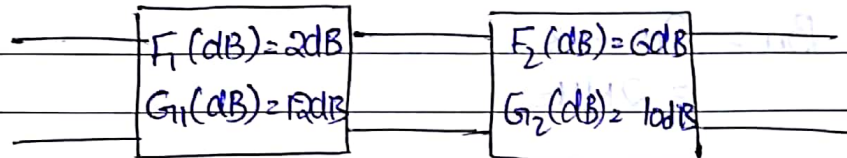
iii)

$$V_n = \sqrt{4KTB_n \frac{R_1 R_2}{R_1 + R_2}} \\ = 4.96 \times 10^{-6} V$$



Ques? A Cascaded two stage amplifier the first stage has noise figure of 2dB and power gain of 12dB. The second stage has noise figure of 6dB and power gain of 10dB and determine the overall noise figure in dB.

Sol.



$$F = F_1 + \frac{F_2 - 1}{G_1} \quad \leftarrow \text{In absolute scale}$$

$$P_1 \times 10^{0.1 \times 12} \times (1 + F_1) \times 10^{0.1 \times 10} = P_2 \times 10^{0.1 \times 12} \times (1 + F_2)$$

$$= 2 +$$

$$F_1 = 10^{\log 2} = 3.010$$

$$F_2 = 10^{\log 6} = 7.78$$

$$F_3 = 10^{\log 12} = 10.79$$

} for absolute to dB

$$F_1 = 10^{2/10} = 10^{0.2} = 1.58$$

$$F_2 = 10^{6/10} = 10^{0.6} = 3.98$$

$$G_1 = 10^{12/10} = 10^{1.2} = 15.84$$

$$G_2 = 10^{10/10} = 10^1 = 10$$

$$F = 1.58 + \frac{3.98 - 1}{15.84} = 1.768 \quad (1.772)$$

$$(1.772) \times 10^{0.1 \times 12} \times (1 + F_2) = (1.772) \times 10^{0.1 \times 12} \times (1 + F_2)$$

$$F_{dB} = 10 \log 1.772 = 2.48 \text{ dB}$$

$$= 2.48 \text{ dB} + 0 \text{ dB} = 2.48 \text{ dB}$$

$$\left(\frac{10^{0.1 \times 12}}{10^{0.1 \times 12}} \right) = 1$$

Ques An amplifier operating on a frequency range of 18 MHz to 20 MHz has $10\text{ k}\Omega$ input resistor. Determine the rms noise voltage at the input of this amplifier at temp. 27°C .

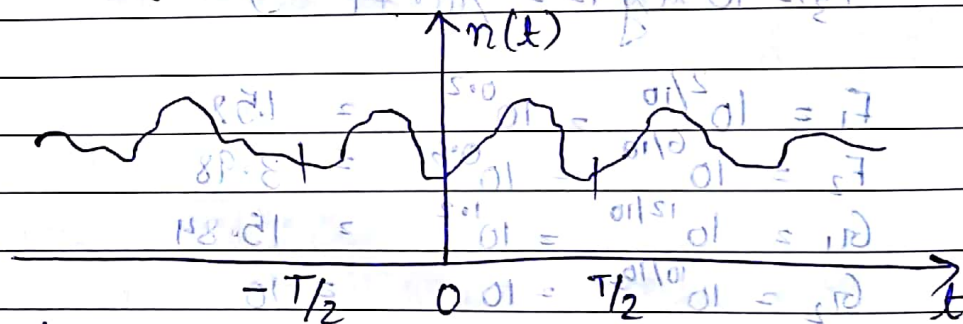
Sol.

$$B_n = 20 - 18 \\ = 2\text{ MHz}$$

$$V_n = \sqrt{4KT B_n R}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times (27 + 273) \times 2 \times 10^6 \times 10^4} \\ = 10.81 \times 10^{-5} \text{ V}$$

Frequency domain representation of noise:-



$$\Delta f = 1$$

(SFF-1)

$$n_s(t) = \sum_{n=1}^{\infty} a_n \cos(2\pi n \Delta f t) + b_n \sin(2\pi n \Delta f t)$$

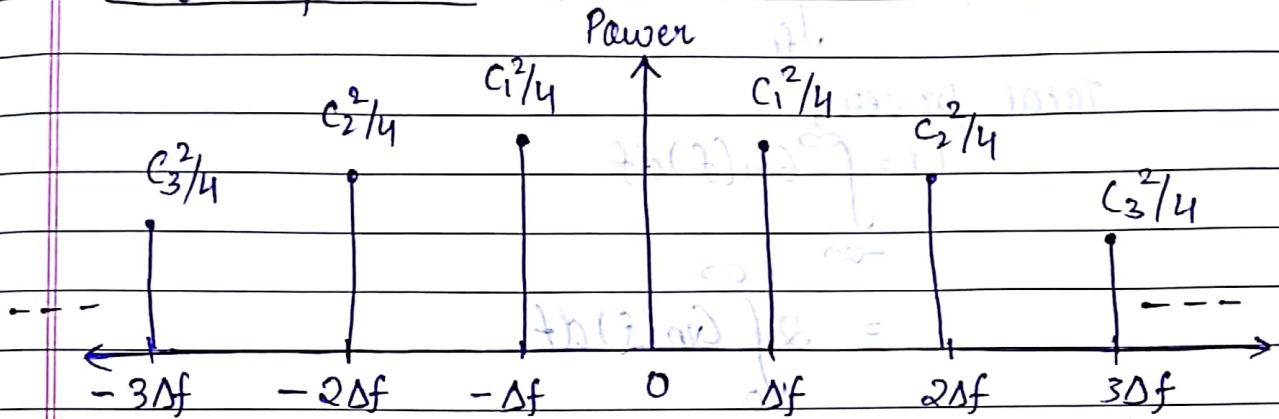
$$n_s(t) = \sum_{n=1}^{\infty} C_n \cos(2\pi n \Delta f t + \phi_n)$$

$$C_n^2 = a_n^2 + b_n^2$$

$$\phi_n = \tan^{-1} \frac{b_n}{a_n}$$

$$n_s(t) = C_1 \cos(2\pi \Delta f t + \theta_1) + C_2 \cos(2\pi(2\Delta f)t + \theta_2) + C_3 \cos(2\pi(3\Delta f)t + \theta_3) + \dots$$

Power spectrum



Power spectral density

$$G_K(K\Delta f) = G_K(-K\Delta f) = \frac{C_K^2}{4\Delta f} = \frac{a_K^2 + b_K^2}{4\Delta f}$$

$$P_K = G_K(K\Delta f) \cdot \Delta f = \frac{C_K^2}{4}$$

$$P(f_1 \rightarrow f_2) = \int_{f_1}^{f_2} G_K(K\Delta f) df + \int_{-f_2}^{-f_1} G_K(-K\Delta f) df$$

$$\lim_{\Delta f \rightarrow 0} \sum_{K=1}^{\infty} C_K \cos(2\pi K\Delta f t + \theta_K)$$

$$\overline{C_K^2} = \overline{a_K^2} + \overline{b_K^2}$$

$$G_n(f) = \lim_{\Delta f \rightarrow 0} \frac{\overline{C_K^2}}{4\Delta f} = \lim_{\Delta f \rightarrow 0} \frac{\overline{a_K^2 + b_K^2}}{4\Delta f}$$

$$P(f_1 \rightarrow f_2) = \int_{f_1}^{f_2} G_n(f) df + \int_{-f_2}^{-f_1} G_n(f) df$$

$$P(f_1 \rightarrow f_2) = 2 \int_{f_1}^{f_2} G_n(f) df$$

Total power

$$P_T = \int_{-\infty}^{\infty} G_n(f) df$$

$$= 2 \int_0^{\infty} G_n(f) df$$

Superposition of Noise:-

$$n_1(t) \rightarrow P_1$$

$$n_2(t) \rightarrow P_2$$

$$n_1(t) + n_2(t) \rightarrow P_{12}$$

$$P_{12} = P_1 + P_2$$

$$P_1 = E[n_1^2(t)]$$

$$P_2 = E[n_2^2(t)]$$

$$P_{12} = E[(n_1(t) + n_2(t))^2]$$

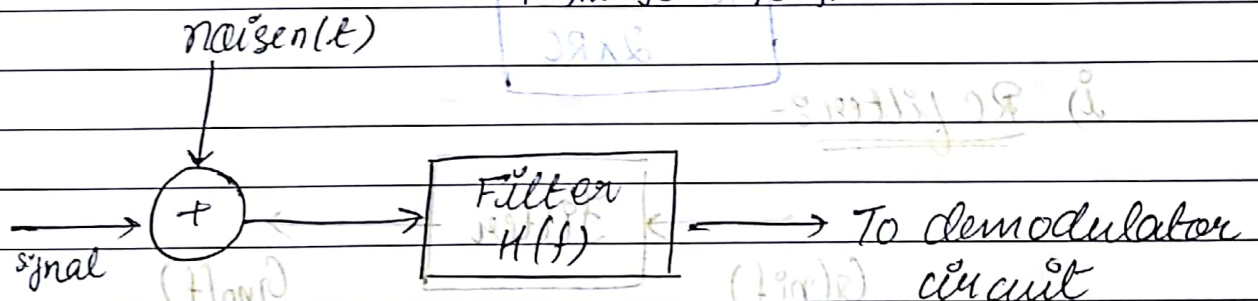
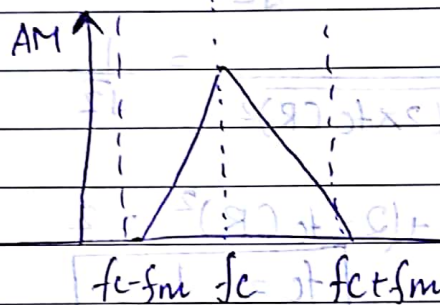
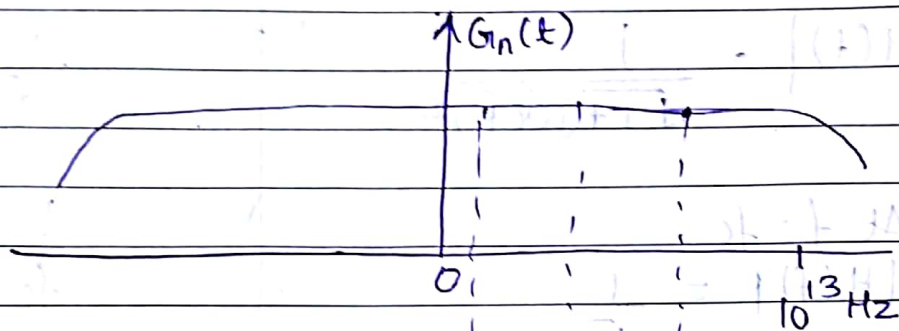
$$= E[n_1^2(t) + 2n_1(t)n_2(t) + n_2^2(t)]$$

$$= E[n_1^2(t)] + E[n_2^2(t)] + 2E[n_1(t)n_2(t)]$$

$$P_{12} = P_1 + P_2$$

Linear filtering of Noise:

- * Gaussian Noise
- * White Noise

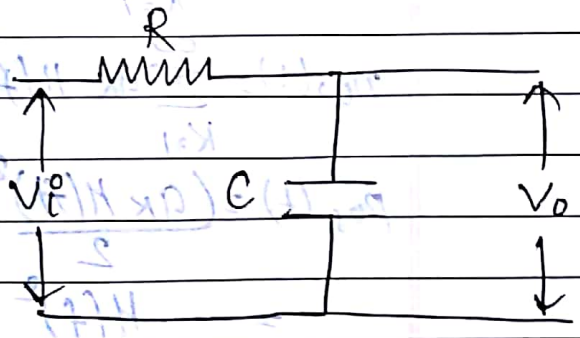


Low pass filter:-

$$|H(f)| = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

$$V_o = \frac{V_i \times 1/SC}{R + \frac{1}{SC}}$$

$$= \frac{V_i}{SCR + 1}$$



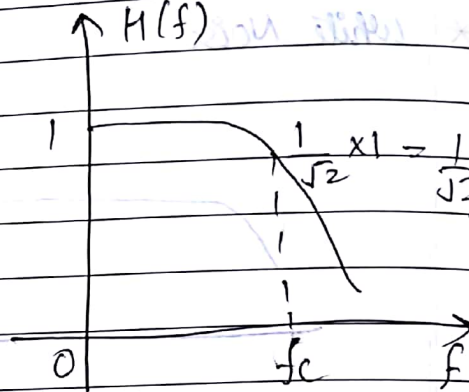
$$H(f) = \frac{V_o}{V_i} = \frac{1}{1 + j\omega CR}$$

$$= \frac{1}{1 + j\omega CR}$$

$$|H(f)| = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

At $f = f_c$

$$|H(f)| = \frac{1}{\sqrt{2}}$$



$$\frac{1}{\sqrt{1 + (2\pi f_c CR)^2}} = \frac{1}{\sqrt{2}} \quad \text{MA}$$

$$1 + (2\pi f_c CR)^2 = 2$$

$$f_c = \frac{1}{2\pi RC}$$

i) RC filter:-



$$G_{mo}(f) = G_{mi}(f) |H(f)|^2$$

$$n_s(t) = \sum_{k=1}^{\infty} a_k \cos(2\pi k f_c t) + b_k \sin(2\pi k f_c t)$$

$$n_{os}(t) = \sum_{k=1}^{\infty} a_k H(f) \cos(2\pi k f_c t) + b_k H(f) \sin(2\pi k f_c t)$$

$$P_{os}(t) = \frac{(a_k H(f))^2 + (b_k H(f))^2}{2} \cdot \frac{1}{1 + \frac{1}{Q^2}}$$

$$= \frac{H(f)^2}{2} (a^2 k + b^2 k) \cdot \frac{1}{1 + \frac{1}{Q^2}}$$

$$P_{is}(t) = \frac{a^2 K}{2} + \frac{b^2 K}{2}$$

$$\frac{P_{os}(t)}{t} = \frac{|H(f)|^2 P_{is}(t)}{f}$$

$$G_{no}(f) = |H(f)|^2 G_{ni}(f)$$

$$G_{no}(f) = \frac{\eta}{2} \left[\frac{1}{\sqrt{1 + (f/f_c)^2}} \right]^2$$

$$G_{no}(f) = \frac{\eta}{2} \left(\frac{1}{1 + (f/f_c)^2} \right)$$

Noise power at the output of the filter

$$N_o = \int_{-\infty}^{\infty} G_{no}(f) df$$

Linear filtering of noise t :-

$$N_o = \int_{-\infty}^{\infty} \frac{\eta}{2} \frac{1}{1 + (f/f_c)^2} df$$

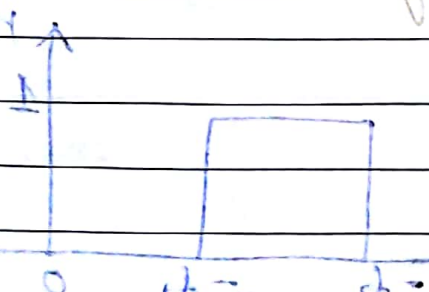
$$\text{Let } f/f_c = x$$

$$N_o = \frac{\eta}{2} \int_{-\infty}^{\infty} \frac{1}{1 + x^2} f_c dx$$

$$= \frac{\eta f_c}{2} [\tan^{-1} x]_{-\infty}^{\infty}$$

$$= \frac{\eta f_c}{2} \left[\frac{\pi}{2} + \frac{\pi}{2} \right]$$

$$N_o = \frac{\eta f_c \pi}{2}$$



ii) Rectangular (Ideal) Low pass filter :-

$$H(f) = \begin{cases} 1 & -B \leq f \leq B \\ 0 & \text{otherwise} \end{cases}$$

$H(f)$ is transfer function of low pass filter

$$G_{no}(f) = G_{ni}(f) |H(f)|^2$$

$$N_o = \int_{-\infty}^{\infty} G_{ni}(f) |H(f)|^2 df = \int_{-B}^B \frac{\eta}{2} \cdot 1 df$$

$$= \frac{\eta}{2} [f]_{-B}^B$$

$$= \frac{\eta}{2} [B - (-B)]$$

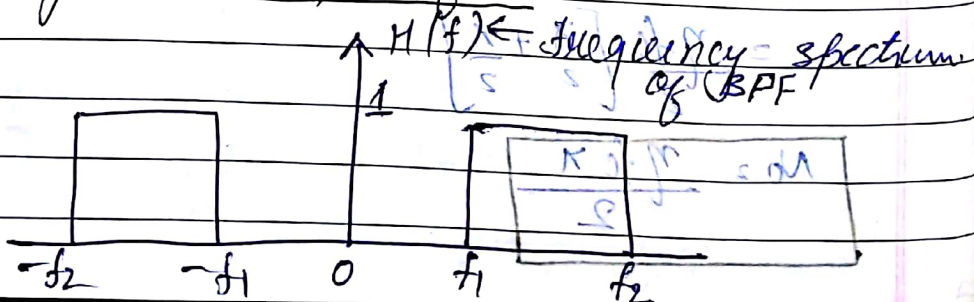
$$G_{no}(f) = \boxed{N_o = \eta B}$$

Noise Bandwidth :-

The rectangular bandwidth B is adjusted so that the real filter transmit the same noise power than the bandwidth B is called as the noise bandwidth of real filter.

$$\boxed{B = \frac{f_c \cdot x}{2}}$$

iii) Rectangular Band pass filter :-



$$G_{no}(f) = G_{ni}(f) |H(f)|^2$$

$$N_0 = \int_{-\infty}^{\infty} G_{ni}(f) |H(f)|^2 df = \int_{-\infty}^{\infty} \frac{\eta}{2} |H(f)|^2 df$$

$$= \frac{\eta}{2} \int_{-f_2}^{-f_1} 1 \cdot df + \frac{\eta}{2} \int_{f_1}^{f_2} 1 \cdot df$$

$$= \frac{\eta}{2} \left[f \right]_{-f_2}^{-f_1} + \frac{\eta}{2} \left[f \right]_{f_1}^{f_2}$$

$$= \frac{\eta}{2} [-f_2 + f_1] + \frac{\eta}{2} [f_2 - f_1]$$

$$\boxed{N_0 = \eta (f_2 - f_1)}$$

iv) Differentiating filter:-

$$V_0 = -RC \frac{dV_i}{dt}$$

taking $-RC$ as time constant τ

$$V_0 = \tau \frac{dV_i}{dt}$$

$$H(f) = j2\pi\tau f$$

$$G_{no}(f) = G_{ni}(f) |H(f)|^2$$

$$N_0 = \int_{-\infty}^{\infty} G_{no}(f) df$$

$$= \int_{-\infty}^{\infty} G_{ni}(f) |H(f)|^2 df$$

$$N_0 = \int_{-B}^B \frac{\eta}{2} (2\pi\tau f)^2 df$$

$$N_0 = \frac{\eta}{2} \times 4\pi^2 \times \tau^2 \left[\frac{f^3}{3} \right]_{-B}^B$$

$$= \frac{\eta}{2} \times 4\pi^2 \times \tau^2 \left[\frac{B^3}{3} + \frac{B^3}{3} \right]$$

$$N_0 = \frac{\eta 4\pi^2 \tau^2 B^3}{3}$$

Integrating filter:-

$$H(f) = \frac{1}{j\omega\tau}$$

$$G_{no}(f) = G_{ne} |H(f)|^2$$

$$= \frac{\eta}{2} \frac{1}{\omega^2 \tau^2}$$

$$N_0 = \int_{-\infty}^{\infty} G_{no}(f) df$$

$$= \int_{-B}^B \frac{\eta}{2} \frac{1}{\omega^2 \tau^2} df$$

$$= \int_{-B}^B \frac{\eta}{2} \frac{1}{4\pi^2 f^2 \tau^2} df$$

$$= \int_{-B}^B \frac{\eta}{8\pi^2 \tau^2} f^{-2} df$$

$$= \frac{\eta}{8\pi^2 \tau^2} \left[\frac{f^{-2+1}}{-1} \right]_{-B}^B$$

$$N_0 = - \frac{\eta}{8\pi^2 \tau^2} \left[\frac{1}{B} - \left(-\frac{1}{B} \right) \right]$$

$$N_0 = - \frac{\eta}{4\pi^2 \tau^2 B}$$

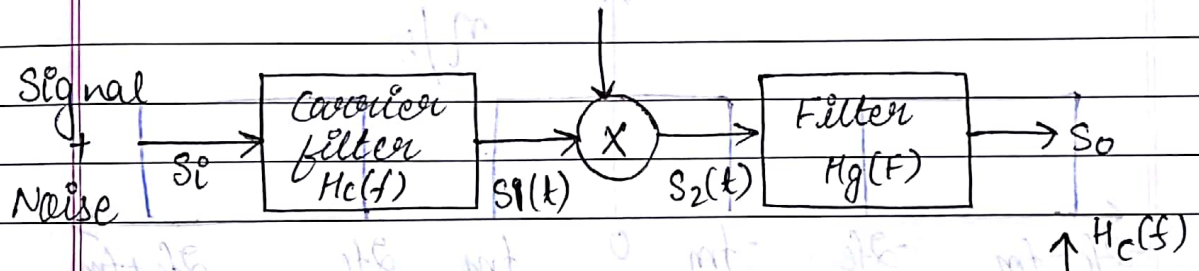
Quadrature component of noise :-

$$n(t) = \lim_{\Delta f \rightarrow 0} \sum_{k=1}^{\infty} C_k \cos(2\pi K \Delta f t + \theta_k)$$

$$\begin{aligned} & \cos(2\pi K \Delta f t + \theta_k) \\ &= \underbrace{\cos \theta_k}_{\text{In phase}} \cos(2\pi K \Delta f t) - \underbrace{\sin \theta_k}_{\text{Quadrature Component}} \sin(2\pi K \Delta f t) \end{aligned}$$

$$n(t) = n_c(t) \cos(2\pi K \Delta f t) - n_s(t) \sin(2\pi K \Delta f t)$$

Noise Calculation for SSB-SC :-



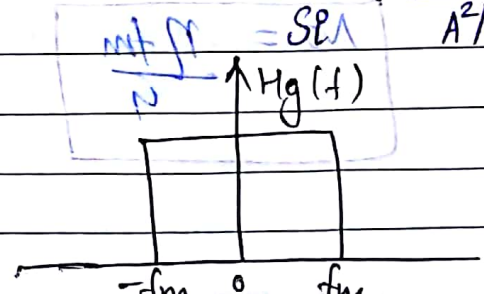
$$S_i = A \cos(2\pi(f_c + f_m)t)$$

$$\begin{aligned} S_2(t) &= S_1(t) \cos 2\pi f_c t \\ &= A \cos(2\pi(f_c + f_m)t) \cos 2\pi f_c t \\ &= \frac{A}{2} \cos(2\pi(2f_c + f_m)t) + \frac{A}{2} \cos(2\pi f_m t) \end{aligned}$$

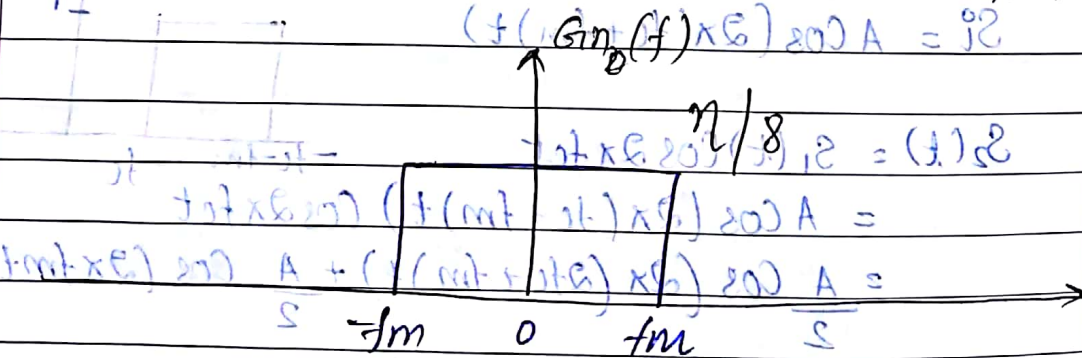
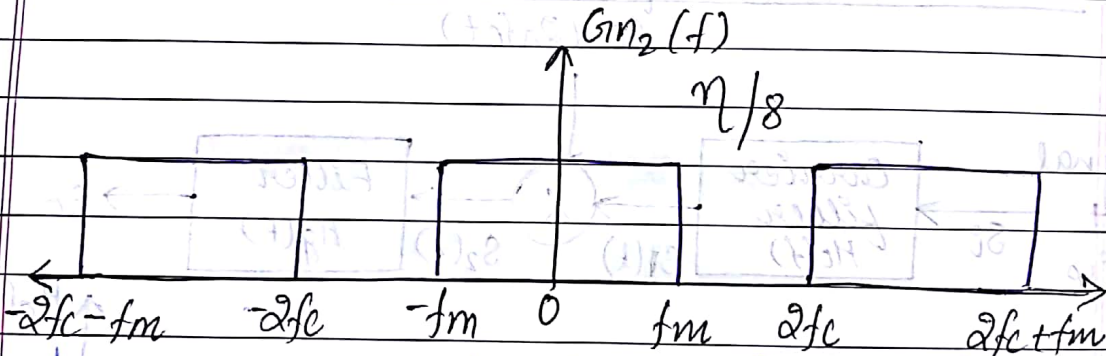
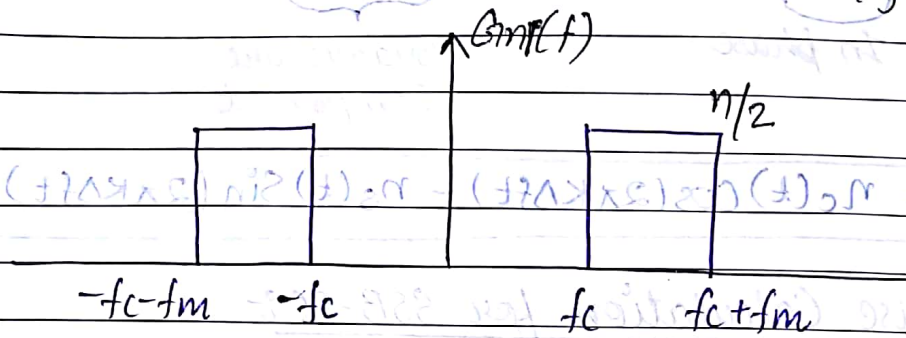
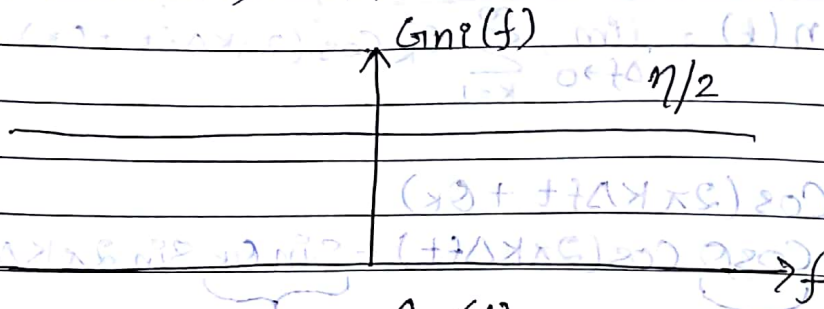
$$S_o(t) = \frac{A}{2} \cos 2\pi f_m t \quad \text{(Figure of merit)}$$

$$S_i^o = \left(\frac{A}{\sqrt{2}} \right)^2 = \frac{A^2}{2} \quad \text{and} \quad S_o = \frac{A^2/8}{A^2/2} = \frac{1}{4}$$

$$S_o = \left(\frac{A}{2\sqrt{2}} \right)^2 = \frac{A^2}{8}$$



$$G_{no}(f) = G_{ni}(f) / |H(f)|^2$$



$$G_{no}(f) = G_{ni}(f) / |H(f)|^2$$

$$N_0 = \frac{\eta}{8} \times 2f_m$$

$$\frac{8f_m}{S} = \frac{N_0}{S}$$

$$N_0 = \frac{\eta f_m}{4}$$

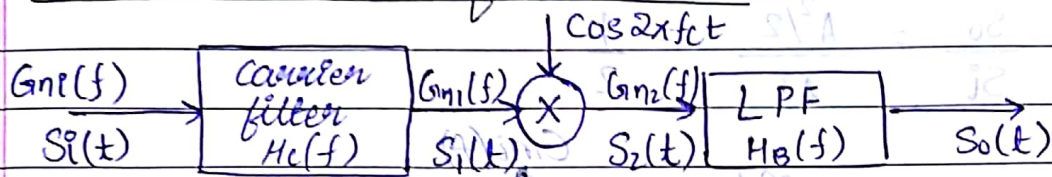
$$\frac{S_A}{S} = \frac{S}{\left(\frac{A}{\sqrt{2}}\right)^2}$$

$$\frac{S_A}{8} = \frac{S}{\left(\frac{A}{\sqrt{2}}\right)^2}$$

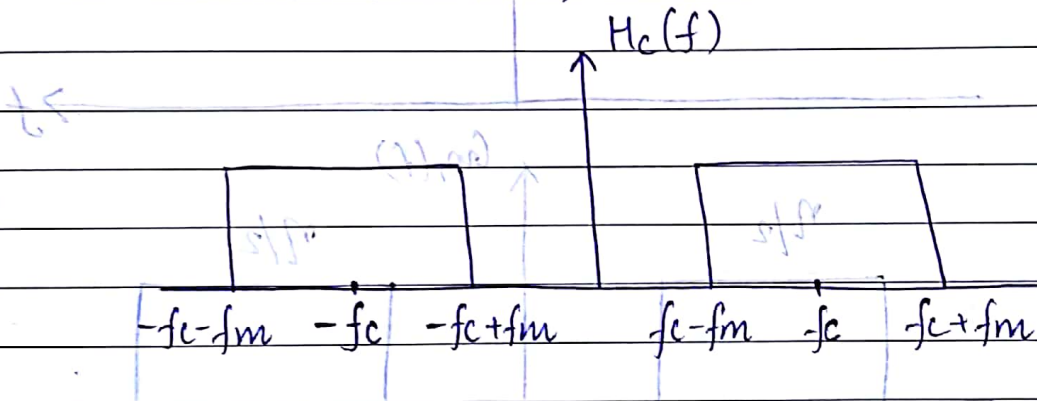
$$\left[\frac{S}{N_0} \right] = \frac{A^2}{8 \eta f_m}$$

$$= \frac{A^2}{2 \eta f_m}$$

Noise calculation for DSB-SC:-



$$S_i(t) = A \cos(2\pi(f_c + f_m)t) + A \cos(2\pi(f_c - f_m)t)$$



$$S_i(t) = A \cos(2\pi(f_c + f_m)t) + A \cos(2\pi(f_c - f_m)t)$$

$$S_2(t) = S_i(t) \cos(2\pi f_c t)$$

$$= \frac{2A}{2} \cos(2\pi(f_c + f_m)t) \cos(2\pi f_c t) + \frac{2A}{2} \cos(2\pi(f_c - f_m)t) \cos(2\pi f_c t)$$

$$= \frac{A}{2} \cos(2\pi(2f_c + f_m)t) + \frac{A}{2} \cos(2\pi f_m t) + \frac{A}{2} \cos(2\pi(2f_c - f_m)t) + \frac{A}{2} \cos(2\pi f_m t)$$

$$S_o(t) = \frac{A}{2} \cos 2\pi f_m t + \frac{A}{2} \cos 2\pi f_m t$$

$$= A \cos 2\pi f_m t$$

$$S_i = \left(\frac{A}{\sqrt{2}}\right)^2 + \left(\frac{A}{\sqrt{2}}\right)^2$$

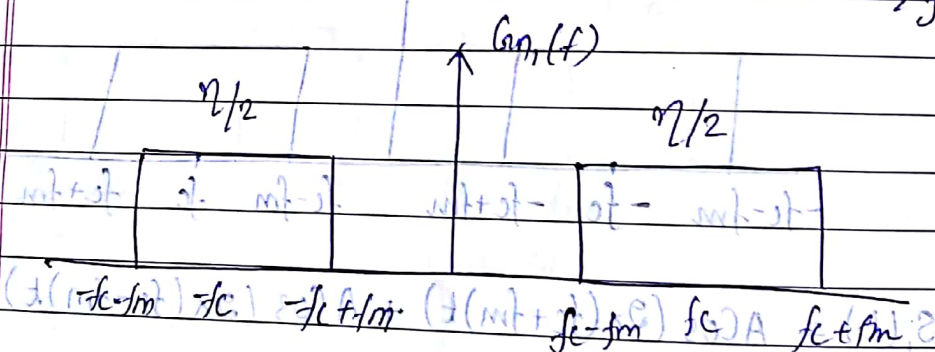
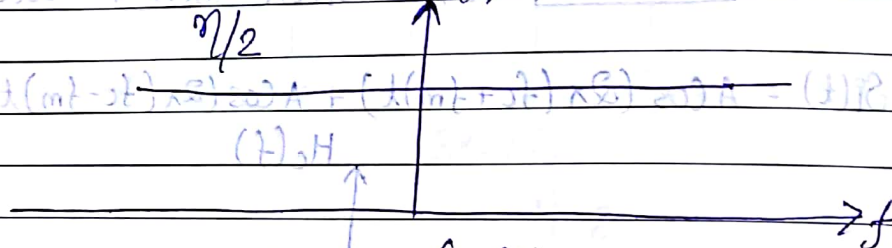
$$= \frac{A^2}{2} + \frac{A^2}{2}$$

$$= A^2$$

$$S_o \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$$

$$S_o = \frac{A^2/2}{A^2} = \frac{1}{2}$$

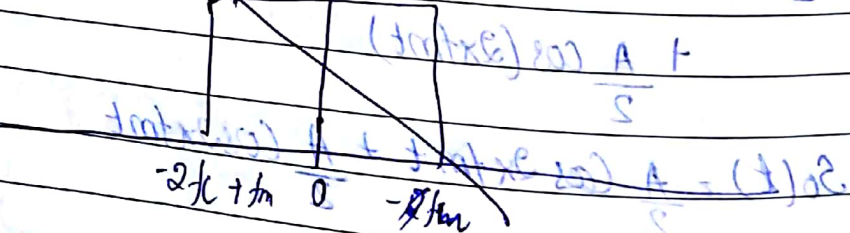
$$\frac{S_o}{S_i} = \frac{A^2/2}{A^2} = \frac{1}{2}$$

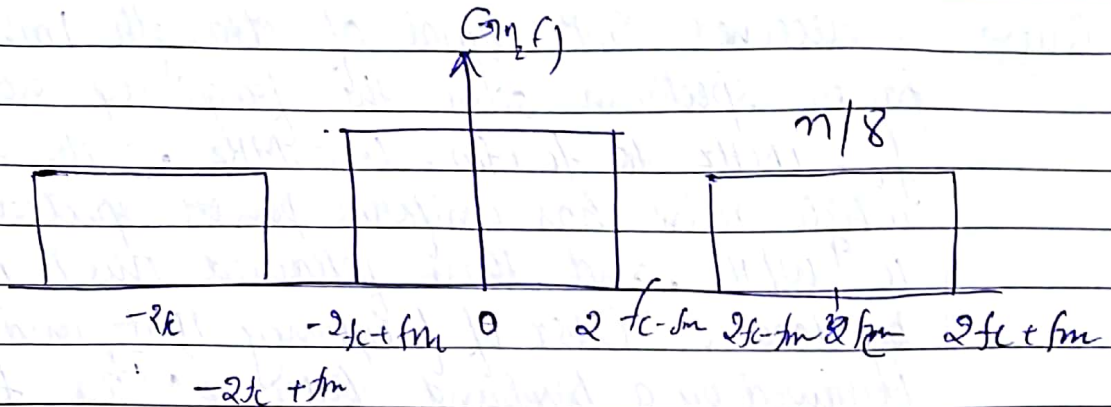


$$\cos(f_c + f_m)t = \cos(2\pi f_c t) \cos(2\pi f_m t)$$

$$= \frac{1}{2} \cos(2\pi f_c t + 2\pi f_m t) + \frac{1}{2} \cos(2\pi f_c t - 2\pi f_m t)$$

$$= \frac{1}{2} \cos(2\pi(f_c + f_m)t) + \frac{1}{2} \cos(2\pi(f_c - f_m)t)$$





$$G_{\eta/2}(f) = \frac{\eta}{2}$$

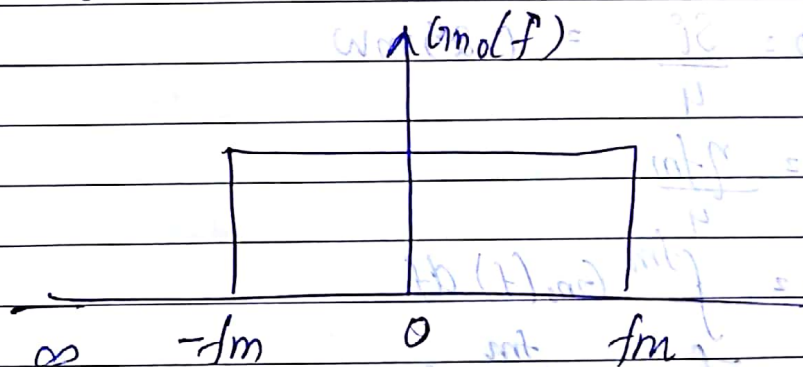
$$A_m = \frac{\eta f}{2}$$

$$v_{m1} = \sqrt{\frac{\eta f}{2}}$$

$$v_{m1}(t) = \sqrt{\frac{\eta f}{2}} \cos(2\pi (2\tau_c f_m t))$$

$$= \frac{1}{2} \sqrt{\frac{\eta f}{2}} \cos(2\pi (2\tau_c f_m t)) + \frac{1}{2} \sqrt{\frac{\eta f}{2}} \cos(2\pi f_m t)$$

$$= \sqrt{\frac{\eta f}{8}} \cos(2\pi (2\tau_c f_m t)) + \sqrt{\frac{\eta f}{8}} \cos(2\pi f_m t)$$



$$N_{02} \int_{-\infty}^{\infty} G_{m0}(f) df = \int_{-fm}^{fm} \frac{(\eta f)}{4} df = \frac{\eta}{4} [fm - (-fm)]$$

$$\frac{\eta f_m}{2} = \boxed{\frac{N_{02} \eta f_m}{2}}$$

Ques 1 A received SSB signal of strength 1mW has power spectrum over the frequency range $f_c = 1\text{MHz}$ to $f_c + f_m = 1.005\text{MHz}$. The accompanying white noise has uniform power spectral density 10^{-9}W/Hz and then followed this is multiplied by local oscillator of frequency 1MHz and then followed by a baseband filter of $f_{\text{cutoff}} = f_m$ what is the message bandwidth. Find the signal and noise strength at the output of baseband filter and calculate SNR. How does the SNR change if bandwidth of message signal is reduced by 25%.

ii)

Sol: $f_c = 1\text{MHz}$ $f_c + f_m = 1.005\text{MHz}$
 $S_i = 1\text{mW}$ $f_c \leftrightarrow f_c + f_m$

$f_m = f_c + f_m - f_c$
 $= 0.005\text{MHz}$

$S_o = \frac{S_i}{4} = 0.25\text{mW}$

$N_o = \frac{\eta f_m}{4}$

$N_o = \int_{-f_m}^{f_m} G_{no}(f) df$
 $= \int_{-f_m}^{f_m} \frac{10^{-9}}{4} df$

$= \frac{10^{-9}}{4} [f_m - (-f_m)]$

$= \frac{10^{-9} \times 2f_m}{4} = \frac{10^{-9} f_m}{2}$

iii)

$$= \frac{10^{-9} \times 5 \times 10^3}{2}$$

$$= 2.5 \times 10^{-6} \text{ W}$$

$$= 2.5 \mu\text{W}$$

$$\frac{S_0}{N_0} = \frac{0.25 \text{ mW}}{2.5 \mu\text{W}} = \frac{10^{-3} \times 10^1}{10^{-6}} = 100$$

$$\left(\frac{S_0}{N_0} \right)_{\text{dB}} = 10 \log_{10} 100$$

$$= 20 \text{ dB}$$

$$\text{ii)} \quad f_m' = \frac{5 \times 10^3 \times 3}{4} = 3.75 \text{ KHz}$$

$$S_0' = \frac{S_0}{4} = 0.25 \text{ mW}$$

$$N_0' = 2.5 \mu\text{W}$$

$$\frac{S_0'}{N_0'} = 100$$

$$\text{iii)} \quad N_0'' = \frac{10^{-9} f_m'}{2} = \frac{10^{-9} \times 3.75 \times 10^3}{2}$$

$$= 1.875 \mu\text{W}$$

$$\frac{S_0''}{N_0''} = \frac{0.25 \text{ mW}}{1.875 \mu\text{W}} = 133.33$$

$$= 21.24 \text{ dB}$$

Ques 2 Repeat the above question for DSB-SC with a frequency range $f_c - f_m = 0.995 \text{ MHz}$, $f_c + f_m = 1.005 \text{ MHz}$. Signal strength and noise power density is same as in question 1.

Sol $f_c - f_m = 0.995 \text{ MHz}$ $f_c + f_m = 1.005 \text{ MHz}$

$$\frac{S_o}{S_i} = \frac{1}{2} = 0.5 \text{ dB} \quad S_o = \frac{S_i}{2}$$

$$\frac{S_o}{S_i} = \frac{1}{2} = 0.5 \text{ mW} \quad f_m = 5 \text{ kHz}$$

$$N_o = \int_{-f_m}^{f_m} G_{n_o}(f) df$$

$$= \int_{-f_m}^{f_m} \frac{10^{-9}}{2} df$$

$$= \frac{10^{-9} \times 2 f_m}{2}$$

$$= 10^{-9} \times 5 \times 10^3$$

$$= 5 \mu\text{W}$$

$$\left(\frac{S_o}{N_o} \right) = \frac{0.5 \text{ mW}}{5 \mu\text{W}}$$

$$= 100$$

⇒ SNR for DSB with ^{full} carrier :- / Amplitude modulation :-

$$\left(\frac{S_o}{N_o} \right) = \frac{S_i^{DSB}}{\eta_{fm}}$$

$$\begin{aligned} S_i(t) &= (A_c + m(t)) \cos \omega_c t \\ &= (A_c + A_m \cos \omega_m t) \cos \omega_c t \\ &= A_c (1 + m \cos \omega_m t) \cos \omega_c t \end{aligned}$$

$$S_i(t) = A (1 + m(t)) \cos \omega_c t$$

$$m(t) = m \cos \omega_m t$$

$$S_i(t) = A \cos \omega_c t + A m(t) \cos \omega_c t$$

$$S_i^2 = \frac{A^2}{2} + \frac{A^2 m^2(t)}{2} = \frac{A^2}{2} (1 + m^2(t))$$

$$S_i^{DSB} = \frac{A^2 m^2(t)}{2}$$

$$\frac{S_i^{DSB}}{S_i} = \frac{\frac{A^2 m^2(t)}{2}}{\frac{A^2}{2} (1 + m^2(t))}$$

$$\frac{A^2 m^2(t)}{2 (1 + m^2(t))}$$

$$S_i^{DSB} = \left(\frac{m^2(t)}{1 + m^2(t)} \right) S_i$$

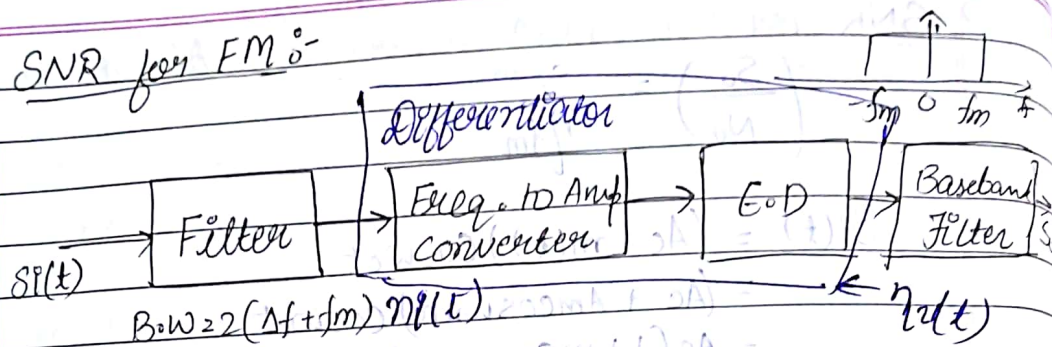
$$\frac{S_o}{N_o} = \frac{m^2(t)}{1 + m^2(t)} \frac{S_i}{\eta_{fm}}$$

$$m(t) = m \cos \omega_m t$$

$$\frac{m^2(t)}{2} = \frac{m^2}{2}$$

$$\frac{S_o}{N_o} = \left(\frac{m^2/2}{1 + m^2/2} \right) \frac{S_i}{\eta_{fm}} = \left(\frac{m^2}{2 + m^2} \right) \frac{S_i}{\eta_{fm}}$$

SNR for FM :-

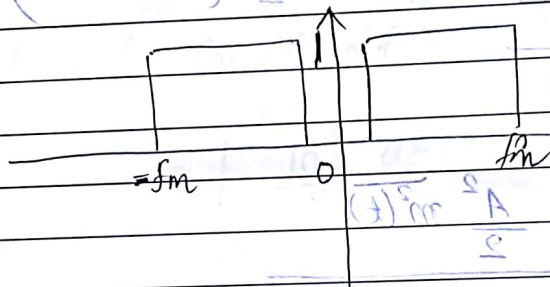


$$S_i(t) = A \cos(\omega_c t + K \int m(t) dt)$$

$$S_1(t) = S_i(t)$$

$$S_2(t) = \alpha(\omega_c + K m(t))$$

$$S_2(t) = \alpha \omega_c + \alpha K m(t)$$



$$S_o(t) = \alpha K m(t) + 1$$

$$S_o = \alpha^2 K^2 m^2(t)$$

$$n(t) = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t$$

$$n(t) = A (A + n_c(t)) \cos(\omega_c t) - n_s(t) \sin \omega_c t$$

$$R(t) = \sqrt{(A + n_c(t))^2 + (n_s(t))^2}$$

$$\theta(t) = \tan^{-1} \frac{n_s(t)}{A + n_c(t)}$$

$$m(t) = \frac{R(t)}{K}$$

$$m(t) = \frac{R(t)}{K}$$

$$m(t) = R(t) \cos(\omega_c t + \theta(t))$$

$$\theta(t) = \tan^{-1} \left(\frac{n_s(t)}{A + n_c(t)} \right)$$

$$\tan \theta = \frac{n_s(t)}{A + n_c(t)} \approx \frac{n_s(t)}{A}$$

θ is small

$$\tan \theta \approx \theta \approx \frac{n_s(t)}{A}$$

$$m(t) = R_s(t) \cos \left[\omega_c t + \frac{n_s(t)}{A} \right]$$

$$n_2(t) = \alpha \left[\omega_c + \frac{1}{A} \frac{d}{dt} n_s(t) \right]$$

$$n_2(t) = \alpha \omega_c + \frac{\alpha}{A} \frac{d}{dt} n_s(t)$$

$$n_0(t) = \frac{\alpha}{A} \frac{d}{dt} n_s(t)$$

$$1 = (\omega_c)_{\text{min}} (\omega_c)_{\text{max}}$$

$$\frac{\eta}{2} \xrightarrow{\text{Differentiator}} \frac{4x^2 \gamma^2 f^2 \eta}{2}$$

$$N_0 = \int_{-f_m}^f \frac{4x^2 \gamma^2 f^2 \eta}{2} \frac{\alpha^2}{A^2} df$$

$$= \frac{2x^2 \gamma^2 \eta \alpha^2}{A^2} \int_{-f_m}^f f^2 df$$

$$= \frac{2x^2 \gamma^2 \eta \alpha^2}{A^2} \left[\frac{(2f_m)^3}{3} \right] = \frac{16x^2 \gamma^2 \eta \alpha^2 f_m^3}{3A^2}$$

$$\eta \gg \alpha$$

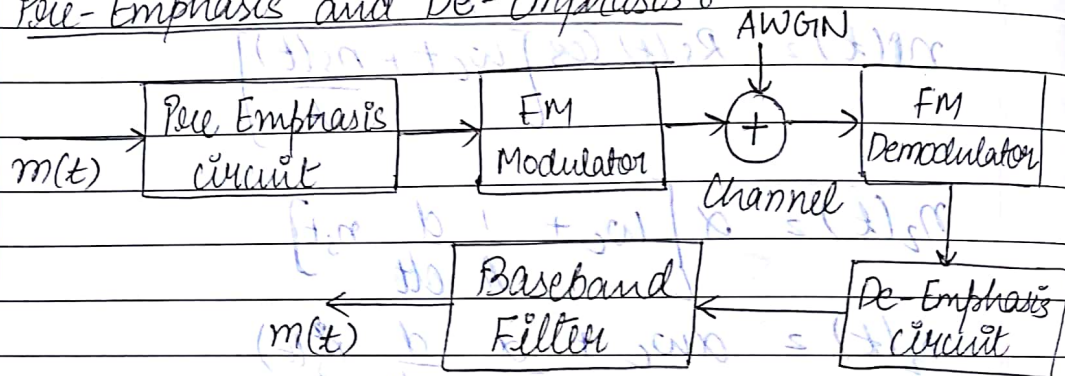
$$N_0 = \frac{4x^2 \gamma^2 \eta \alpha^2 f_m^3}{3A^2}$$

$$\frac{S_o}{N_o} = \frac{\alpha^2 k^2 m^2 t}{4\pi^2 \eta \alpha^2 f m^3 \frac{3A^2}{2}}$$

$$\boxed{\frac{S_o}{N_o} = \frac{3A^2 k^2 \overline{m^2(t)}}{4\pi^2 \eta f m^3}}$$

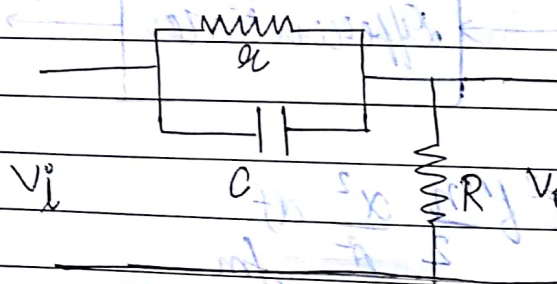
SNR should always be greater.

Pre-Emphasis and De-Emphasis :-



$$H_p(\omega) H_d(\omega) = 1$$

Pre-Emphasis circuit :-



$$H_p(\omega) = \frac{V_o}{V_i} = \frac{R}{R + \frac{1}{j\omega C}}$$

$$R \ll \frac{1}{\omega C}$$

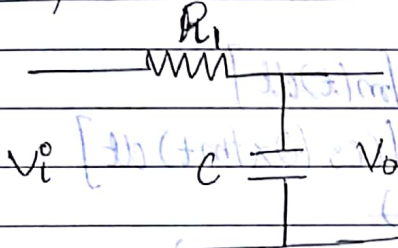
$$= \frac{R}{\frac{R}{sC}} = \frac{R}{R + \frac{1}{sC}}$$

$$= \frac{R}{R + \frac{1}{sC}} \left(\frac{R + \frac{1}{sC}}{sC} \right)$$

$$H_p(\omega) = \frac{R}{R} [sRC + 1]$$

$$H_p(\omega) = \frac{R}{R} \left[1 + \frac{j\omega}{\omega_c} \right] \quad \text{where } \omega_c = \frac{1}{RC}$$

De-emphasis circuit :-



$$H_d(\omega) = \frac{1/sC}{R + 1/sC} = \frac{1}{R + 1/sC}$$

$$H_d(\omega) = \frac{1}{\left(\frac{j\omega}{\omega_c} + 1 \right)} \quad \text{where } \omega_c = \frac{1}{RC}$$

$$H_p(\omega) H_d(\omega) = \frac{R}{R} \left[1 + \frac{j\omega}{\omega_c} \right] \times \left[\frac{1}{1 + \frac{j\omega}{\omega_c}} \right] \quad \text{where } R = R_1$$

$$H = \frac{R}{R}$$

$$\frac{1}{1 + \frac{j\omega}{\omega_c}}$$

$$q = \frac{1}{\omega_c} \text{ where } \omega_c = \frac{1}{RC}$$

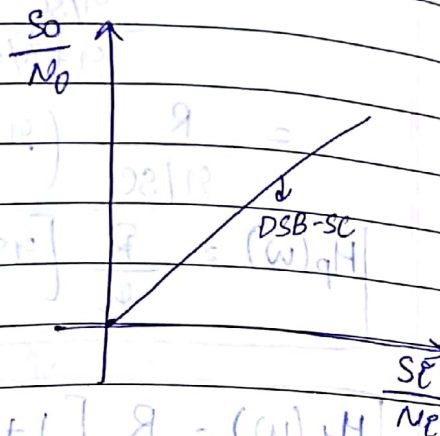
$$\frac{1}{1 + \frac{j\omega}{\omega_c}} = \frac{1}{1 + \frac{j\omega}{\omega_c}}$$

Threshold in FM

DSB-SC

$$\frac{S_o}{N_o} = \frac{S_i}{\eta f_m} = \frac{S_i}{N_i}$$

$$N_i = \frac{\eta}{2} \times 2f_m = \eta f_m$$



$$10 \log \frac{S_o}{N_o} = 10 \log \frac{S_i}{N_i}$$

$$\frac{S_o}{N_o} = \frac{3A^2 K^2 m^2(t)}{4\chi^2 \gamma^2 f_m^3 \eta}$$

$$S_i(t) = A \cos [\omega_c t + K \int m(t) dt]$$

$$= A \cos [\omega_c t + K \int \cos(2\pi f_m t) dt]$$

$$S_i(t) = \omega_c + K \cos(2\pi f_m t)$$

$$S_i(t) = \omega_c + K A_m \cos(2\pi f_m t)$$

$$A_m K \cos 2\pi f_m t = K m(t)$$

$$K m(t) = 2\chi K_f A_m \cos(2\pi f_m t)$$

$$= 2\chi \Delta f \cos(2\pi f_m t)$$

$$K^2 m^2(t) = \left(\frac{2\chi \Delta f}{\sqrt{2}} \right)^2$$

$$= \frac{4\chi^2 \Delta f^2}{2} = 2\chi^2 \Delta f^2$$

$$\frac{S_o}{N_o} = \frac{3}{2} \frac{\omega_c^2 (2\chi^2 \Delta f^2) A^2 / 2}{\chi^2 \gamma^2 f_m^2 \eta f_m}$$

$$= \frac{3}{2} \left(\frac{\Delta f}{\gamma f_m} \right)^2 \frac{S_i}{N_i}$$

$$\frac{\eta f_m}{\gamma} = N_i$$

$$\boxed{\frac{S_o}{N_o} = \frac{3}{2} \beta^2 \frac{S_i}{N_i}}$$

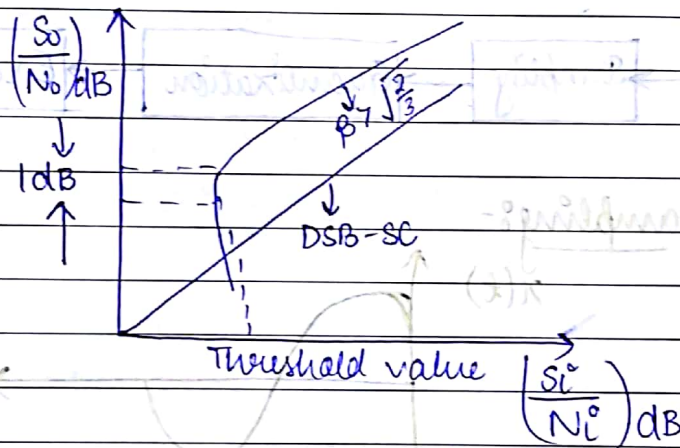
$$\gamma = \sqrt{2} \text{ and } \frac{\Delta f}{f_m} = \beta$$

$$\left(\frac{S_o}{N_o}\right)_{dB} = 10 \log \frac{3}{2} \beta^2 + \left(\frac{S_i}{N_i}\right)_{dB}$$

$$\left(\frac{S_o}{N_o}\right) > \left(\frac{S_i}{N_i}\right)$$

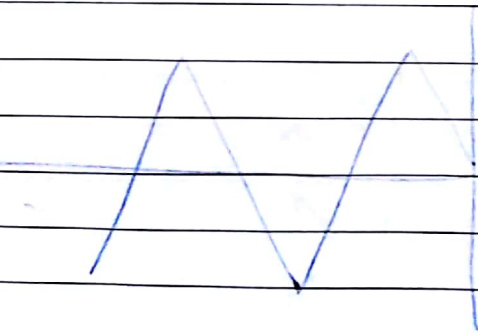
$$\frac{3}{2} \beta^2 > 1$$

$$\beta^2 > \frac{2}{3} \Rightarrow \boxed{\beta > \sqrt{\frac{2}{3}}}$$

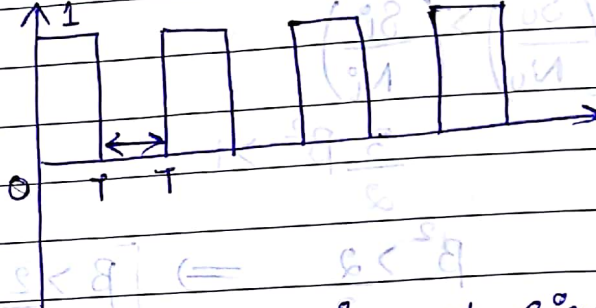


The value at which the graph decrease 1dB from linear value is called threshold value.

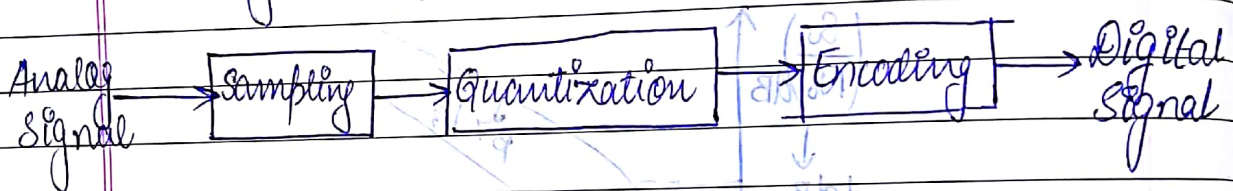
SNR = 22
SNR = 11



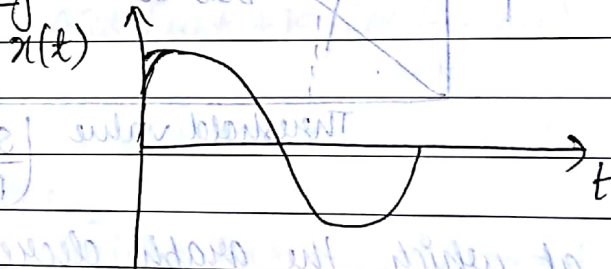
Pulse modulation and digital transmission of Analog Signal:-



Analog to Digital conversion of signals:-



Sampling:-



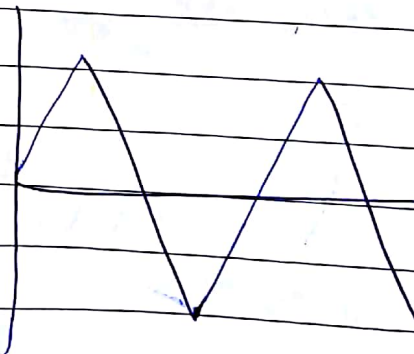
Sampling theorem

Nyquist criteria

$$f_s \geq 2f_m$$

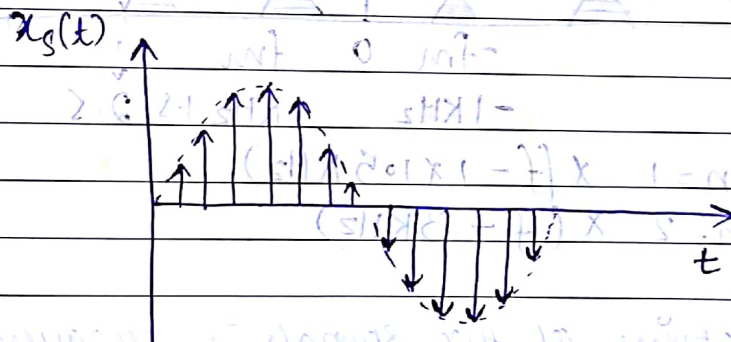
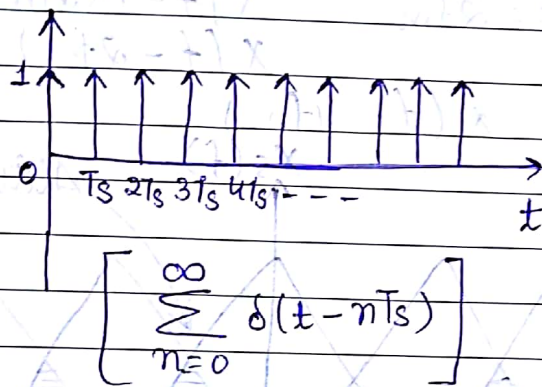
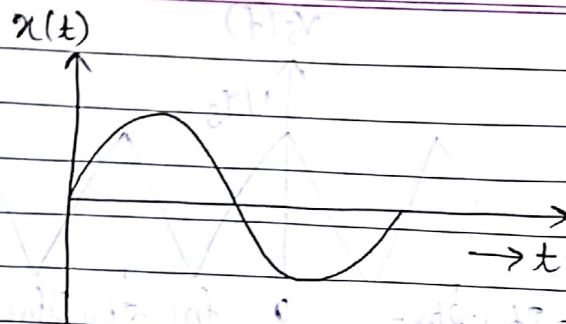
f_s = Sampling frequency

Maximum frequency of modulating signal



$$f_s = 2 \text{ kHz}$$

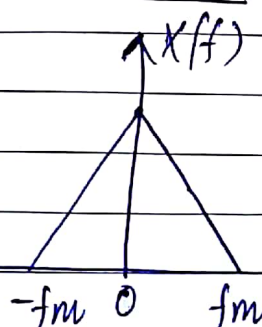
$$f_m = 1 \text{ kHz}$$

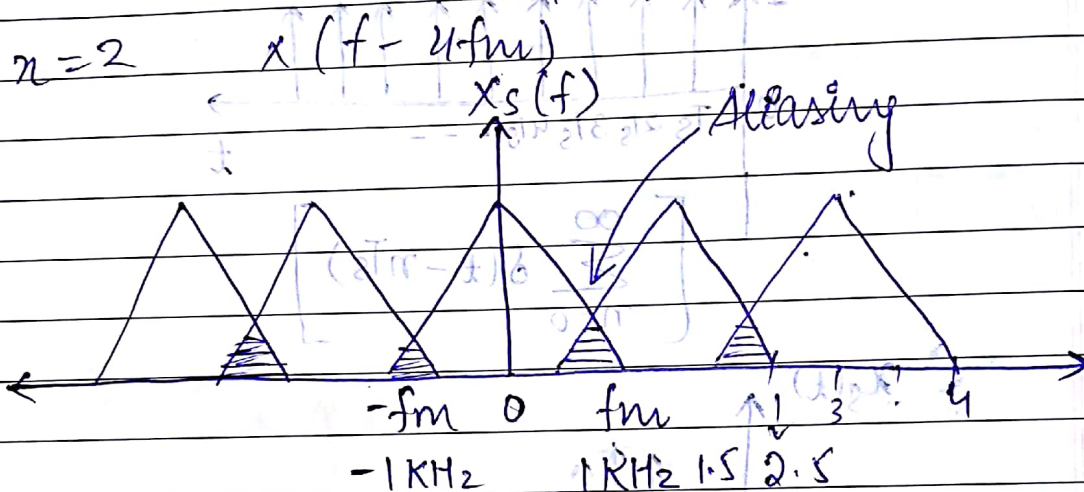
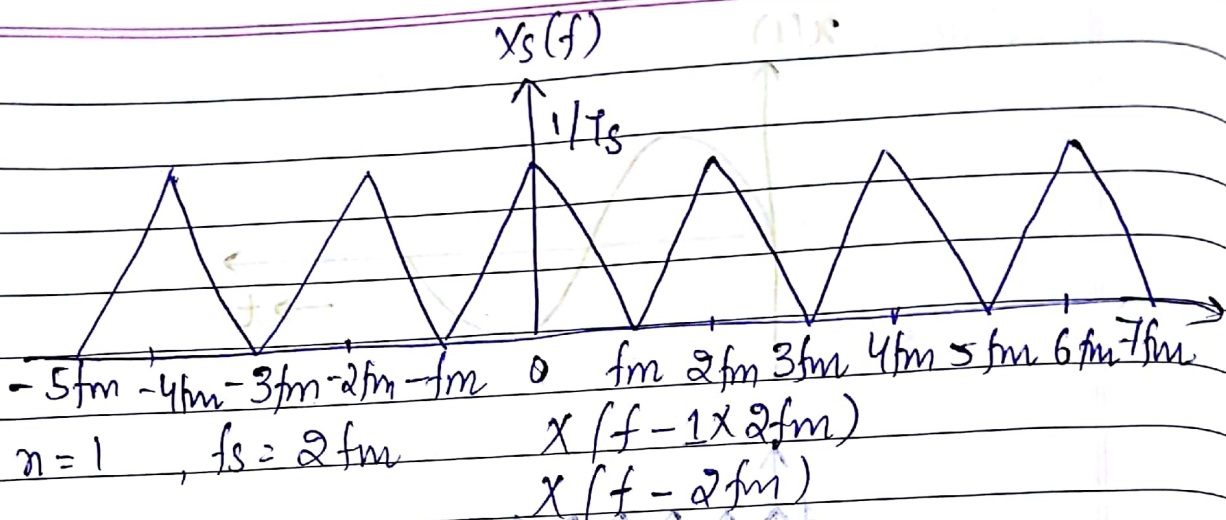


$$x_s(t) = x(t) \sum_{n=0}^{\infty} \delta(t - nTs)$$

$$X_s(f) = X(f) * \frac{1}{T_s} \sum_{n=0}^{\infty} \delta(f - n f_s)$$

$$X_s(f) = \frac{1}{T_s} \sum_{n=0}^{\infty} X(f - n f_s)$$





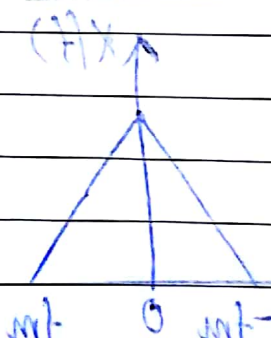
overlapping of two signals in frequency domain is aliasing.

Types of Sampling

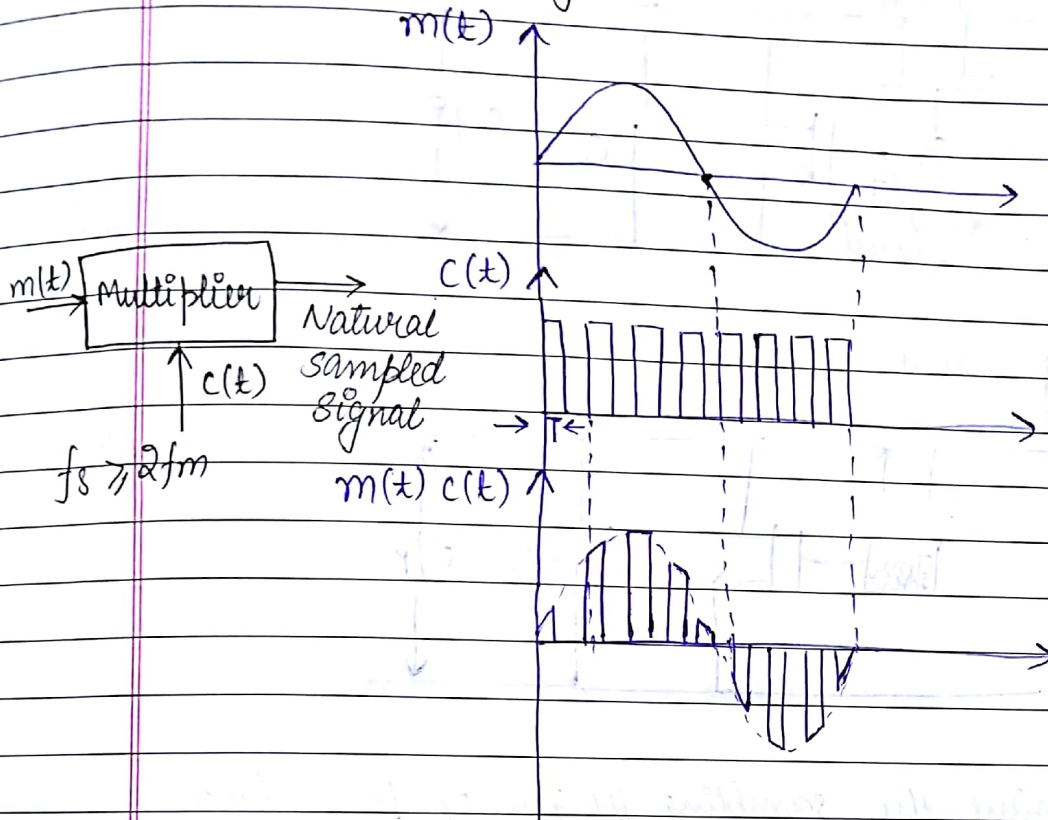
- i) Natural Sampling
- ii) Flat-top Sampling

$$\sum_{n=-\infty}^{\infty} \frac{1}{2T} X = (f)_0 X$$

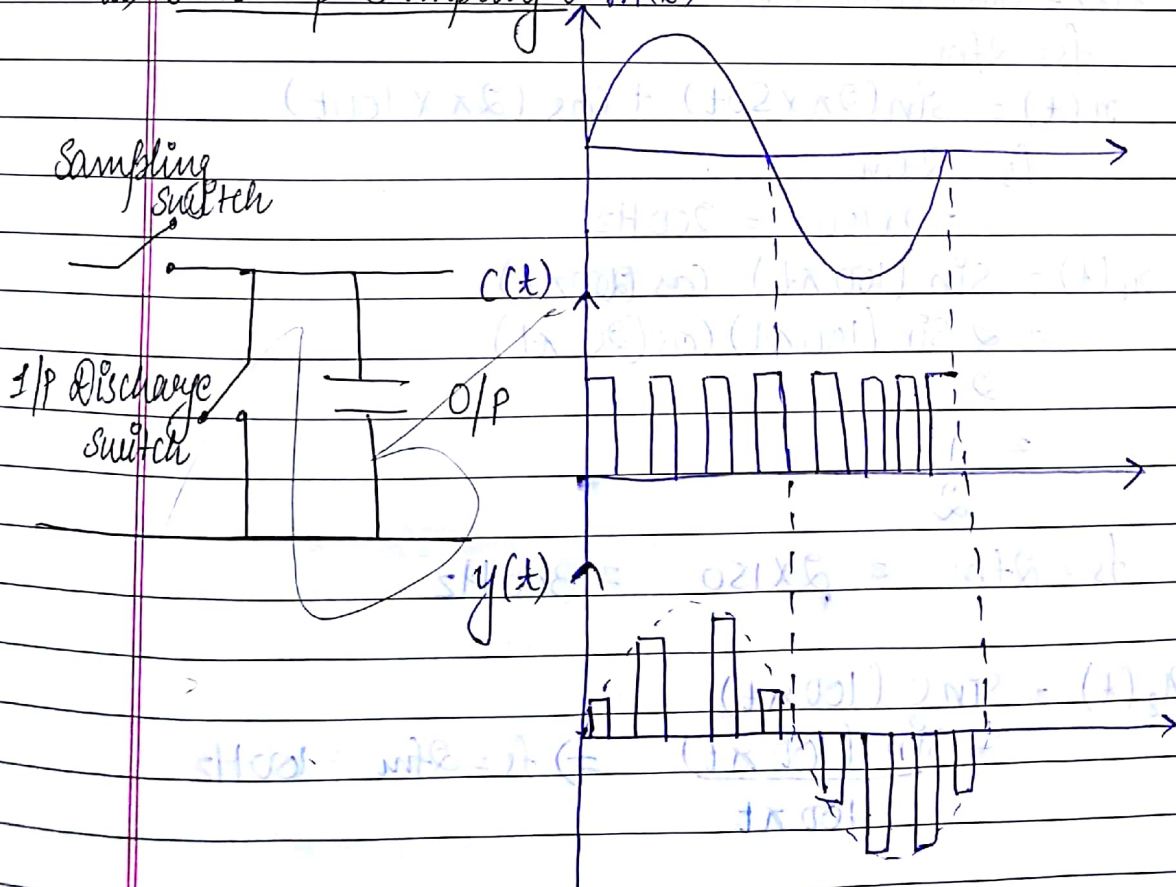
$$(2f_m - 1) X \sum_{n=-\infty}^{\infty} \frac{1}{2T} = (f)_0 X$$

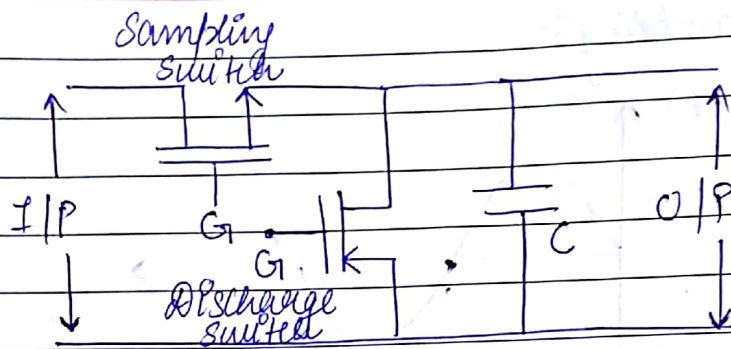


i) Natural Sampling

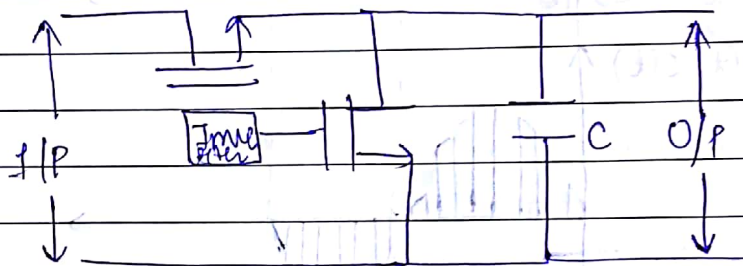


ii) Flat-top Sampling





Using same mosfet.



Ques → Determine the sampling frequency for signal $x(t) = \sin(100\pi t) + \cos(200\pi t)$

$$f_s = 2f_m$$

Sol. $x(t) = \sin(2\pi \times 50t) + \cos(2\pi \times 100t)$

$$f_s = 2f_m$$

$$= 2 \times 100 = 200 \text{ Hz}$$

$$x_1(t) = \sin(100\pi t) \cos(200\pi t)$$

$$= \frac{2}{2} \sin(100\pi t) \cos(200\pi t)$$

$$= \frac{1}{2}$$

$$f_s = 2f_m = 2 \times 150 = 300 \text{ Hz}$$

$$x_2(t) = \sin(100\pi t)$$

$$= \sin(100\pi t) \Rightarrow f_s = 2f_m = 100 \text{ Hz}$$

$$\begin{aligned} x_3(t) &= \text{Sinc}(100\pi t) \text{Sinc}(200\pi t) \\ &= \frac{\sin(100\pi t)}{100\pi t} \times \frac{\sin(200\pi t)}{200\pi t} \end{aligned}$$

$$f_s = 300 \text{ Hz}$$

Ques Determine the sampling time period of a message signal having bandwidth of 1 KHz is to be sampled at 1.25 times the Nyquist wave.

Sol

$$B.W = 1 \text{ KHz}$$

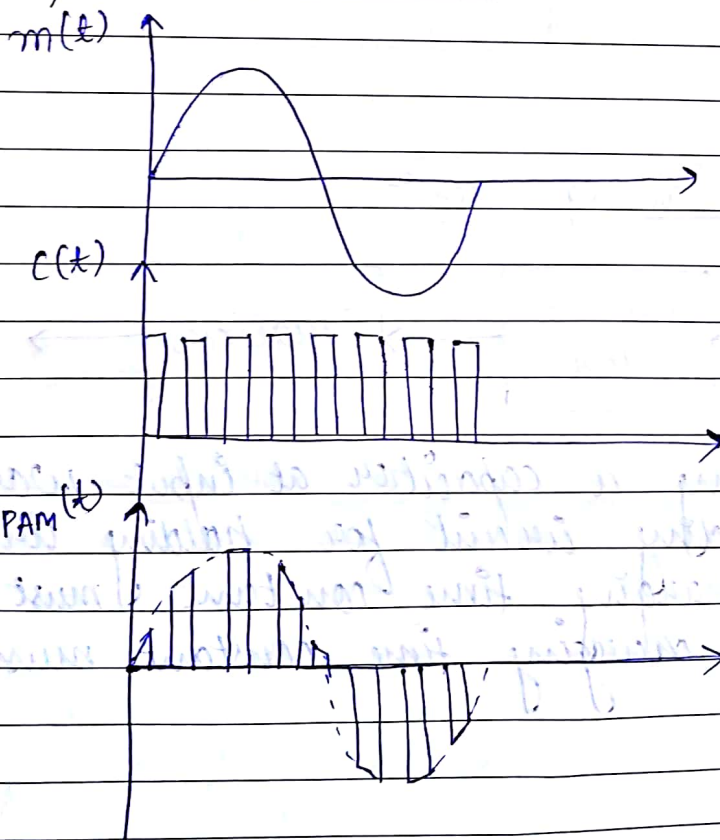
$$f_s = 2f_m \times 1.25$$

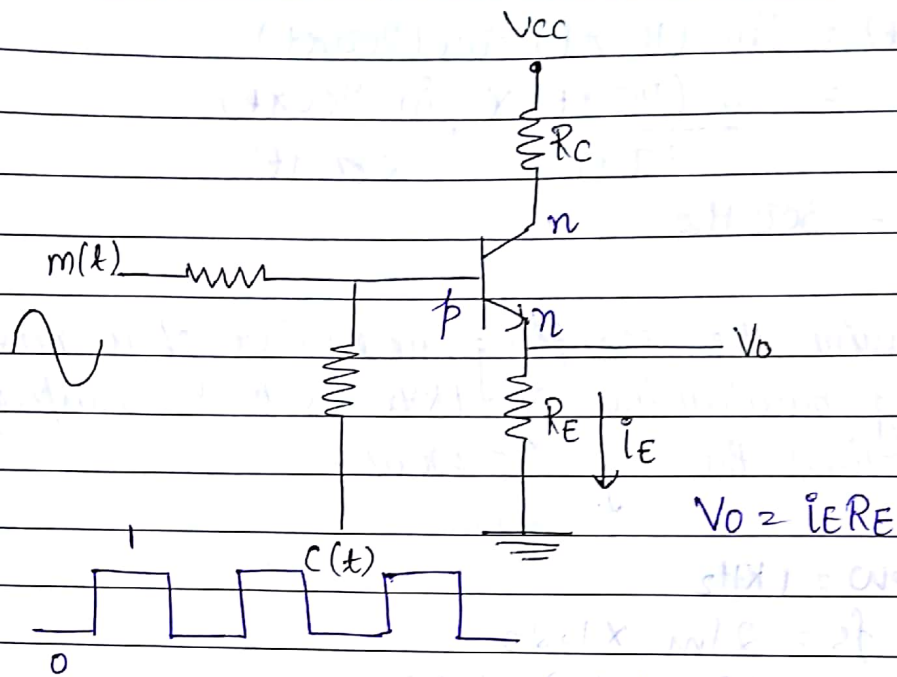
$$= (2 \times 1 \text{ KHz}) \times 1.25$$

$$f_s = 2.5 \text{ KHz}$$

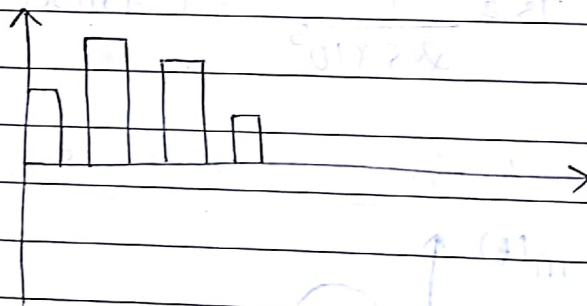
$$T_s = \frac{1}{2.5 \times 10^3} = 0.4 \text{ msec}$$

Pulse Amplitude modulation:-





PAM demodulation:-

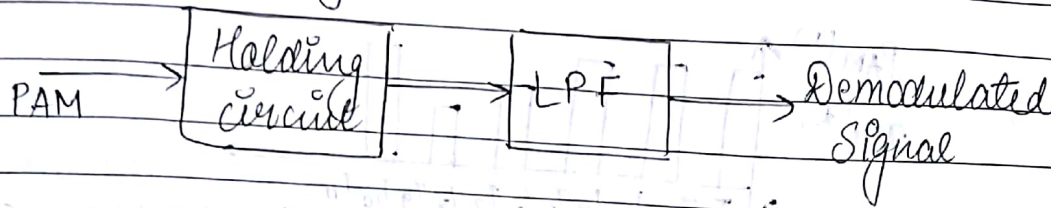


i) Using an equalizer:-



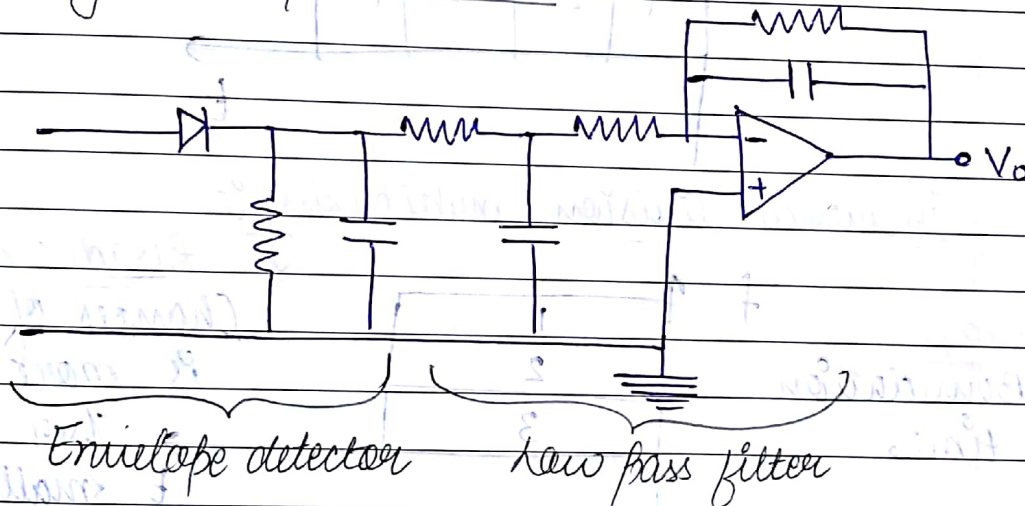
Adding a capacitor at input works as holding circuit for holding circuit
 discharging time constant must be high
 and charging time constant must be low.

ii) using holding circuit:-



Zero-order holding circuit

iii) using envelope detector:-



$$V_o = -\frac{1}{RC} \int V_i dt$$

$$I_1 + I_2 = 0$$

$$\frac{V_i - 0}{R} + \frac{V_o - 0}{RSC} = 0$$

$$V_o = -\frac{V_i}{RSC}$$

$$\frac{V_o}{V_i} = -\frac{1}{RSC}$$

-ve sign indicates 180° phase difference

Time division multiplexing:-

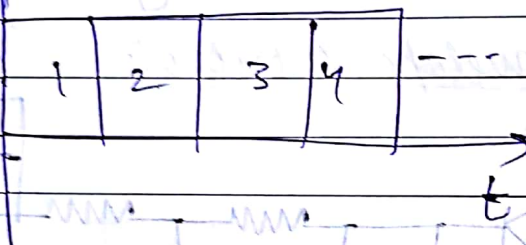
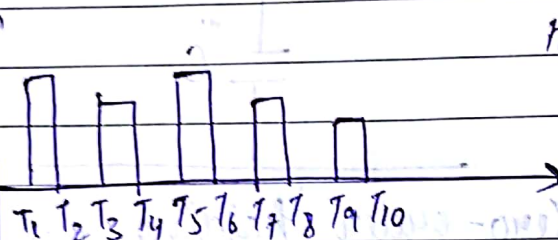


Advantages

Possibility of interference is less.

User utilize the whole bandwidth

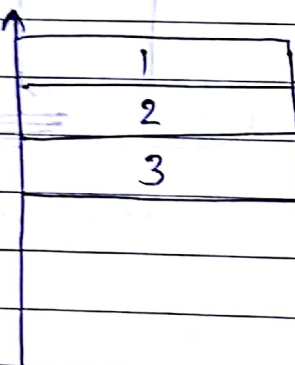
Disadvantages
Message signal has to wait for its time.



Frequency division multiplexing:-

Advantages

No Boundation of time.



Disadvantages
Chances of interference is more.
User has to small bandwidth

used for continuous data transmission.

all the message signal should be sampled at the same Nyquist rate for transmission in time-division multiplexing.

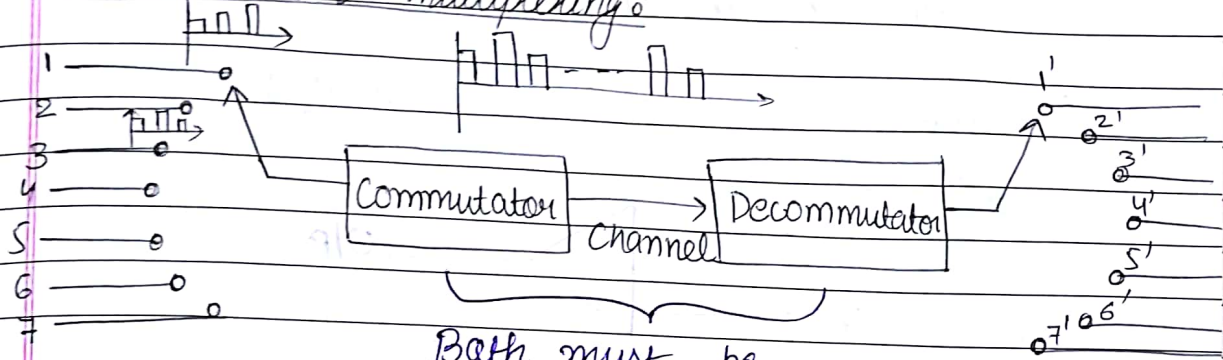


~~Akshay~~
~~Sharma~~

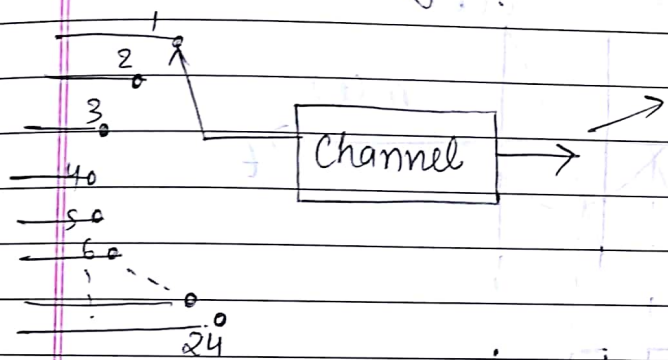
Cheeta Saxena

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Page No.
Shivalal

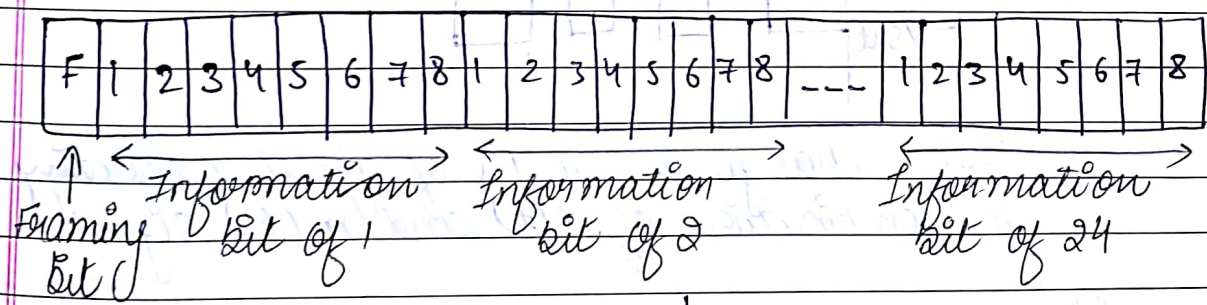
Time-division multiplexing:-



T1 Carrier System:- Used for digital data transmission



Frame Structure:-

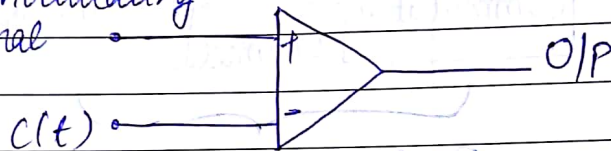


$\left. \begin{array}{l} \text{Total no of} \\ \text{bits in} \\ \text{a frame} \end{array} \right\} = (24 \times 8) + 1$ $= 193 \text{ bits}$	$\text{Sampling frequency} = 8000 \text{ samples/sec}$ $\text{Sampling time} = \frac{1}{8000} = 125 \mu\text{s}$
$\text{Bit Rate} = \frac{1}{125 \mu\text{s}} \times 193$ $= \frac{193}{125 \times 10^{-6}}$	$= 1.544 \times 10^6 \text{ bps}$ $= 1.544 \text{ Mbps}$

Pulse Time modulation :-

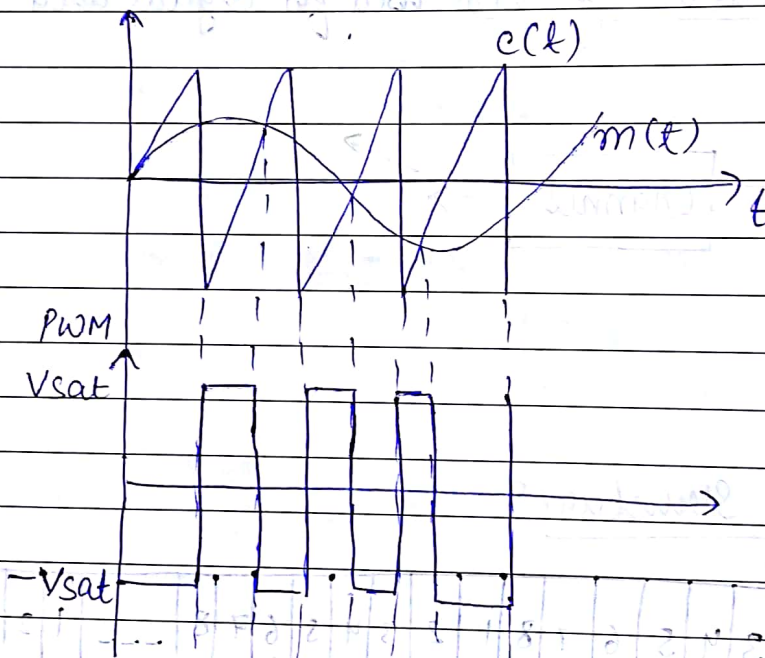
i) Pulse width modulation (PWM) :-

$m(t)$ modulating signal



$c(t) > m(t) \Rightarrow -ve \text{ O/P}$

$c(t) < m(t) \Rightarrow +ve \text{ O/P}$

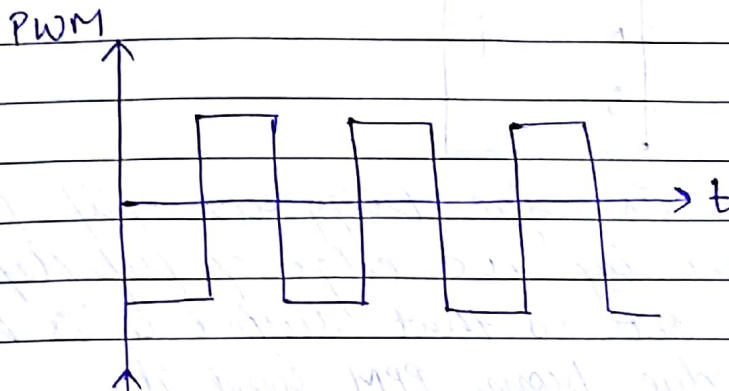
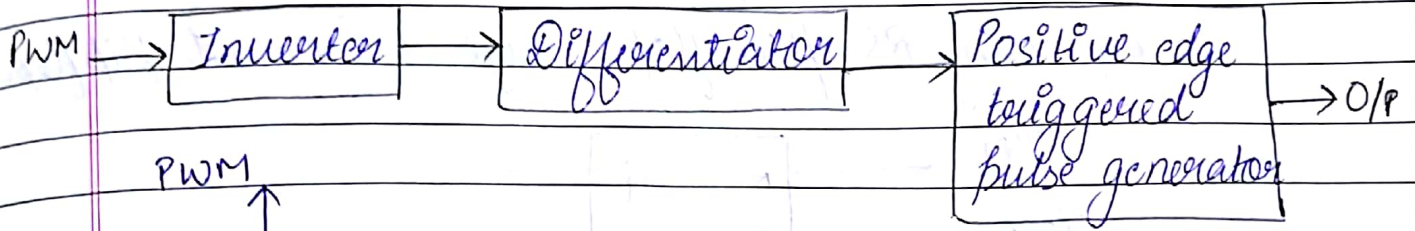


PWM = change in width of signal according to the combination of $c(t)$ and $m(t)$ signal.

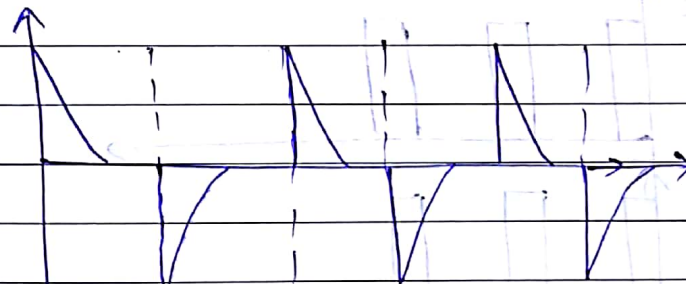
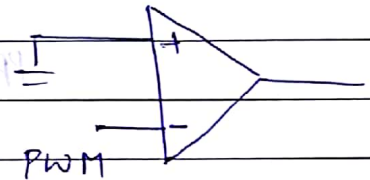
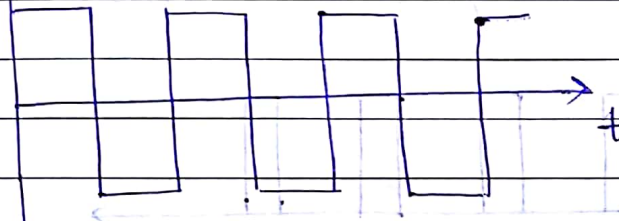
ii) Pulse Position Modulation (PPM) :-

a) Pulse position of the signal is dependent on the position of signal.

b) PPM is generated from PWM.



After inverting



after passing from differentiator (sinc)

PPM
O/P

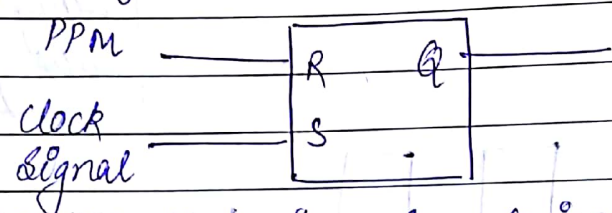
after passing from +ve edge triggered pulse generator

here amplitude and width remains same but position is different.

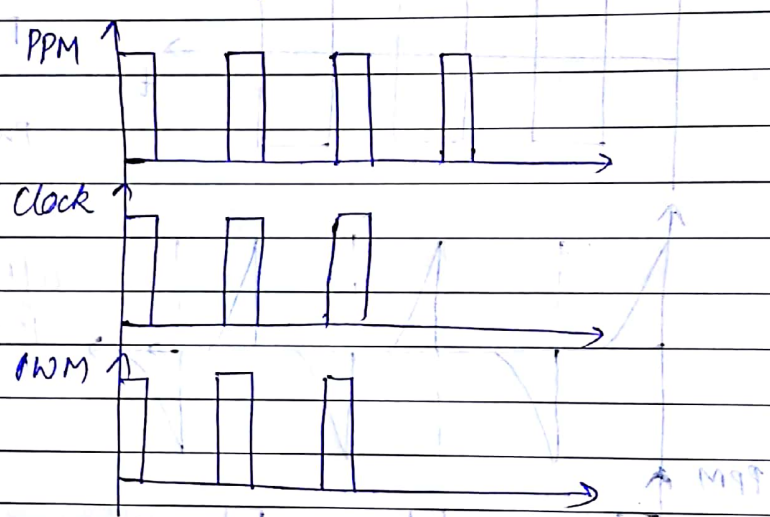
Effect of noise is more in PAM, less in PWM and least in PPM.

Limitation:- It creates problem during demodulation.

PPM to PWM:-
by using RS flip flop, edge triggered flip flop.

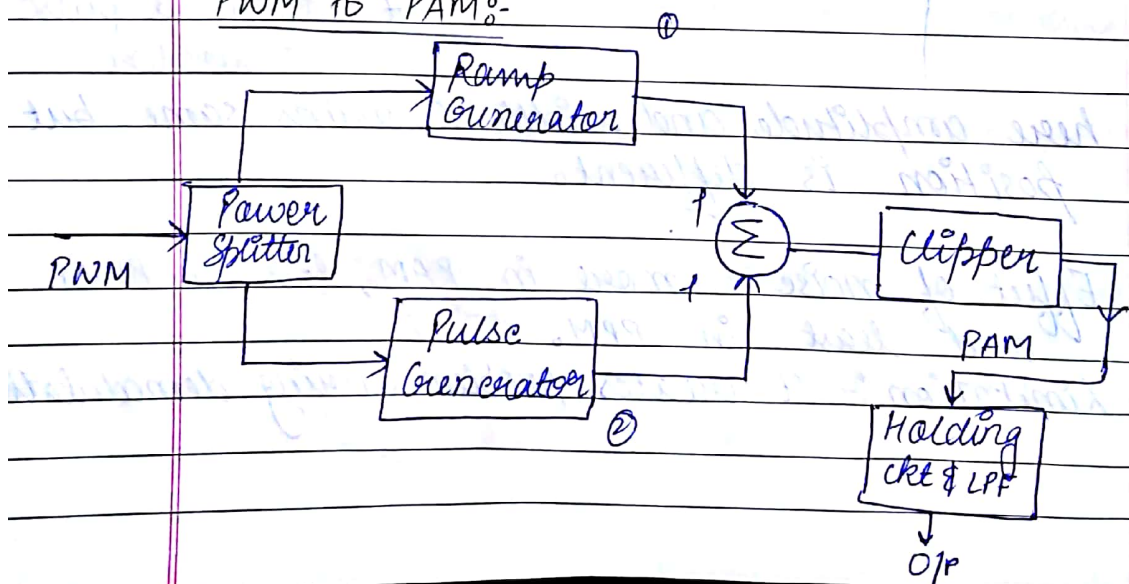


SR flip flop is edge triggered flip flop which is set by +ve edge of flip flop. It remains set so that output Q is high till a +ve edge from PPM reset it.



Demodulation:-

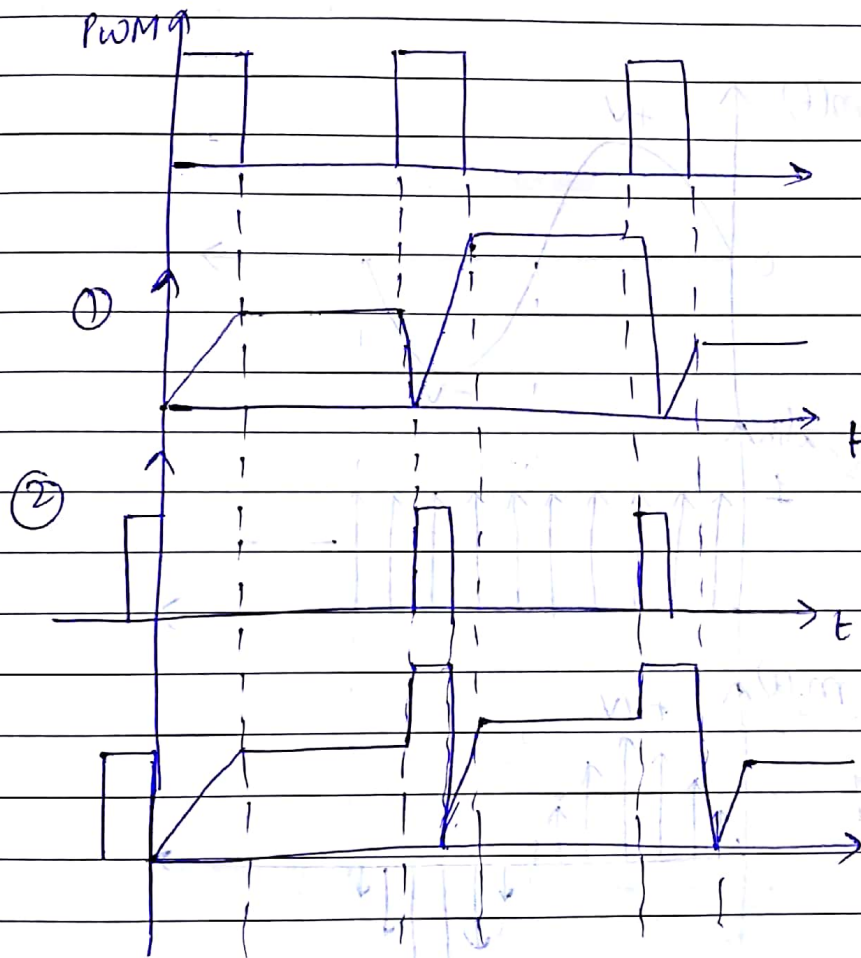
PWM to PAM:-



Power Splitter:- It splits the power in equal division but the signal remains same.

Ramp generator = Integrator circuit

Pulse generator =

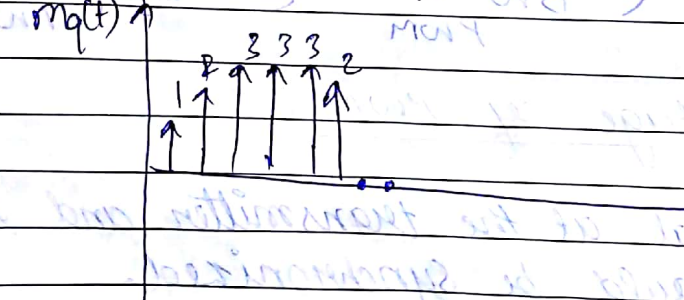
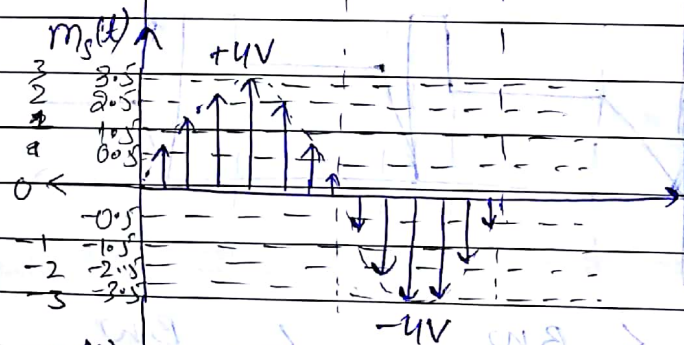
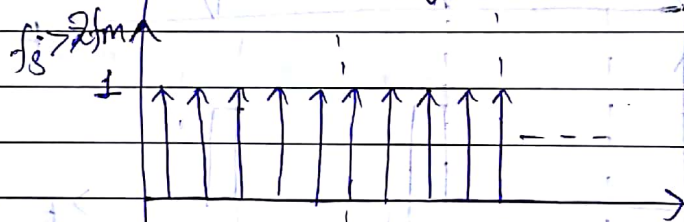
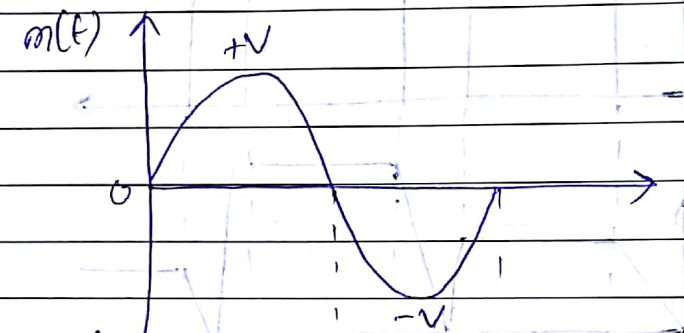
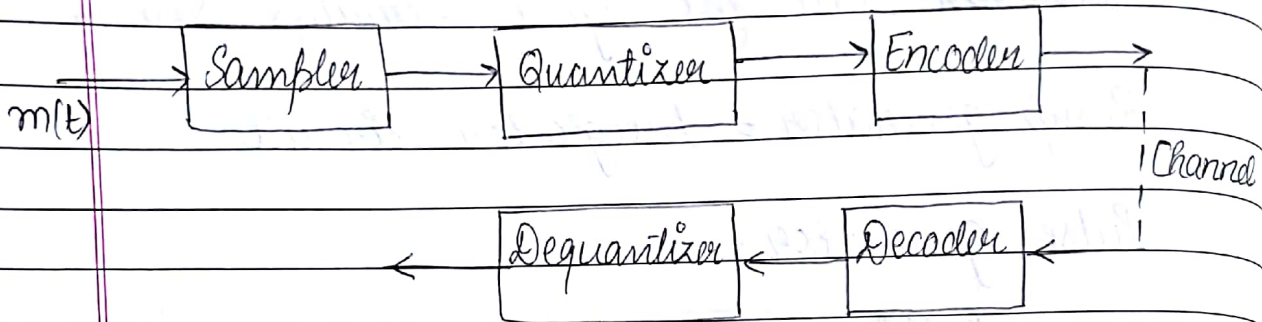


$$BW_{PAM} < BW_{PWM} < BW_{PPM}$$

Disadvantage of PWM is:-

Pulse signal at the transmitter and receiver side should be synchronized.

Pulse Code modulation:-



Quantization:-

i) Uniform quantization:-

$$\text{Step Size} = \Delta = \frac{V_{pp}}{L} = \frac{V_{pp}}{2^n} = \frac{2V_p}{2^n}$$

L = no. of levels

n = no. of bits

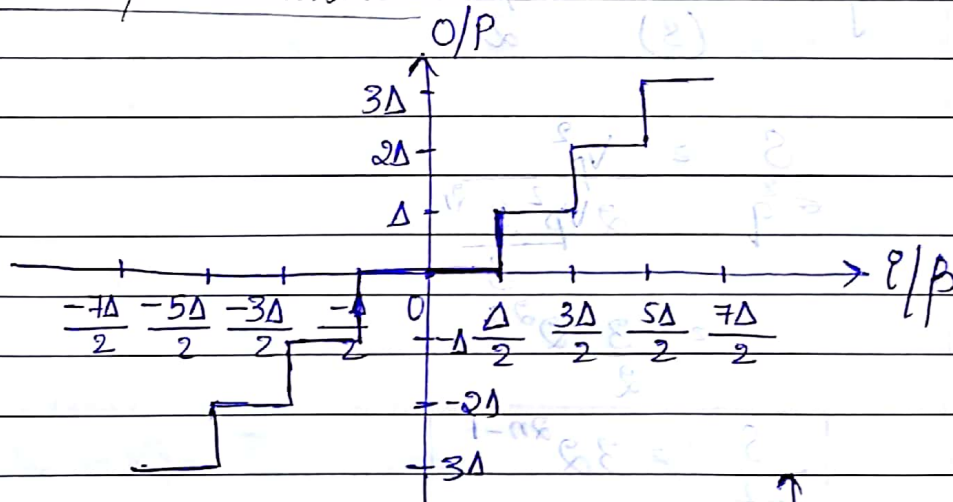
Quantization error:-

$$N_{qmax} = \text{Maximum quantization error} = \frac{\Delta}{2}$$

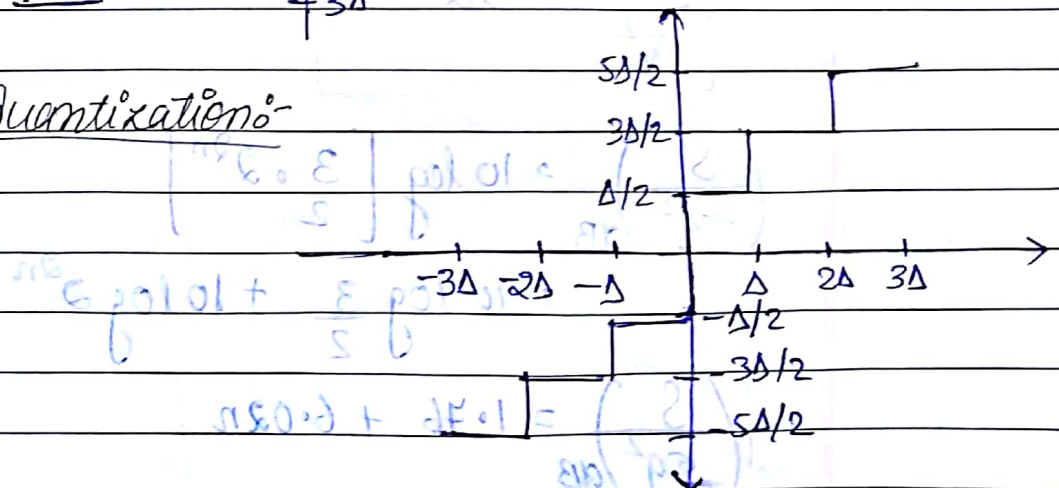
N_{qmax} = maximum quantization noise

If $N_q \downarrow$, $\frac{\Delta}{2} \downarrow$, $2^n \uparrow$, $L \uparrow$

a) Midtread quantization:-



b) Midrise Quantization:-



Quantization noise power :-

$$\sigma_q^2 = \left[\frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2 dq \right]$$

$$\sigma_q^2 = \frac{1}{\Delta} \left[\frac{q^3}{3} \right]_{-\Delta/2}^{\Delta/2}$$

$$\sigma_q^2 = \frac{\Delta^2}{12}$$

where $\Delta = \frac{2V_p}{2^n}$

$$\begin{aligned} \sigma_q^2 &= \frac{(2V_p)^2}{2^{2n} \cdot 12} \\ &= \frac{V_p^2}{3} 2^{-2n} \end{aligned}$$

Signal power = $\frac{V_p^2}{2}$
(S)

$$\begin{aligned} \frac{S}{\sigma_q^2} &= \frac{V_p^2}{2 V_p^2 2^{-2n} \cdot 3} \\ &= \frac{3}{2} 2^{2n} \end{aligned}$$

$$\frac{S}{\sigma_q^2} = 3 \cdot 2^{2n-1}$$

$$\left(\frac{S}{\sigma_q^2} \right)_{dB} = 10 \log \left[\frac{3 \cdot 2^{2n}}{2} \right]$$

$$= 10 \log \frac{3}{2} + 10 \log 2^{2n}$$

$$\left(\frac{S}{\sigma_q^2} \right)_{dB} = 1.76 + 6.02n$$

Ques

Sol.

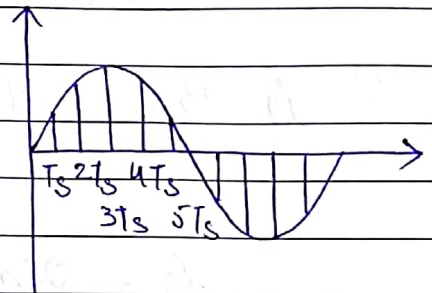
Bandwidth of PCM system:-

Sampling frequency = f_s

η = No. of bits required to encode the sample

$$\text{Sampling Period } (T_s) = \frac{1}{f_s}$$

$$\text{bit rate} = \frac{\eta}{T_s} \text{ (bps)}$$



$$\text{Signaling Rate} = \frac{\eta}{T_s}$$

$$\text{Bandwidth} = \frac{\eta}{2} f_s$$

Bandwidth for PCM in TDM:-

N = No. of channels

η = No. of bits in TDM commutator side

$$\text{Bit rate} = \frac{(\eta N + 1)}{T_s} = (\eta N + 1) f_s$$

If $f_s = 2f_m$

$$\boxed{\text{Bandwidth} = 2f_m (\eta N + 1)}$$

Ques 24 channel each bandlimited to 4 kHz are to be TDM by using PCM calculate the bandwidth of PCM for 128 quantization levels and 8 kHz sampling frequency.

Sol. $N = 24$ $f_s = 8 \times 10^3$ $128 = 2^n$ $n = 7$

$$\text{BW} = f_s (\eta N + 1)$$

$$= 8 \times 10^3 (7(24) + 1)$$

$$= 8 \times 10^3 (169)$$

$$= 10352 \text{ Mb/s.}$$

- Ques → A television signal has bandwidth of 5 MHz
- Determine sampling rate if signal is sampled at a rate 25% above Nyquist rate.
 - Determine the binary pulse rate of the binary coded signal 512 are the quantization levels.

Sol i) Bandwidth = 5×10^6 Hz

$$= \frac{n}{T_s}$$

$$T_s = 2f_m$$

$$= 2f_m (1.25)$$

$$= 12.5 \text{ MHz}$$

ii) Bit rate = nT_s

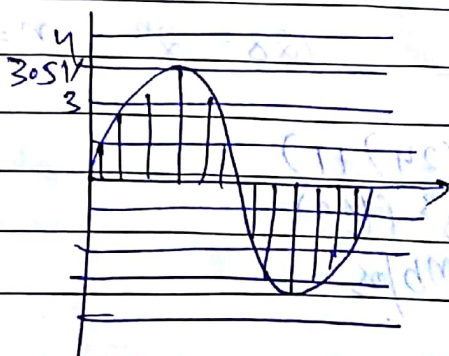
$$= 2 \times 12.5 \text{ Mbps}$$

$$= 112.5 \text{ Mbps}$$

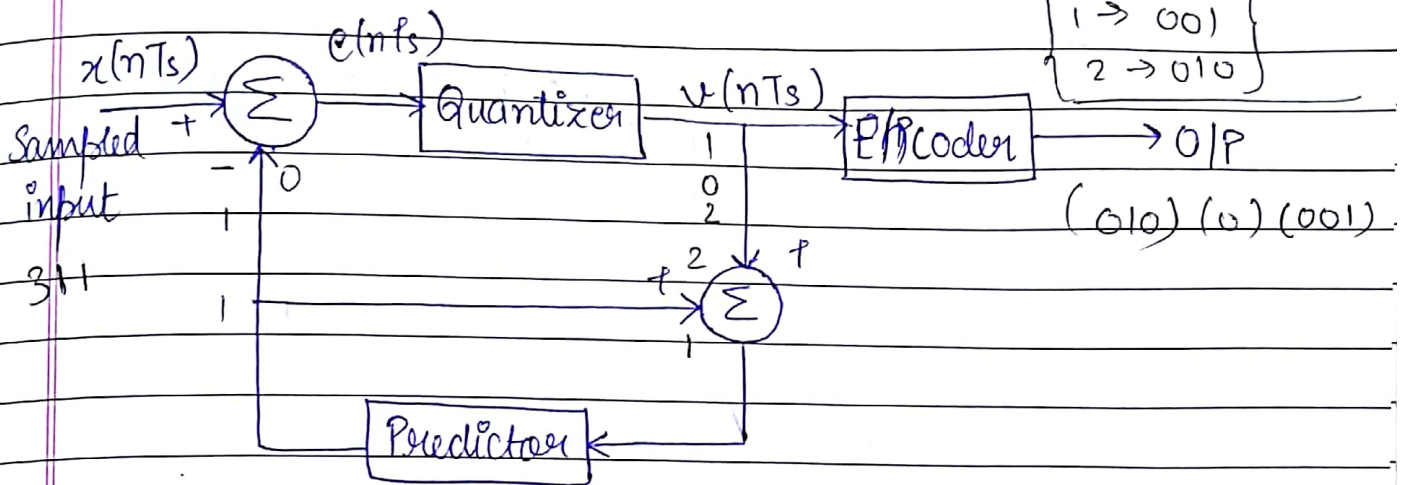
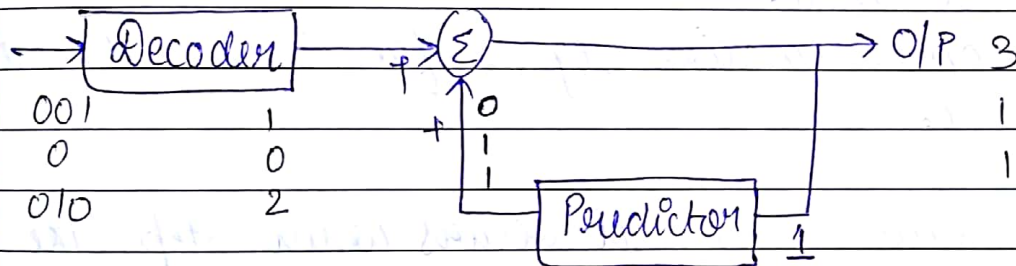
Disadvantage of PCM using TDM is Bandwidth required is very high. But we use it because data transmission is also very high.

Differential Pulse code modulation:-
 used to overcome the drawback of PCM using TDM of Bandwidth.

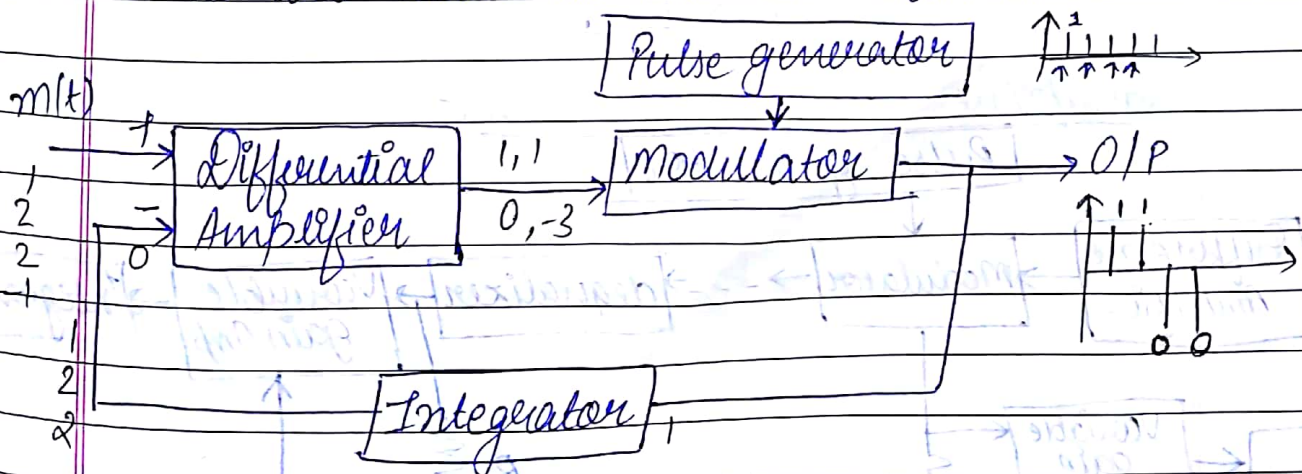
$$\downarrow L = 2^n$$



- Redundancy decrease
- two same signal only one of them is passed.
- Encoder is more than PCM.
- But bandwidth is decreased

Transmitter :-Receiver :-Delta modulation :-

It is also called as one bit modulator.

Transmitter side

Shivalal

Diagram of the system blocks and input/output sequences:

Block diagram: $\text{Input} \rightarrow \text{Dekuantizer} \rightarrow \text{Integrator} \rightarrow \text{LPF} \rightarrow \text{Output}$

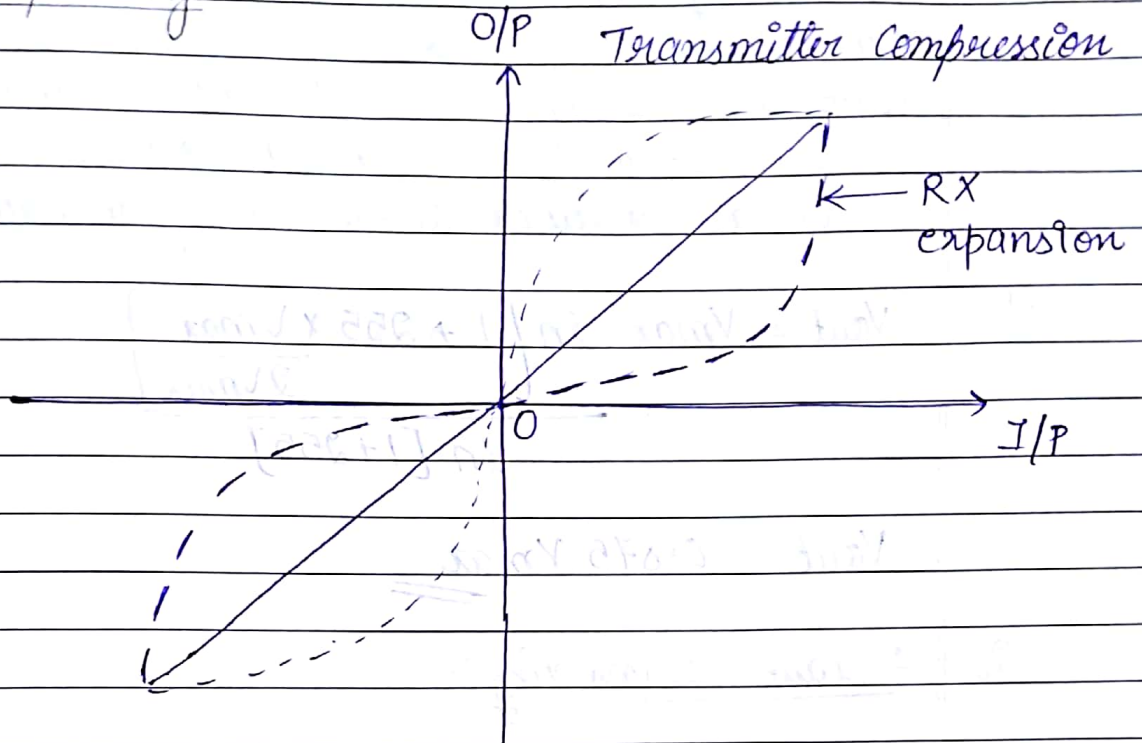
Input	Dekuantizer	Integrator	LPF	Output
1	1	1	1	1
1	1	1	2	2
0	0	0	2	2
0	0	-3	-1	-1

- i.) Slope Overload noise
- ii.) Granular noise

The approximation signal falls behind the message signal.

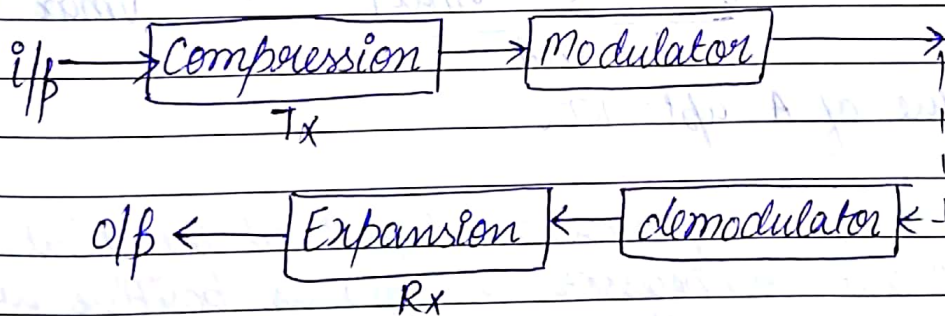
The diagram illustrates a closed-loop control system for a motor speed. The main control path consists of the following blocks in series: Pulse generator, Difference Amplifier, Modulator, dequalizer, Variable gain Amp., and Integrator. There are two feedback paths. The first feedback path goes from the output of the Integrator block through an Integrator and then a Variable gain amp back to the input of the Difference Amplifier. The second feedback path goes from the output of the Integrator block through a Square law device and then a Variable gain amp back to the input of the Difference Amplifier. Additionally, there is an RC network (resistor R and capacitor C) connected to the output of the Integrator block, which is used for frequency compensation.

Companding:-

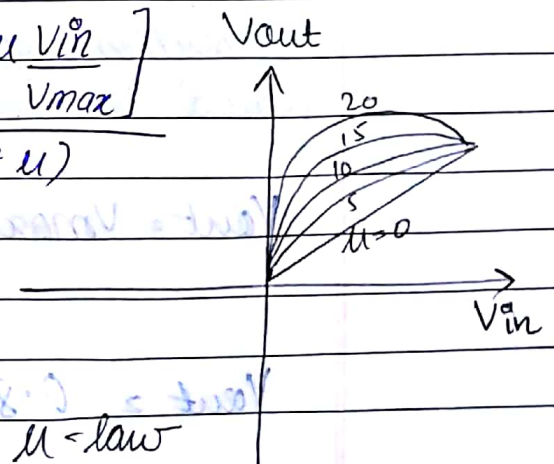


low $\rightarrow$ high
high $\rightarrow$ low

1. μ law companding:-



$$\text{O/P voltage} = \frac{V_{\max} \ln \left[1 + \mu \frac{V_{\text{in}}}{V_{\max}} \right]}{\ln (1 + \mu)}$$



Ques

An analog information signal at the input of a μ law compressor ($\mu=255$) is positive with its voltage level 1 half the max value. What prop of the max O/P voltage level would be produced at the O/P of the compressor.

Sol.

$$V_{out} = V_{max} \frac{\ln \left[1 + \frac{255 \times V_{max}}{2V_{max}} \right]}{\ln [1+255]}$$

$$V_{out} = 0.875 V_{max}$$

2. A law companding:-

$$V_{out} = V_{max} \left[\frac{A \frac{V_{in}}{V_{max}}}{1+A} \right] \quad \text{or } \frac{V_{in}}{V_{max}} = \frac{1}{A}$$

$$V_{out} = V_{max} \left[\frac{1 + \ln \left[A \frac{V_{in}}{V_{max}} \right]}{1 + \ln A} \right] \quad \frac{1}{A} < \frac{V_{in}}{V_{max}} < 1$$

Value of A upto 100.

Ques

An analog information signal at the input to a μ law compressor ($A=100$) is positive with voltage level $\frac{1}{2}$ the max value. What proportion of the max O/P voltage level would be produced at the O/P of the compressor.

Sol.

$$V_{out} = V_{max} \left[\frac{1 + \ln \left[100 \frac{V_{max}}{2V_{max}} \right]}{1 + \ln 100} \right]$$

$$V_{out} = 0.875 V_{max}$$

Ques → A composite video signal with basement frequency ranges from 0-4 MHz is transmitted by linear PCM using 8 bit per sample and sampling rate of 10 MHz.

- i) determine no. of quantization level
- ii) transmission bit rate
- iii) Signal to Quantization noise ratio

$$V_{255} = 0$$

$$V_{255} = 1$$

$$V_{255} = 1$$

$$0$$

$$V_{255} = 1$$

$$V_{255} = 1$$

$$V_{255} = 0$$

$$V_{255} = 1$$

$$V_{255} = 0$$

$$V_{255} = 0$$

$$V_{255} = 1$$

$$V_{255} = 1$$

$$V_{255} = 0$$

$$V_{255} = 1$$

$$V_{255} = 1$$

$$V_{255} = 1 \quad R = 8 \times 10$$

$$R = 8 \times 10$$

$$\frac{S(0.0) + S(1.0) + S(2.0) + S(3.0) + S(4.0)}{5}$$

$$S(0.0) = 0$$

Ques The measurement of 5 consecutive samples 1.2V, 1.0V, 0.95V, 1.41V and 1.65V the no. of quantization levels are 8 and dynamic range of input signals is 2V determine the quantization error in terms of its mean-square value.

Sol.

	Quantized value	Quantization error
1.2V	1.25V	0.05V
1.0V	1.0V	0
0.95V	1.0V	0.05V
1.41V	1.5V	0.09V
1.65V	1.75V	0.10V

$$\Delta = \frac{2}{L} = \frac{2}{8} = 0.25V$$

$$\frac{(0.05)^2 + (0)^2 + (0.05)^2 + (0.09)^2 + (0.10)^2}{5}$$

$$= 0.00462$$

Ques Consider the test signal $m(t)$ defined by a hyperbolic tangent function where A and B are the constants. Determine the minimum step size for delta modulation of this signal which is required to avoid slope overload distortion. $m(t) = A \tanh(Bt)$

Sol.

$$\frac{\Delta}{T_s} \geq \frac{d}{dt} m(t)$$

$$= \frac{d}{dt} (A \tanh(Bt))$$

$$= A \operatorname{sech}^2(Bt) \cdot B$$

$$f_m = \frac{B}{2\pi}$$

$$f_s = 2f_m \Rightarrow f_s = \frac{B}{\pi}$$

$$T_s = \frac{\pi}{B}$$

$$\frac{d}{dt} m(t) = A - A \tanh^2(Bt) \cdot B$$

$$= A - BA \tanh^2(Bt)$$

$$\frac{\Delta B}{\pi} \geq (A - BA \tanh^2(Bt))$$

$$\frac{\Delta B}{\pi} \geq A \Rightarrow \Delta = \frac{\pi A}{B}$$

Ques Two signals 3KHz and 5KHz are sampled at Nyquist rate in PAM using TDM. determine the time gap between two successive samples

Sol.

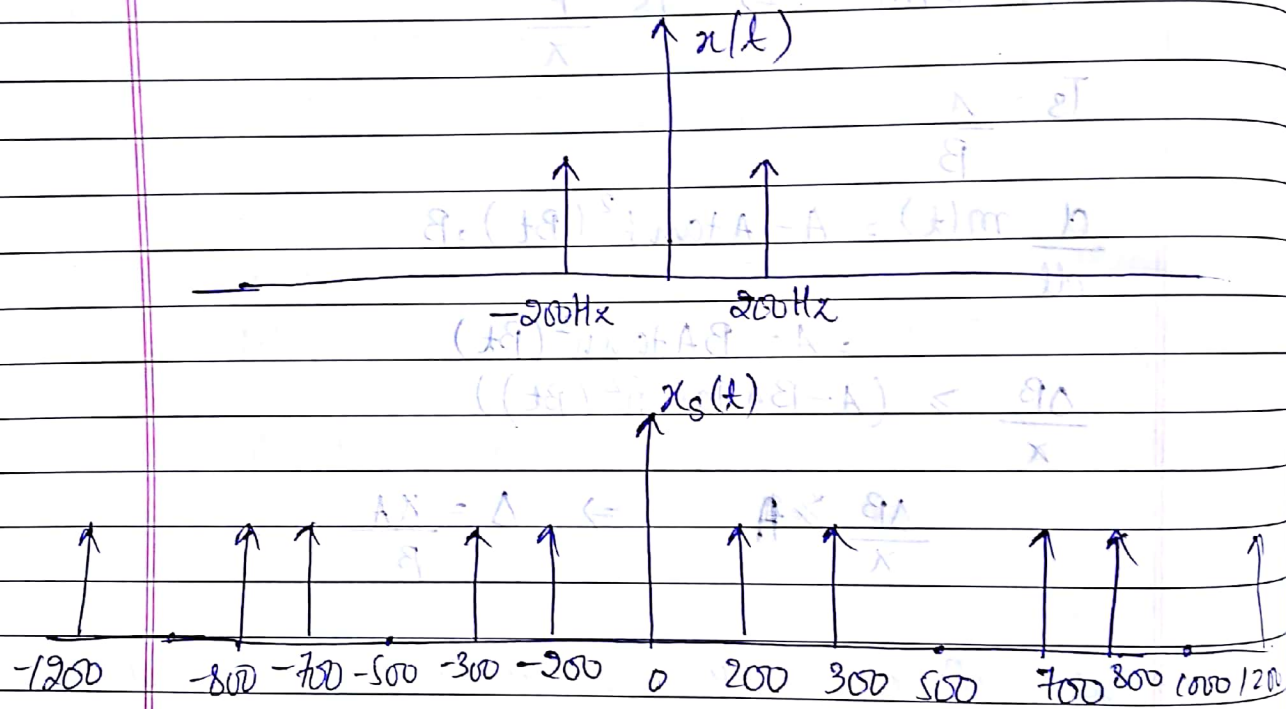
$$f_s = 10 \text{ KHz} \quad \because f_s = 2f_m = 2 \times 5 = 10 \text{ KHz.}$$

$$T_s = \frac{1}{f_s} = \frac{1}{10 \times 10^3} = 10^{-4} \text{ sec.}$$

Ques A signal $x(t) = 2\cos 400\pi t$ is ideally sampled at $f_s = 500$ Hz. Draw the spectrum of the sampled signal.

Sol.

$f_m = 200$ Hz $f_s = 500$ Hz



Ques

In a PCM system the signal $m(t) = \sin 100\pi t + \cos 100\pi t$ is sampled at Nyquist rate.

The samples are processed by uniform quantizer with stepsize $0.75V$. Determine the minimum data rate of PCM system in bits per second.

Sol.

$$m(t) = \sin 100\pi t + \cos 100\pi t$$

$$\text{Step size} = 0.75V$$

$$\text{Nyquist rate} \Rightarrow f_s = 2f_m$$

$$= 2 \times 50$$

$$= 100 \text{ Hz}$$

$$0.75 = \frac{2}{2^n}$$

$$2^n = \frac{2V_p}{0.75} = \frac{4}{3} \cdot 2\sqrt{2}$$

$$2^n = 3.77$$

$$2^n = 2^{1.2}$$

$$\log 2^n = \log 2^{1.2} = 3^{-1}$$

$$n = \log_2 \left(\frac{2\sqrt{2}}{0.75} \right)$$

$$n = \frac{\log \left(\frac{2\sqrt{2}}{0.75} \right)}{\log_{10} 2}$$

$$= \frac{0.576}{0.301}$$

$$= 1.915$$

$$\underline{n \approx 2}$$

$$\text{Bit rate} = 2 \times 100 = 200 \text{ Kbps} \quad \therefore n f_s$$

Ques A bandwidth of 4KHz is sampled at Nyquist rate using 4 levels. If we transmit two quantize samples per sec then calculate information rate?

Sol.

$$\text{Bandwidth} = 4\text{KHz} = f_m$$

$$f_s = 2f_m = 8\text{KHz}$$

$$L = 4 \quad 2^n$$

$$n = 2 \text{ bit}$$

$$\text{bit rate} = n f_s$$

$$= 2 \times 8$$

$$= 16 \text{ Kbps}$$

but if we transmit two quantize sample per sec.

$$\text{Information rate} = 2 \times 8$$

$$= 16 \text{ bps.}$$

Ques

write the condition for reducing slope overload distortion.

Sol.

$$T_s \geq \frac{1}{f_s} \quad \frac{\Delta}{T_s} \geq \frac{d m(t)}{dt}$$

Ques

The input to a linear delta modulation having step size $\Delta = 0.628\text{V}$ is a sine wave with frequency f_m and peak amplitude E_m . If $f_s = 40\text{KHz}$, the combination of f_m and E_m where slope overload will take place is

E_m	f_m	
0.3V	6KHz	15.07
1.5V	4KHz	37.69
1.5V	3KHz	28.29
3.0V	1KHz	18.84

$$\frac{0.628}{6.025} \approx \frac{d}{dt} 0.3 \sin 16\pi t$$

$$= 0.3 \cos 16\pi t \cdot 16\pi$$

$$25.12 \approx 95.079 \cos 16\pi t$$

Given max voltage amplitude of 3.0V
determine the compressed value for an
input voltage of 1.026V using

- a law compander
- A-law compander.

$$V_{out} = V_{max} \frac{\ln \left[1 + \frac{V_m}{V_{max}} \right]}{\ln(1+\mu)}$$

$$= 3.0 \frac{\ln \left[1 + \frac{255 \times 1.026}{3.0} \right]}{\ln(1+255)}$$

$$= 3.0 \frac{\ln[90.25]}{\ln[256]}$$

$$= \frac{3.0 \times 4.5025}{5.545}$$

$$V_{out} \approx 2.923 \text{ V}$$

$$V_{out} = 3.6 \left[\frac{1 + \ln \left(\frac{100 \times 1.026}{3.6} \right)}{1 + \ln 100} \right]$$

$$= 3.6 \left[\frac{1 + \ln(35)}{1 + \ln 100} \right]$$

$$= 3.6 \left[\frac{4.553}{5.6051} \right]$$

$$V_{out} = 2.925 \text{ V}$$

Ques

Determine the min step size required for delta modulation operating at 32 K sample per seconds to track the signal $x(t)$:

$$x(t) = 125t [u(t) - u(t-1)] + (250 - 125t) [u(t-1) - u(t-2)]$$

Sol.Sol.

$$\text{Sample or bit} = 32 \text{ K sample/sec} = f_s$$

$$f_s = \frac{1}{T_s} \quad \text{or} \quad T_s = \frac{1}{f_s}$$

$$= \frac{1}{32 \times 10^3}$$

$$T_s = 31.25 \text{ } \mu\text{sec}$$

$$x(t) = 125tu(t) - 125t(u(t-1)) + 250[u(t-1) - 250u(t-2) - 125t(u(t-1)) + 125t(u(t-2))]$$

$$\frac{d}{dt} x(t) = u(t) \cdot 125 + 125t \delta t - 125t \delta(t-1) - u(t-1)(125) + 250 \delta(t-1) - 250 \delta(t-2) - 125t \delta(t-1) - u(t-1)125 + 125 \delta(t-2) + 125 u(t-2)$$

Ans $125 (u(t) - 2u(t-1) + u(t-2))$

$$\frac{\Delta}{T_s} \geq 125 \text{ max}$$

$$\Delta \geq \frac{125 \times 10^{-3}}{32}$$

$$\Delta_{\min} \geq 3.9 \times 10^{-3} \text{ V}$$

Ques Discrete sample of an analog signal are uniformly quantize to PCM if maximum value of analog sample is to be represented within 0.1% accuracy find minimum number of binary digits required per sample.

Sol. $\frac{\Delta}{2} = 0.1\% \text{ of } V_{\max}$

$$\Delta = \frac{0.1}{100} \times 2 \times V_{\max}$$

$$\Delta = 0.002 V_{\max}$$

$$\frac{2}{0.002} = 2^n \Rightarrow \frac{2 \times 1000}{2} = 2^n$$

$$1000 = 2^n$$

$$n \log 2 = 3$$

$$n = \frac{3}{\log 2}$$

$$n = 9.96$$

Ques Consider a PCM system with a law compander having $\mu = 255$. Let the max input voltage $4V$. Determine the output voltage and amount of compression in dB when input voltage is 35% of V_{max} .

Sol. $V_{in} = \frac{35}{100} V_{max} = \frac{7}{20} V_{max} = 1.4$

$$V_o = V_{max} \left[\frac{\ln(1 + \mu V_{in}/V_{max})}{\ln(1 + \mu)} \right]$$

$$= \frac{4 \ln(1 + 255 \times 0.35)}{\ln(256)}$$

$$= \frac{4 \ln(90.25)}{\ln 256}$$

$$V_{out} = 3.247V$$

$$\begin{aligned} \text{Amount of compression} &= 10 \log(3.247 - 1.4) \\ &= 10 \log 1.847 \\ &= 2.667 \text{ dB} \end{aligned}$$

Ques An audio signal $x(t) = 3 \cos(2\pi \times 500t)$ is quantized using 10 PCM determine the signal to quantization noise ratio.

$$\left(\frac{S}{N_0} \right)_{dB} = 1.76 + 6.02 \times 10 = 61.96 \text{ dB}$$

Ques 1. Total 24 voice signals are sampled uniformly and then to the time division division multiplexed the highest frequency component for each voice signal is 3.4 KHz .

ii) If signals are PAM using Nyquist rate what could be the minimum number channel required.

iii) If signals are PCM using 8 bit encode what would be the sampling rate.

sol. $f_m = 3.4 \text{ KHz}$

$$f_c = 2 f_m$$

$$= 2 \times 3.4 = 6.8 \text{ KHz}$$

$$B.W = 24 \times f_c$$

$$= 24 \times 6.8$$

$$= 163.2 \text{ KHz}$$

$$n = 8$$

$$\text{bit rate} = n N f_c$$

$$= 8 \times 24 \times 6.8$$

$$= 8 \times 163.2$$

$$= 1305.6 \text{ Kbps}$$

$$= 1.3056 \text{ Mbps}$$

Consider the analog information signal is 3.2 KHz a binary channel of 36 Kbps available for PCM voice transmission determine the sampling frequency if sampled at 50% above Nyquist rate no. of bits per sample, no. of quantized level.

Sol.

$$f_m = 3.2 \text{ KHz}$$

$$\text{bit rate} = 36 \text{ Kbps}$$

$$f_c = n f_s = 36$$

$$n (2 \times 3.2 \times 10^3 \times 1.5) = 36 \times 10^3$$

$$n (6.4 \times 1.5 \times 10^3) = 36 \times 10^3$$

$$n = \frac{3.6}{1.6} = 3.75 \approx 4 \text{ bits}$$

$$\text{quantization level} = 2^4 = 16 \text{ levels.}$$

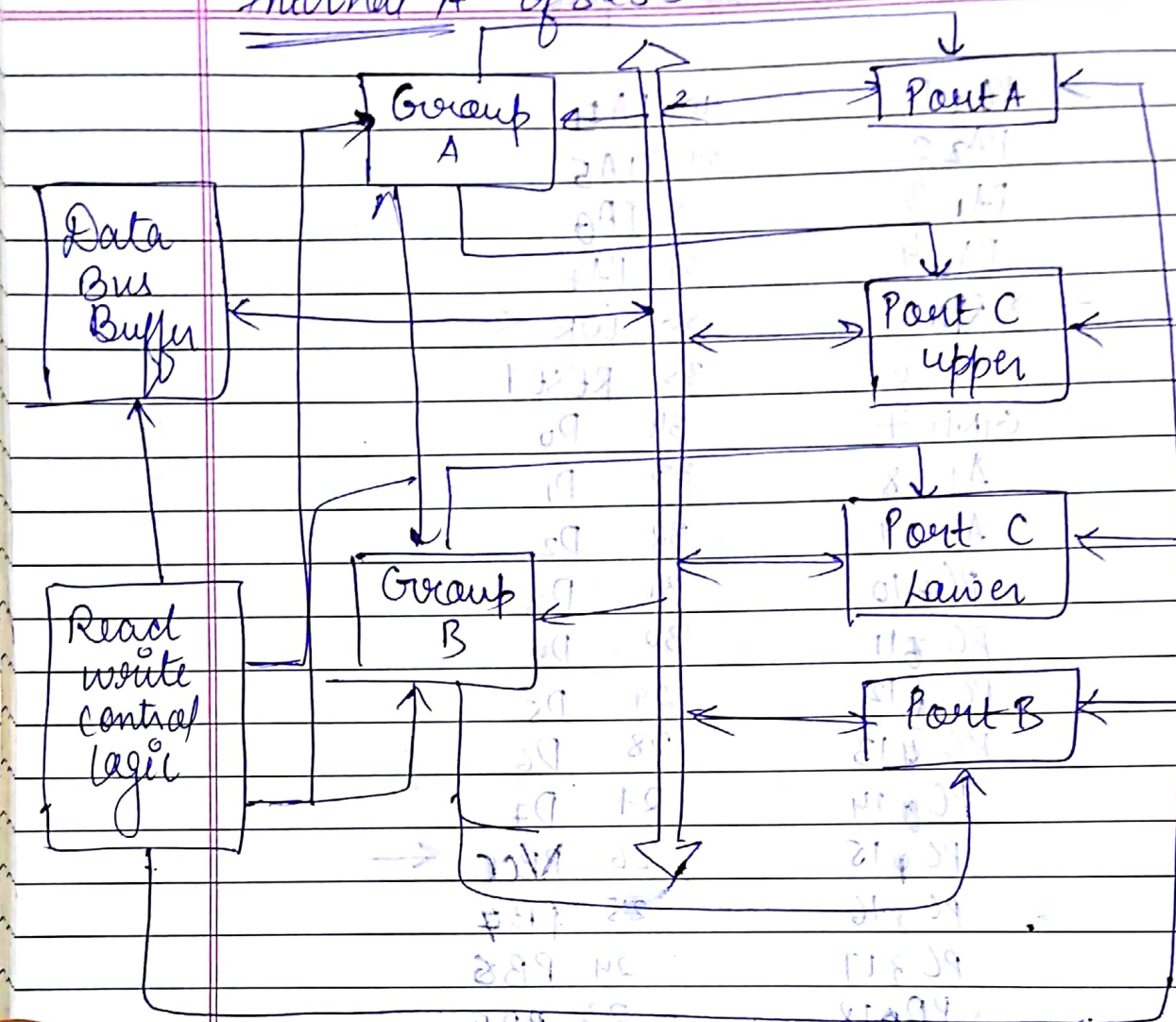
PA4	PA5	PA6	PA7	WR	RESET	D0	D1	D2	D3	D4	D5	D6	D7	VCC	PB7	PB6	PB5	PB4	PB3	
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	
PA3	PA2	PA1	PA0	RDS	CS	GND	A7	A0	PC4	PC5	PC6	PC7	PC13	PC14	PC15	PC16	PC17	PB18	PB19	PB20
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

8255 PPI → Ports 40 pin

PA ₃ 1	40 PA ₄
PA ₂ 2	39 PA ₅
PA ₁ 3	38 PA ₆
PA ₀ 4	37 PA ₇
→ RD 5	36 WR ←
CS 6	35 RESET
GND 7	34 D ₀
A ₁ 8	33 D ₁
A ₀ 9	32 D ₂
P _C 10	31 D ₃
PC 11	30 D ₄
PC 12	29 D ₅
PC 13	28 D ₆
PC 14	27 D ₇
PC 15	26 RD ←
PC 16	25 PB ₇
PC 17	24 PB ₆
PB 18	23 PB ₅
PB 19	22 PB ₄
PB 20	21 PB ₃

PA ₃ 1	40 PA ₄
PA ₂ 2	39 PA ₅
PA ₁ 3	38 PA ₆
PA ₀ 4	37 PA ₇
RD 5	36 WR
CS 6	35 RESET
GND 7	34 D ₀
A ₁ 8	33 D ₁
A ₀ 9	32 D ₂
P _C 10	31 D ₃
PC 11	30 D ₄
PC 12	29 D ₅
PC 13	28 D ₆
PC 14	27 D ₇
PC 15	26 VCC
PC 16	25 PB ₇
PC 17	24 PB ₆
PB 18	23 PB ₅
PB 19	22 PB ₄
PB 20	21 PB ₃

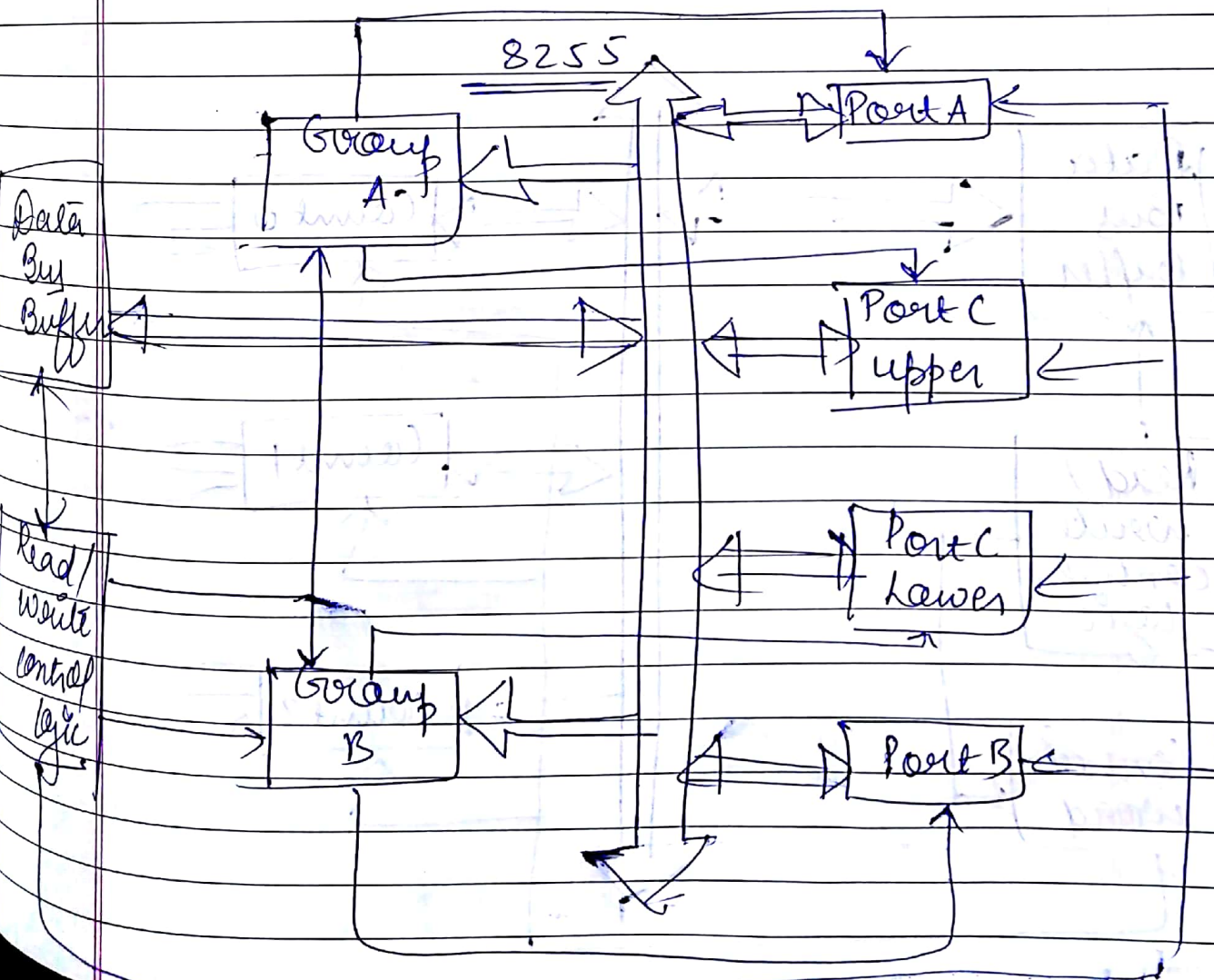
Internal A of 8255



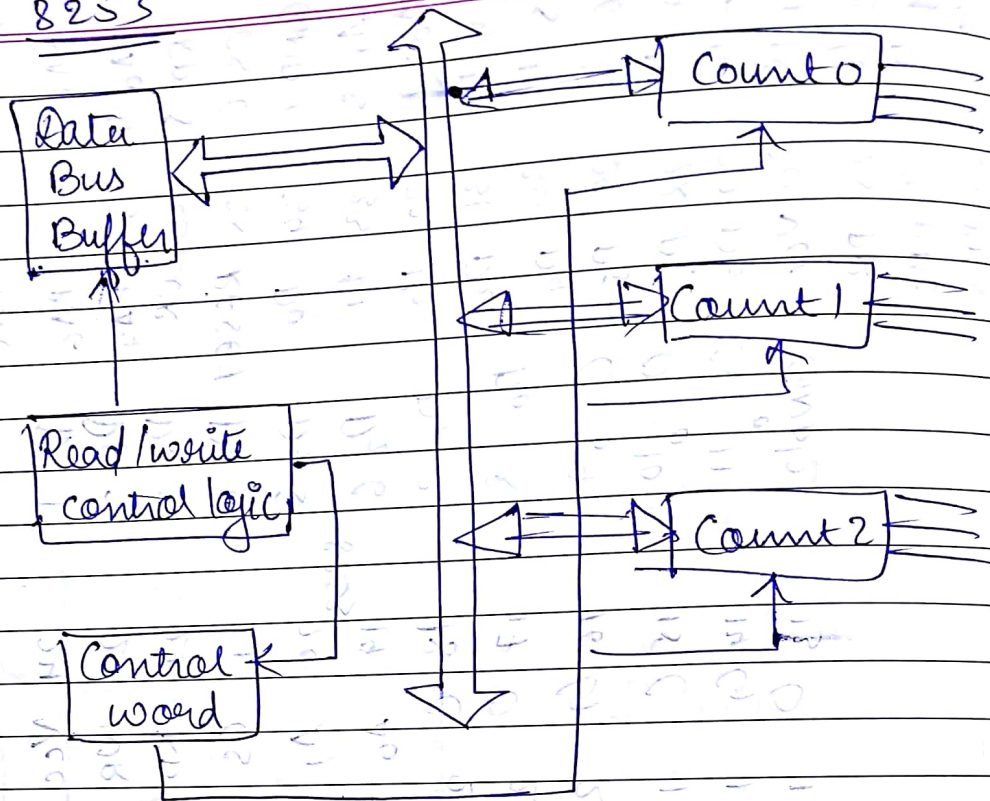
8253 → PPI PPI

D ₀ 1	24	VCC	D ₇ 1	24	VCC
D ₁ 2	23	$\overline{WR}$	D ₆ 2	23	$\overline{WR}$
D ₂ 3	22	$\overline{RD}$	D ₅ 3	22	$\overline{RD}$
D ₃ 4	21	$\overline{CS}$	D ₄ 4	21	$\overline{CS}$
D ₄ 5	20	A ₁	D ₃ 5	20	A ₁
D ₅ 6	19	A ₀	D ₂ 6	19	A ₀
D ₆ 7	18	CLK ₂	D ₁ 7	18	C
D ₇ 8	17	OUT ₂	D ₀ 8	17	O
CLK ₁	16	GATE ₂	C	16	G
OUT ₀	15	CLK ₁	O	15	C
GATE ₀	14	GATE ₁	G	14	G
GNP ₁₂	13	OUT ₁	GNP ₁₂	13	O

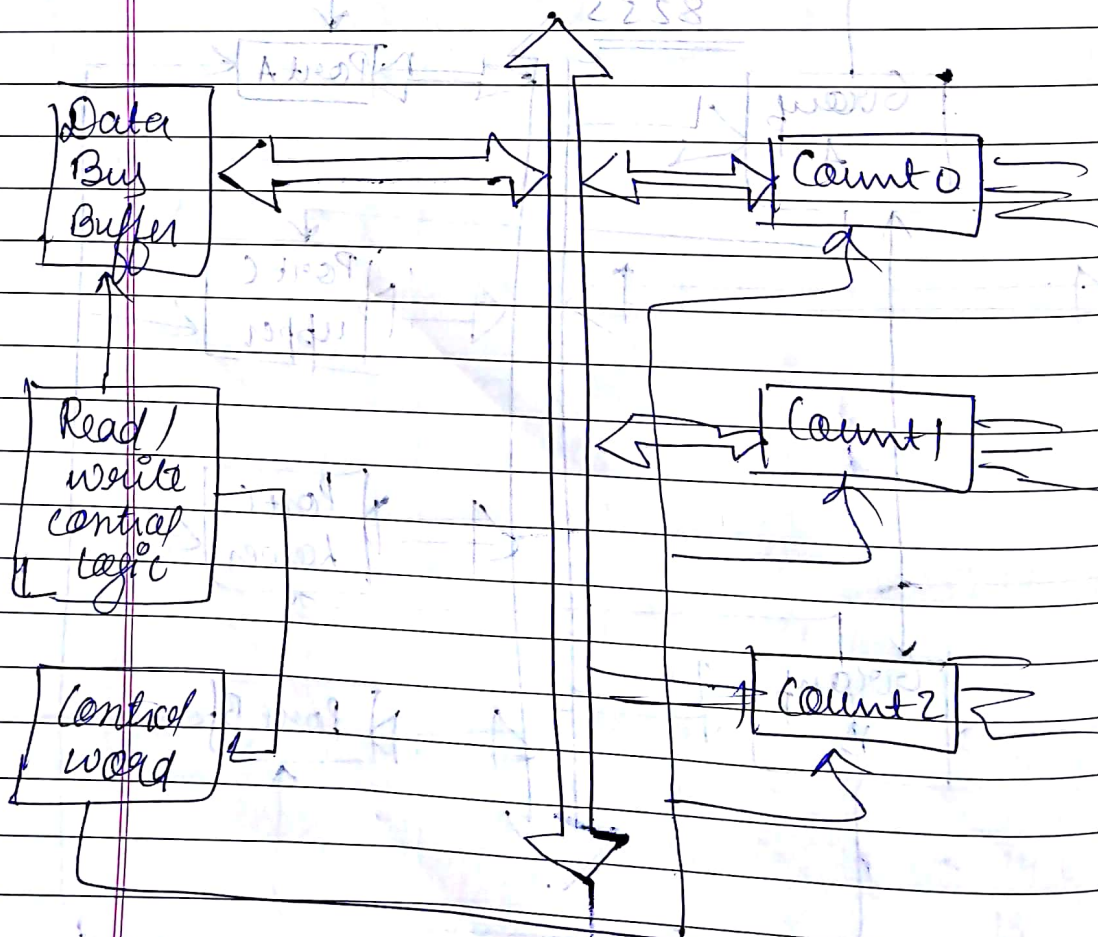
8255		8253	
		<u>24 pin</u>	
PA ₃ 1	no PA ₄	D ₇ 1	24 Vcc
PA ₂ 2	PA ₅	D ₆ 2	23 WR
PA ₁ 3	PA ₆	D ₅ 3	22 RD
PA ₀ 4	PA ₇	D ₄ 4	21 CS
RD 5	WR	D ₃ 5	20 A ₁
CS 6	RESET	D ₂ 6	19 A ₀
GND 7	D ₀	D ₁ 7	18 C
A ₁ 8	D ₁		17 0
A ₀ 9	D ₂		16 1
PC ₄ 10	D ₃		15 C
5 11	D ₄		14 1
6 12	D ₅		13 1
7 13	D ₆		12 2
14 14	D ₇		11 2
15 15	Vcc		10 2
			9 2
			8 2
			7 2
			6 2
			5 2
			4 2
			3 2
			2 2
			1 2



8253



2258



8259 → 28 pin

$\overline{CS}$ 1	28 Vcc	$\overline{CS}$ 1	28 Vcc
WR 2	27 A ₀	$\overline{WR}$ 2	27 A ₀
RD 3	26 INTA	Rd 3	26 INTA
D ₇ 4	25 IR ₇	D ₇ 4	25 IR
D ₆ 5	24 IR ₆	D ₆ 5	24 IR
D ₅ 6	23 IR ₅	D ₅ 6	23 IR
D ₄ 7	22 IR ₄	D ₄ 7	22 IR
D ₃ 8	21 IR ₃	D ₃ 8	21 IR
D ₂ 9	20 IR ₂	D ₂ 9	20 IR
D ₁ 10	19 IR ₁	D ₁ 10	19 IR
D ₀ 11	18 IR ₀	D ₀ 11	18 IR
CAS ₀ 12	17 IN ₁	CAS ₀ 12	17 IN ₁
CAS ₁ 13	16 SP/EN	CAS ₁ 13	16 SP/EN
GND 14	15 CAS ₂	GND 14	15 CAS ₂

8255
PA3 1
PA2 2
PA1 3
PA0 4
RD 5
GND 6
GND 7
A₀ 8
A₁ 9
A₂ 10
A₃ 11
A₄ 12
A₅ 13
A₆ 14
A₇ 15
16
17
18
19
20

NO PA4
5
6
WR
RESET
34
35
36
37
38
39
40

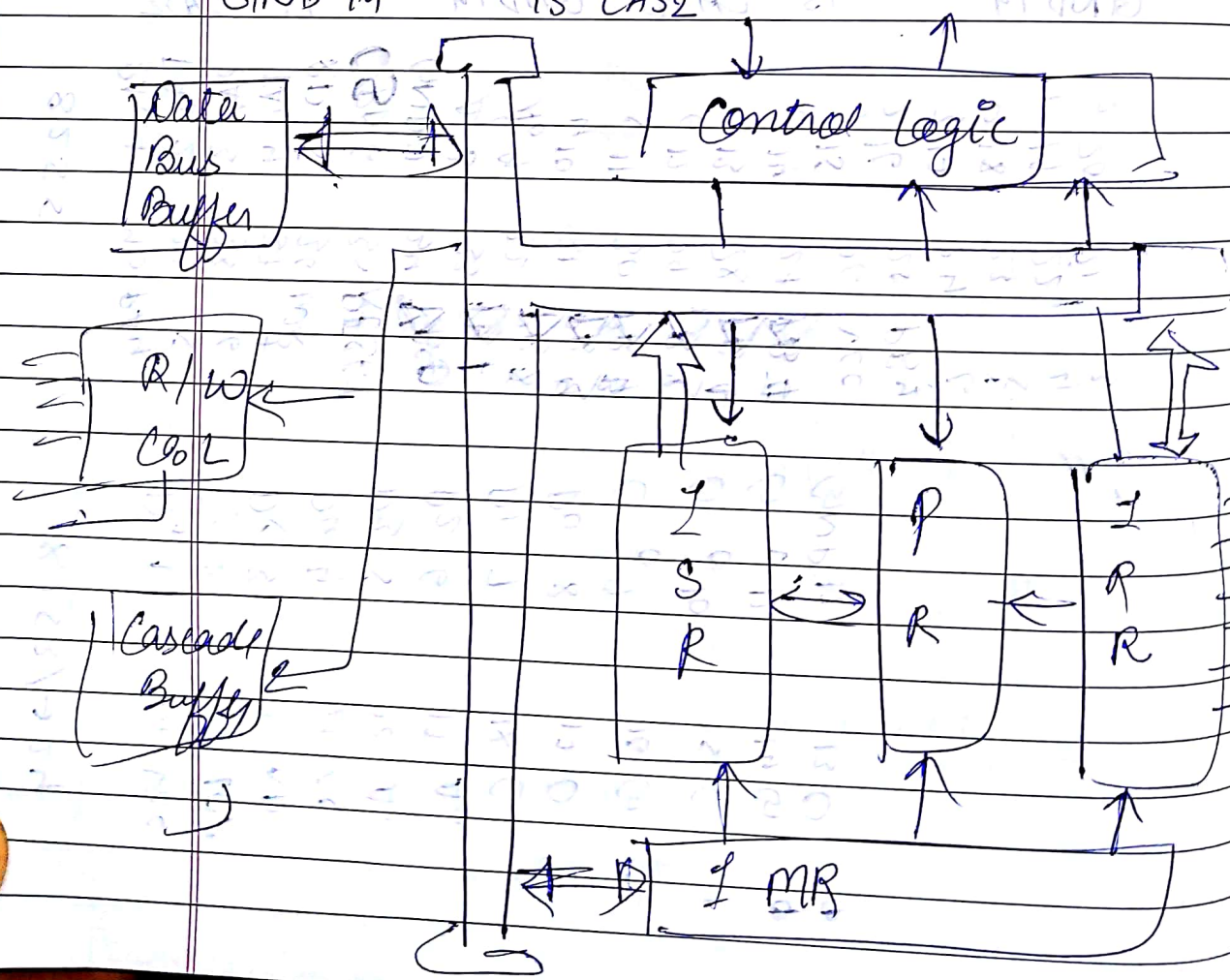
8253/24
D₇ 1
D₆ 2
D₅ 3
D₄ 4
D₃ 5
D₂ 6
D₁ 7
D₀ 8
C 9
D 10
G 11
GND 12

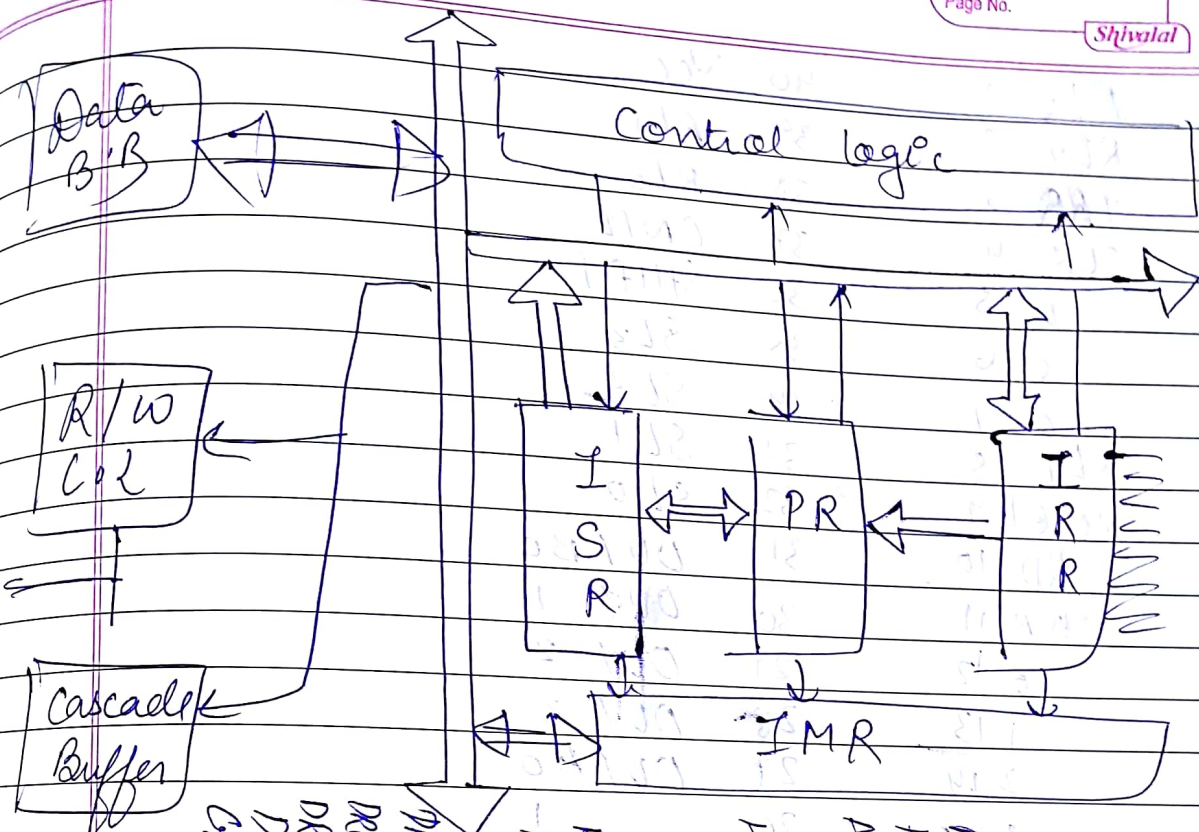
24 Vcc
23 WR
22 RD
21 CS
20 A₁
19 A₀
18 C
17 D
16 G
15 C
14 G
13 D

2
2
2
1
1
1

8259

CS 1	28	VCC
WR 2	27	A0
RD 3	26	INTA
D7 4	25	IR7
D6 5	24	IR6
D5 6	23	IR5
D4 7	22	IR4
D3 8	21	IR3
D2 9	20	IR2
D1 10	19	IR1
D0 11	18	IR0
CAS0 12	17	INT
CAS1 13	16	SP/EN
GND 14	15	CAS2





IO R 1
IO W 2
R3
Memory
MARKS
READY 6
HLD 7
ADSTB 8
AEN 9
HRQ 10
CS 11
CLK 12
RESET 13
PAC 14
PAC 15
PAC 16
DRQ 2 17
DRQ 1 18
DRQ 0 19
GND 20

40 A0
39 A1
38 A2
37 A3
36 TC
35 A4
34 A5
33 A6
32 A7
31 VCC
30 D0
29 D1
28 D2
27 D3
26 D4
25 DACK0
24 DACK1
23 D5
22 D6
21 D7

IO R 1
IO W 2
R3
Memory
MARKS
READY 6
HLD 7
ADSTB 8
AEN 9
HRQ 10
CS 11
CLK 12
RESET 13
PAC 14
PAC 15
PAC 16
DRQ 2 17
DRQ 1 18
DRQ 0 19
GND 20

40 A0
39 A1
38 A2
37 A3
36 TC
35 A4
34 A5
33 A6
32 A7
31 VCC
30 D0
29 D1
28 D2
27 D3
26 D4
25 DACK1
24 DACK0
23 D5
22 D6
21 D7

RL3	1	40	VCC
RL2	2	39	RL1
IRRA	3	38	RL0
CLK	4	37	CNT2
4	5	36	SHIFT
5	6	35	SL3
6	7	34	SL2
RL7	8	33	SL1
RESET	9	32	SL0
RD	10	31	OUT B0
WR	11	30	OUT 1
0	12	29	OUT 2
1	13	28	OUT 3
2	14	27	OUT A0
3	15	26	1
4	16	25	2
5	17	24	3
6	18	23	BD
DB7	19	22	IS
GND	20	21	DO